Optimization of Loneys Solenoid Design Using a Dynamic Search Based Technique

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Abstract **─** Particle swarm optimizer is one of the searched based stochastic technique that has a weakness of being trapped into local optima. Thus, to tradeoff between the local and global searches and to avoid premature convergence in PSO, a new dynamic quantum-based particle swarm optimization (DQPSO) method is proposed in this work. In the proposed method a beta probability distribution technique is used to mutate the particle with the global best position of the swarm. The proposed method can ensure the particles to escape from local optima and will achieve the global optimum solution more easily. Also, to enhance the global searching capability of the proposed method, a dynamic updated formula is proposed that will keep a good balance between the local and global searches. To evaluate the merit and efficiency of the proposed DQPSO method, it has been tested on some well-known mathematical test functions and a standard benchmark problem known as Loney's solenoid design.

Index Terms **─** Design optimization, probability distribution, quantum mechanics, searched based technique.

I. INTRODUCTION

Over the last few decades, random optimization methods, including evolutionary technique and swarm intelligence, such as genetic algorithm (GA), evolutionary strategies (ES), genetic programming and evolutionary programming (EP) have been used to solve different global optimization problems. These techniques are inspired by different natural evolutionary phenomena.

Many efforts have been made to develop a global stochastic technique for hard optimization problems. Some of the latest updated literature is recorded in the following paragraph.

An adaptive null-steering beamformer based on Bat Algorithm for the uniform linear array was presented in [1]. The optimization of radome-enclosed antenna arrays was proposed to compensate the distortion error of radome-enclosed antenna arrays [2]. A radiation pattern synthesis of non-uniformly excited planar arrays was presented [3]. An updated version of artificial immune system algorithm was proposed for electromagnetic applications [4]. A new quantum-based approach was proposed for the electromagnetic applications in [5]. An improved particle swarm optimization was applied to electromagnetic devices [6]. A hybrid harmony search method and ring theory based evolutionary algorithm was presented for feature selection [7]. A newly emerging nature inspired optimization algorithms were reviewed

Submitted On: July 29, 2020
Accepted On: December 6, 2020
https://doi.org/10.47037/2020.ACES.J.360105

in [8]. A pareto optimal characterization of a microwave transistor was presented [9]. A multiple black hole inspired meta heuristic searching method was proposed to optimize the combinatorial testing [10]. In [11], a whale optimization technique based on lamarckian learning was proposed to solve global optimization problems. A quantum inspired particle swarm method with enhanced strategy was applied to optimizing the electromagnetic devices [12]. An improved quantum particle swarm method was applied to solve the electromagnetic design problem [13].

Thus, a continued research and development is needed to search a global optimizer to optimize hard engineering design problems. However, according to the no free lunch theorem, all these optimizers are problem oriented, so, an effort is required to seek a global optimizer. In this context, some modification has been proposed in this work for the optimization of Loney solenoid design.

Moreover, particle swarm optimization is an essential member of a broader class of swarm intelligence. This method originated in 1995 by John Kennedy and Robert Hart as an imitation of insect's social behavior [14].

Since 1995, many efforts have been made to make PSO a global optimizer. Consequently, introducing the quantum mechanics into PSO, called quantum behaved particle swarm optimizer (QPSO) [15]. The numerical results on some widely used benchmark problems have demonstrated the superiority of QPSO over basic PSO. However, there are still many issues in QPSO that needs to be addressed. In this context, a new mutation phenomenon is applied to the particle with the global best position of the swarm that will avoid the premature convergence and significantly improves its global searching capability. Also, a parameter updated formula is proposed that will bring a good balance between the local and global searches. Thus, a dynamic quantum-inspired particle swarm method (DQPSO) is applied to Loney's solenoid design as reported in this work.

II. QPSO APPROACH

The trajectory analysis [16] illustrates that the PSO convergence behavior can be guaranteed if each particle converges to its local attractor $pi = (p_{i,1}, p_{i,2}, ..., p_{i,d})$, of which the coordinates are:

$$p_i(t) = (c_1 p_i(t) + c_2 p_g(t)) / (c_1 + c_2), \quad (1)$$

$$\text{or } \; p_i(t) = \varphi \cdot p_i(t) + (1-\varphi) \cdot p_g(t),$$

where $\varphi = c_1 r_1 / (c_1 r_1 + c_2 r_2)$. It has been shown that the local attractor is stochastic of particle i and lies in a hyper rectangle with p_i and p_g are the two ends of its diagonal.

