A Novel Multidirectional Strain Sensor Realized by a 3D Microstrip-Line Fed Near-Circular Patch Antenna

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Abstract **─** In this paper, we propose a newly developed impedance method to gauge the variations of multi-directional strain using a near-circular-patch based sensor. A novel three-dimensional (3D) feeding mechanism realized by a 90-deg bended microstrip line is devised for allowing strain detection along any direction in the azimuth plane of a metallic surface. The simulated results, verified by experimental results, demonstrate that there is a linear relationship between normalized impedance and multi-directional strain with high sensitivities of about 100 ppm/με. The relationship between sensitivity and sensor orientation is derived as a cosine function, which is a useful feature for estimating principal strain direction.

Index Terms **─** 3D feeding mechanism, multidirectional strain monitoring, near-circular-patch, strain sensor.

I. INTRODUCTION

The aging of steel structures, such as high-speed train, bridge, railways and airplanes has become a social problem. Recently, engineering accidents are of great concern. On Aug. 14, 2018, the Morandi Bridge of Italy collapsed sending vehicles and tons of rubble to the ground 150 feet below and 43 people were killed. In structural health monitoring (SHM), accurate assessment of deterioration demands efficient sensing techniques [1-3]. Strain sensors are used extensively in applications including construction engineering, bridge fabrication and railroading project. Resistive strain gauge is known as one of the most common strain sensors; its low cost and easy installation are the most prominent advantages. Nevertheless, it suffers from low sensitivity of 2, cable installation and low immunity to environmental conditions such as temperature and humidity variations. Besides, one resistive strain gauge can only detect strain occurring in one direction which is known beforehand. For strain detection of unknown direction(s), multiple gauges have to be combined in the form of rosettes. Thus, a sensor of single sensing element that can detect unknown directional strain beforehand is highly anticipated.

Recently, there has been a rapid development of antenna technologies, which are gradually used for sensors [4-5]. Patch antenna-based sensors are a passive and cost-effective method for locally monitoring defects immediately. To apply antennas as sensors for strain and crack detecting in SHM [6], linear-polarization (LP) patch antennas with different shapes and sizes have been proposed [7-10]. Relationships between resonant frequency and length/size of patch were used. Good performance of the LP patch antenna sensor has been proved, but the sensitivities are unfortunately found to be limited to 1 ppm/με and the sizes are not small enough [11-13]. The unidirectional sensing and low sensitivity are the main disadvantages of LP patch antenna-based sensors. A comparative study into the directional sensitivity of two coaxial probe-fed LP-patch sensors and circular-polarization (CP) patch was firstly conducted by authors [14]. Mathematical derivation and experiment validation of CP patch-based strain sensor with size of 80 mm×80 mm was first elaborated in [15]. Moreover, with the rapid development of 5G sub-6-GHz base-stations [16], their application scenarios will greatly expand, which also brings some special requirements to sensor performance. Linearity and sensitivity are two important parameters of strain sensors. In some instruments, small strain monitoring is required that demands for high sensitivity in order to obtain better

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resolution. For the sake of achieving precise detection of subtle strains, high linearity and high sensitivity are simultaneously required. This is a key research challenge that has not been sufficiently reported to date.