In quantum potential delta model [15], the position of a particle is given by:

$$x_i(t) = p_i(t) \pm \frac{L(t)}{2} \ln(1/u) . \quad (2)$$

In [14], a parameter $L(t)$ is defined as:

$$L(t) = 2 \cdot \beta \cdot |p_i(t) - x_i(t)| . \quad (3)$$

Then the position is updated as follows:

$$x_i(t+1) = p_i(t) \pm \beta \cdot |p_i(t) - x_i(t)| \cdot \ln(1/u) , \quad (4)$$

where u is a random number within the interval [0,1], β is the contraction expansion coefficient (CE) parameter and is used to control the convergence behavior of the QPSO and is represented by:

$$\beta = 0.5 + (1.0 - 0.5)(Maxiter - t) / Maxiter . \quad (5)$$

This parameter is initially set to 1 and then linearly decreased to 0.5.

To evaluate $L(t)$, the mean best position is defined as the average of the personal best position of the swarm, i.e.,

$$m(t) = (m_1(t), m_2(t), ..., m_d(t)) =$$
$$\left(\frac{1}{N} \sum_{i=1}^{N} p_{i,1}(t), \frac{1}{N} \sum_{i=1}^{N} p_{i,2}(t), ..., \frac{1}{N} \sum_{i=1}^{N} p_{i,d}(t) \right), \quad (6)$$

where N represents the swarm size. The parameter L will become,

$$L(t) = 2 \cdot \beta \cdot |m(t) - x_i(t)| . \quad (7)$$

Thus, the position of particles will be updated as:

$$x_i(t+1) = p_i(t) \pm \beta \cdot |m(t) - x_i(t)| \cdot \ln(1/u) . \quad (8)$$

The equation (8) is called the position updated equation of the quantum particle swarm algorithm.

III. PROPOSED METHOD

The quantum inspired particle swarm method has many issues, especially when dealing with complex optimization problems. Because at the start of the search process, the diversity of the algorithm is high, however, it reduces quickly at the later stage of the evolution process. Thus, to improve the QPSO performance in terms of the final solution searched and convergence speed much effort has been made and different variants of QPSO have been developed. However, most of these methods are problem oriented. Thus, a continued research and development is needed to develop a global optimizer to optimize complex design problems.

The mutation phenomena were brought from the evolutionary algorithm to maintain the diversity of the population. Thus, it plays a vital role in exploring the searching capability of the algorithm. Different approaches such as Gaussian, exponential, Cauchy etc., and other probability distributions methods are used to produce random numbers and improve the position updated equation of QPSO. Thus, a new outcome has been presented for the mutation operator in QPSO by using the beta probability distribution method. The proposed DQPSO method will be ensured to keep the high diversity and avoid trapping into local optima. The flow chart of a proposed DQPSO is given in Fig. 1.

A. Introduction of a mutation operation

A beta mutation mechanism is applied to the global best (G_{best}) position of the particle to intensifying the diversity of the swarm and avoid the particles from being trapped into local optima. Hence, it will also improve the QPSO performance in terms of the solution quality and global searching capability.

In this method, the beta mutation is combined with the G_{best} particle as follows:

$$G_{mutated} = G_{best} + beta(rand), \tag{9}$$

where *beta(rand)* is the random number generated with the beta probability distribution method.

The proposed mutation strategy will enhance the searching capability of the proposed DQPSO method. Hence, the mutated particle will explore more searched area to achieve the best optimal outcomes and avoid the algorithm to trap into local minima.

B. Parameter updating mechanism

The contraction expansion coefficient (CE) β is the control parameter used for tuning the proposed DQPSO. It plays a vital role in controlling the convergence speed of the proposed DQPSO method. Therefore, different researchers have proposed different mechanisms to tune this parameter [16]. The general mechanism for this parameter is to set to 1 and reduced linearly to 0.5 initially. It also plays a vital role in maintaining a right balance between the local and global searches. However, if the parameter is not adjusted correctly then it may disturb the local and global searches and the algorithm will be trapped into local minima. Thus, to address this issue, it is significantly essential to adjust the value of β parameter properly.

Therefore, a new dynamic updated strategy for β parameter is proposed to maintain a good cooperation between the exploration and exploitation searches and avoid the proposed DQPSO method to stuck into local minima:

$$\beta = 1 + \frac{0.5}{\log(G_{mutated} + 0.2)}. \tag{10}$$

The relationship between the β and $G_{mutated}$ parameters is shown in Fig. 2. It should be noticed from Fig. 2, that if the individual (particle) is far away from the mean best position, then one expects a small value of β to help it come back; In contrast if the particle is just near to the mean best position, then one prefers a high β to force it to bounce away. This will bring a good balance between the local and global searches and avoid the algorithm to trap into local minima.

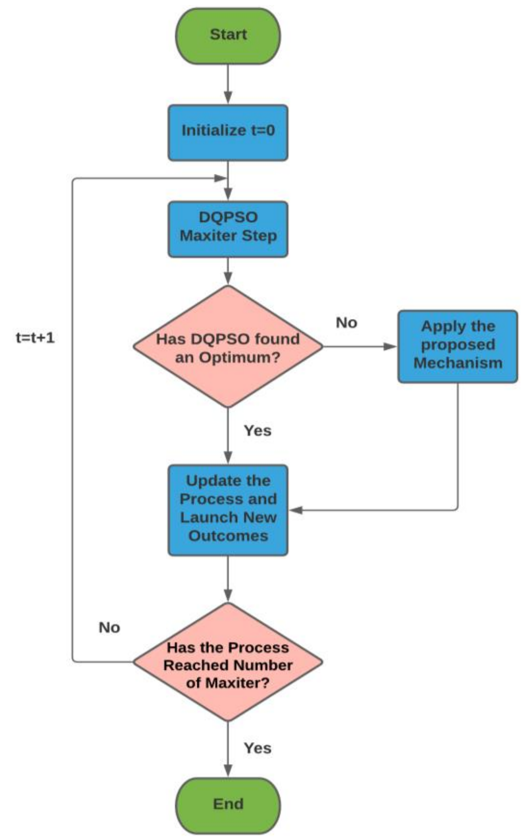

Fig. 1. Flow chart of the proposed DQPSO method.

IV. NUMERICAL RESULTS

To evaluate the performance and global searching capability of the proposed DQPSO algorithm, first three benchmarks shifted versions of mathematical test functions [17] are used as reported:

$$f_1(x) = \sum_{i=1}^{D}(\prod_{j=1}^{i} z_j)^2 + f\_bias_1, z = x - 0, x = [x_1, x_2, ..., x_D],$$

$$x \in [-100, 100]^D.$$

Global optimum: $x* = 0, f_1(x*) = f\_bias_1 = -450$, (11)

$$f_2(x) = \sum_{i=1}^{D}(100.(z_{i+1} - z_i^2)^2 + (z_i - 1)^2) + f\_bias_2,$$

$$z = x - 0 + 1, x = [x_1, x_2, ..., x_D], x \in [-100, 100]^D.$$

Global optimum: $x* = 0, f_2(x*) = f\_bias_2 = 390$, (12)

$$f_3(x) = \frac{1}{4000}\sum_{i=1}^{D} z_i^2 - \prod_{i=1}^{D}\cos(\frac{z_i}{\sqrt{i}}) + 1 + f\_bias_3,$$

$$z = x - 0, x = [x_1, x_2, ..., x_D], x \in [-600, 600]^D.$$

Global optimum: $x* = 0, f_3(x*) = f\_bias_3 = 390$. (13)

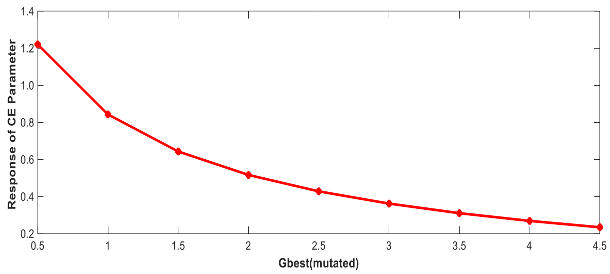

Fig. 2. Relationship between β and Gbest (mutated).

The proposed DQPSO is then compared with the basic QPSO [15], LTQPSO [18] and MQPSO [19] methods.

All these functions are minimization problems and the minimum value for each objective function is zero. Table 1 presents the average performance comparison of different optimizers. Moreover, Figs. 3~5 gives the convergence comparison of the proposed DQPSO with other optimal methods in a logarithmic scale of the best objective function on three well-known test problems using the swarm size of 80 with the number of iterations is 2000 for the 30-dimension problem.