A high-sensitive antenna-based strain sensor called the proximity-fed circularly polarized (PFCP) patch has been initiated by the authors in [17]. High dielectric-constant substrate was used as the substrate for sensor-size reduction. Instead of the resonant-frequency-shift approach, wherein the shifts of resonant frequency represented the strain variations, the new approach of "phase-area" method using CP-patch-based sensor was investigated for strain monitoring. The simulation evidence has shown a liner correlation between normalized phase area change and micro-strain. High linearity for multi-directional strain was validated with favorable values of goodness-of-fit. However, alike the other wired sensors, for instance, the LP-patch based sensors [9-10], PFCP-patch sensor [17] also faced placement difficulty due to the Sub-Miniature-A (SMA) connector. Namely, the SMA connector needs to be installed perpendicular to the cross section of the metallic surface which limits the freedom of sensor's placement. In [9], a LP-patch sensor was installed for monitoring the strain on the central structural surface, a coaxial-feed patch antenna sensor was bonded at the center of a metallic plate through a hole on the metallic plate. The coaxial-feed patch antenna sensor has smaller overall size, but requires an "artificial hole" in the monitored structure. This method is impractical in real-world application. For all types of rectangular patch-based strain sensors presented in [14], [15] and [17], discrepancies were found in the two sensitivities under 0° and 90° direction. The perturbation segment, viz., the difference between the length and width of the rectangle patch is found to be the source of this discrepancies. To solve this problem, a near-circular patch is purposely used for eliminating the discrepancies.

In this study, our motivation is erected for building an antenna sensor with freedom placement for unknown strain detection. A near-circular-patch-based strain sensor with a novel feeding mechanism is designed that provides a way to solve the placement problem. The proposed strain sensor can be attached on any azimuth position of the monitored surface regardless of strain direction, wherein the variation of input impedance was used as the measurand for strain sensing. Moreover, strain direction can be determined based on the polarity and magnitude of the obtained sensitivity.

II. FEASIBILITY VERIFICATION OF 3D MICROSTRIP LINE FEED

Although the PFCP proof-of-concept prototype using "phase-area" method achieves good performances of high sensing sensitivity and linearity. Due to the fact that the SMA connector needs to be installed perpendicular to the cross section of monitored metallic surface, the installation location of sensor is limited. In order to solve the deficiency of the PFCP antenna sensor, a modified 3D feeding mechanism realized by a 90°-bended microstrip line is designed. This type of 3D fed near-circular-patch-based strain sensor provides a way to detect the strain in any direction of the structure surface.

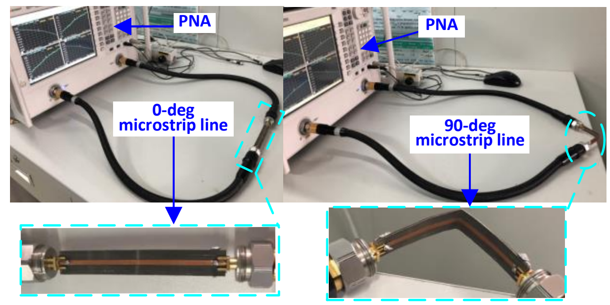
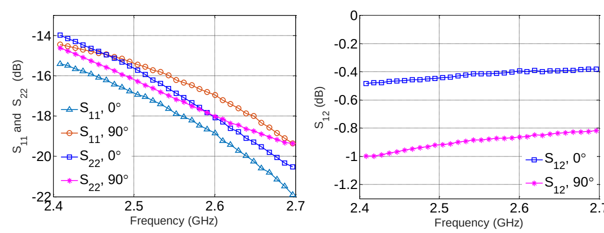


Fig. 1. Performance comparison of two 50-Ω microstrip lines: straight one versus 90°-bended one.



(a) (b)

Fig. 2. Comparison of S-parameters.

In order to verify the feasibility for applying the 3D feeding structure to patch-based strain sensor, two 50-Ω microstrip lines in length of 85 mm were fabricated. One is the straight microstrip line whereas the other is a 90° *L*-shaped microstrip line, which is expected to be used as the 3D feeding mechanism. As shown in Fig. 1, scattering parameters of two-port 50-Ohm microstrip lines were measured by a Keysight Performance Network Analyzer (PNA, N5222A) after calibration. Fig. 2 shows the comparison of S-parameters between 0-deg and 90-deg microstrip lines. The comparison result proves that the shape of microstrip line does not affect the impedance matching performance of it, which is promising for 3D microstrip line fed near-circular-patch-based strain sensors used to be attached to any position on the monitored surface along any direction.

III. STRAIN SENSOR USING NEAR-CIRCULAR PATCH

A. Multiphysics simulation of strain sensor

Figure 3 shows the geometry of the near-circular-patch with 90-deg bended microstrip line. RT/Duroid 5880 dielectric laminate (ε_r=2.2±0.02) with a thickness of 0.787 mm was chosen as the substrate for the near-circular patch having a major (*v*-) and minor (*u*-) length of *a* and *b*, respectively, backed by a square ground-plane of 60 mm×60 mm. A 90-deg bended microstrip line was deliberated to excite the patch so that the SMA connector can be connected along the *w*-axis.

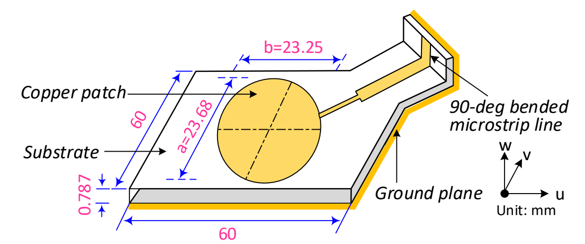


Fig. 3. Geometry of the 3D-line fed near-circular-patch-based strain sensor.

Finite-element-method based COMSOL Multiphysics has been adopted for not only designing antenna sensor but also predicting strain sensing performance. Figure 4 shows the domains including the perfectly matched layer (PML), an air sphere, a metallic surface and a 3D microstrip line fed near-circular-patch-based strain sensor. The PML is used for setting termination boundaries to simulate the simulation domain from an open structure to a finite domain. The near-circular-patch-based strain sensor consists of an upper copper patch and a dielectric substrate used for separating the copper patch from monitored metallic surface. The whole antenna sensor and the monitored metallic structure is placed inside an air sphere. The radiation patch is idealized as the perfect electric conductor (PEC).

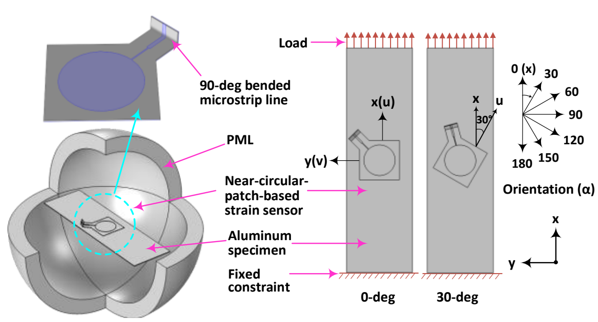


Fig. 4. COMSOL model of the 3D Tx-line fed near-circular-patch-based strain sensor.

The near-circular-patch-based strain sensor with 3D feeding mechanism bonded on the monitored aluminium specimen shown in Fig. 4 was built using the "mechanical-electromagnetic" double physical field coupling in COMSOL. In the mechanical simulation, only the patch-based strain sensor and aluminium specimen are involved. Firstly, the mechanical simulation was conducted at different strain levels from -1400 με to 1400 με at an increment of 350 με applied at the two ends of the aluminium specimen along *x*-axis. Strain is applied along *x*-axis, while the sensor orientation α is defined as the sensor angle with respect to *x*-axis (Fig. 4). Seven orientations of near-circular-patch-based strain sensor (0°, 30°, 60°, 90°, 120°, 150°, and 180°) were undertaken, in which two sensor orientations of 0° and 30° are illustrated as examples in Fig. 4. After the mechanical simulation at each strain level, the newly deformed configuration and meshing of the strain sensor was directly used for the next investigation via electromagnetic (EM) simulation. This convenience was achieved by coupling moving meshes with mechanical and EM field. For the EM simulation, a frequency domain solver is used to achieve the antenna responses at various orientations.