It can be illustrated in Table 1 that the proposed DQPSO and MQPSO have a good performance on most of the shifted version problems. However, the QPSO and LTQPSO could not produce better outcomes and falls into local minima. Thus, it is concluded that the proposed DQPSO improved significantly as compared to other tested optimizers.

Table 1: Mean (first row) and variance (second row) of different optimizer for 30-dimension problems

Algorithms	$f_1(x)$	$f_2(x)$	$f_3(x)$
QPSO	5.9342×10^{-6} 2.8627×10^{-10}	0.1376 3.8629×10^{-2}	3.0716×10^{-2} 4.8620×10^{-3}
LIQPSO	1.23924 0.38211	5.09190 0.52149	2.9314×10^{-2} 7.6824×10^{-4}
MQPSO	1.1339×10^{-7} 2.7820×10^{-13}	7.8273×10^{-2} 5.7432×10^{-5}	5.8461×10^{-3} 1.6560×10^{-7}
DQPSO	3.4182×10^{-9} 1.5086×10^{-17}	8.2016×10^{-4} 5.2791×10^{-8}	6.2041×10^{-5} 5.3942×10^{-10}

V. LONEY'S SOLENOID BENCHMARK PROBLEM

To validate the proposed DQPSO performance for electromagnetic design. It is used to solve the Loney's solenoid problem. The literature has several references to optimization techniques that have been applied to Loney's solenoid design [20]-[22].

Loney's solenoid is a standard nonlinear benchmark problem in magnetostatics inverse problems [20]. Figure 6 show the upper half plane of the axial cross section of the system. Loney's solenoid problem's key point is to find the position and size of two correcting coils to produce an approximate constant magnetic field in the interval of the axis.

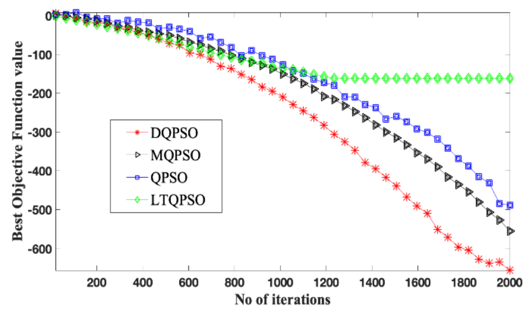

Fig. 3. Convergence plots of different optimization methods solving function f_1.

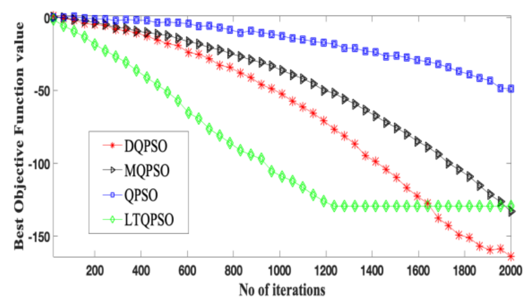

Fig. 4. Convergence plot of different optimization methods solving function f_2.

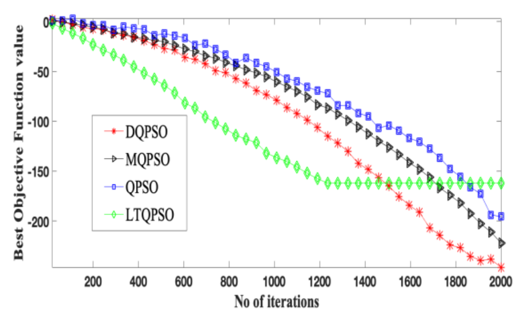

Fig. 5. Convergence plot of different optimization methods solving function f_3.

The Loney's solenoid problem has two variables, which are s and l, and the optimization problem is aiming to find the global minima of $f(s, l)$, i.e.,

$$\min f(s,l). \tag{14}$$

The objective function f can be formulated as:

$$f(s,l)=\frac{B_{\max}-B_{\min}}{B_0}, \tag{15}$$

where Bmax and Bmin represent the maximum and minimum value of the magnetic flux density within the interval $(-z_0, z_0)$, respectively; and B_0 is the magnetic flux density at $z_0 = 0$.