B. Simulated results for directional strain sensing

For the PFCP-patch strain sensor using the "phase-area" method achieved good performance of sensitivity and linearity when compared to the resistive strain gauge and LP patch-based strain sensors [17]. Nevertheless, the "phase-area" method is limited in identifying the strain direction when strain direction is unknown. Hence, a new method using the variation of degenerative mode's input impedance was proposed for identifying the strain direction. Figure 5 shows the variations of input impedance at orientations 0°, 30°, 60° and 90°. Based on the cavity-model theory, the copper patch and the ground plane form a resonant cavity. The patch antenna generates simultaneously two fundamental modes: TM_{01} and TM_{10} due to the perturbation of ellipticity (b/a=0.9818). Unlike the "phase-area" method, impedance curves under each strain level were bimodal wave curves. More interestingly, the maxima of TM_{01} shifts in frequency when sensor orientated at 0° and 30°, while a consistent shift of TM_{01} was observed when sensor orientation is at 60° and 90° placement. This phenomenon allows the determination of strain direction. Another observation is that there is a minimum between two modes and it changes obviously with strain variation. For sensor at 0° and 90° orientation, the minima of impedance curves vary obviously with strain variation. For sensor at 30° and 60° orientation, the minima of impedance curves vary slightly with strain variation.

After extracting the minima from impedance curves at each strain level, linear regression is performed between normalized minimum impedance ($Z_{\text{nor}} = \frac{\Delta|Z|}{|Z_0|}$) and strain to obtain the strain sensitivity (Fig. 6). As seen, fitting curves have good linearity between Z_{nor} and microstrain in multi-directional as verified by R-square (R^2) (all>0.95). Linear models of between simulated Z_{nor} and multi-directional strain verified with the experimental results are shown in Table 1. Very high strain sensitivities denoted by K (in unit of ppm/με) were achieved as indicated by the slopes of the regression lines. Sensitivities of 101.5, 56.11, -46.18, -100.5, -62.15, 40.63 and 98.88 ppm/με are achieved when sensor is placed in 0°, 30°, 60°, 90°, 120°, 150° and 180° orientation, respectively. The overall sensitivities are symmetrically distributed with the sensitivity of 90°.

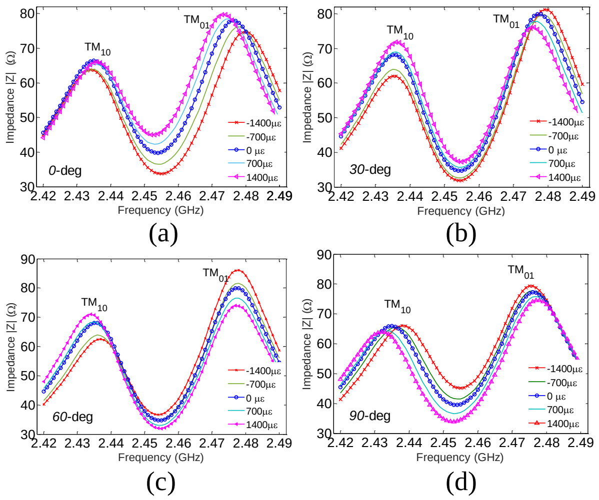


Fig. 5. Simulated input impedance with different sensor orientations: (a) 0°, (b) 30°, (c) 60°, and (d) 90°.

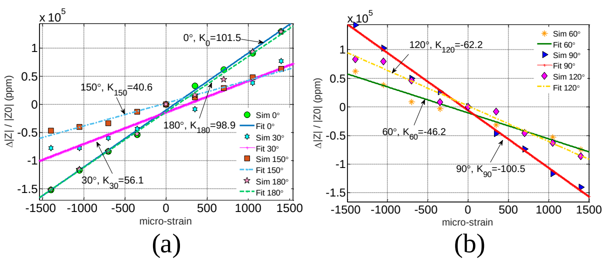


Fig. 6. Simulated multidirectional strain sensing sensitivities: (a) 0°, 30°, 150° and 180°; (b) 60°, 90°, and 120°.