In particular, three different "areas" can be recognized in the domain of f with values of $f > 4 \cdot 10^{-8}$ (high level region), $3 \cdot 10^{-8} < f < 4 \cdot 10^{-8}$ (low level region), and $f < 3 \cdot 10^{-8}$ (very low-level region—global minimum region). The very low-level region is a small ellipsoidal shaped area within the thin low-level valley. In both very low- and low-level small changes in one of the parameters can give rise to changes in objective function values of several orders of magnitude.

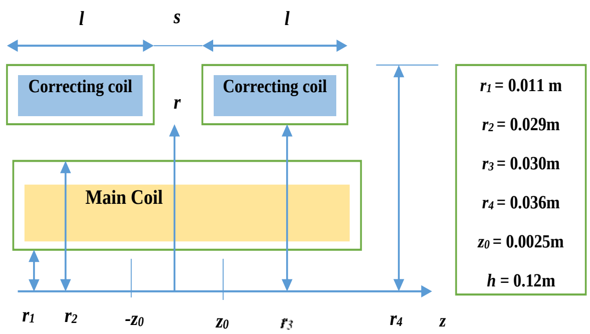

Fig. 6. Upper half plane of the axial cross section of Loney's solenoid problem.

Table 2: Comparison of different optimizer for Loney solenoid problem

Optimizer	$f(s,l) \times 10^{-8}$			
	Minimum (Best)	Mean	SD	Maximum
LTQPSO	3.7416	8.5294	3.8926	14.7682
QPSO	3.6792	5.2867	2.2496	8.7293
MQPSO	3.5728	6.5934	1.7454	8.6719
QBSO	3.3990	3.5749	0.7295	4.7614
DQPSO	3.3876	3.4982	0.9837	4.7428

Table 3: Best solution for Loney solenoid problem

Optimizer	Parameters			Computation Time
	$s(cm)$	$l(cm)$	$f(s,l) \times 10^{-8}$	
LTQPSO	14.60984	15.8997	3.7416	1729
QPSO	13.9675	5.4006	3.6792	1688
MQPSO	13.2813	3.3107	3.5728	1667
DQPSO	11.8301	1.6496	3.3876	1598

For a fair comparison, this case study is solved using the proposed DQPSO method, original QPSO [15], LTQPSO [18], MQPSO [19] and the results obtained by the QBSO method [22] is taken from literature for comparison.

Moreover, each optimizer is run independently 30 times with the corresponding number of maximum generations is 200. The optimal outcomes of different algorithms are tabulated in Table 2.

It can be concluded from the outcomes of Table 2 that the proposed DQPSO outperforms LTQPSO, QPSO, MQPSO and QBSO methods on minimum (best) objective functions values.

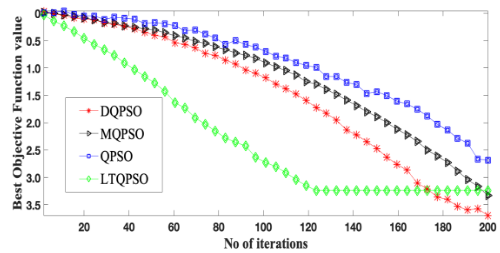

Fig. 7. Comparison of different optimal methods solving Loney Solenoid design.

Since the iterative number is an appropriate parameter to measure the computational time, and one can evaluate the computational efficiency using this parameter as shown in Table 3.

One can also analyze from the statistics of Fig. 7, that QPSO has the weakness of slow convergence behavior and is pertinent to trap into local optima. Though DQPSO has a fast convergence behavior, it is easy to avoid the local optimum. Thus, it can be illustrated that the proposed DQPSO avoids a possible local stuck and tradeoff between local and global searches. The new mechanism of mutation methodology and dynamic parameter can enhance the diversity of population and solution quality. As a result, the proposed DQPSO performance is much better than other tested optimizers.

VI. CONCLUSION

An improved version of quantum inspired particle swarm method has been presented in this work for the optimization of loneys solenoid design. The proposed dynamic QPSO was tested on some shifted version benchmark functions and loneys solenoid problem. The numerical outcomes and statistical analysis illustrate the merit and efficiency of the proposed DQPSO method as compared to other tested optimizers. Thus, the proposed method has significantly improved its solution quality and achieved an optimal solution for the tested problems. However, for future work, the proposed DQPSO method will be applied to other engineering design problems.

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