IV. EXPERIMENTAL VERIFICATION

To validate the multi-directional strain sensing of 3D microstrip line fed patch-based strain sensor, a cantilever beam made by aluminum was designed as shown in Fig. 7, where the sensor orientation of 30° is illustrated as an example. We can see that without the limitation of SMA connector, antenna-based strain sensor can be attached to virtually any position on the monitored surface along any direction. Due to the symmetry of sensitivity distribution, we undertook only the sensor at 0°, 30°, 60° and 90° orientation. The antenna sensor was connected to the aluminum surface by using superglue. One side of the cantilever was fixed to the work bench using C-clamp while the other side was loaded progressively using the load-weights with the increment of 100 g used for simulating different strain levels. Upon every load progress, the impedance of the patch sensor was measured using the PNA. Meanwhile, three metal foil strain gauges were attached onto the back of aluminum beam across the centerline of patch sensor to avoid interference from the metal strain gauges. Strain acquisition system (CF3820, Cheng Fu Electronics, China) was used for strain collection.

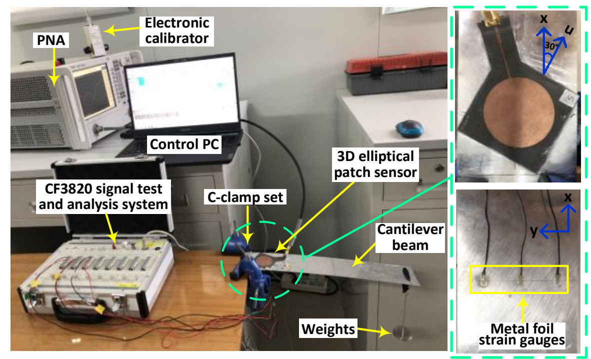


Fig. 7. Experimental set up of simultaneous measurement of multidirectional strain and input impedance.

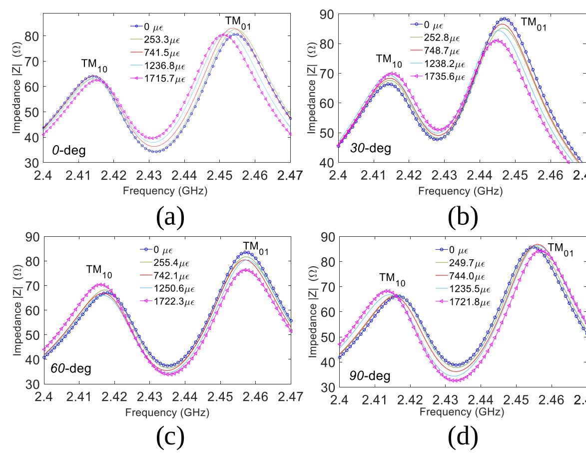


Fig. 8. Measured input impedance on strain arisen at sensor orientations of 0°, 30°, 60° and 90°.

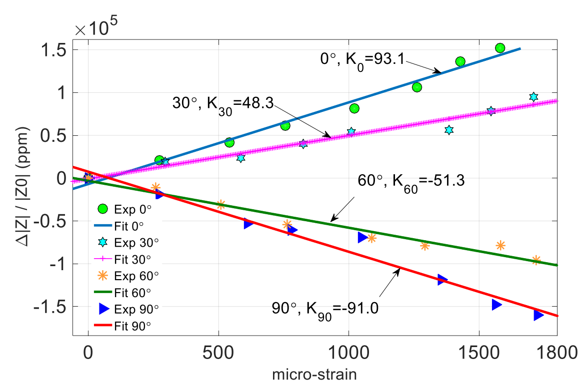


Fig. 9. Measured strain sensitivity at various sensor orientations.

The experimented impedance responses to multi-directional strain are shown in Fig. 8. As agreed with simulated results described in III-B, the TM_{01} shifts when sensor orientation is *0*-deg. We took this idea and reversed it: when the TM_{01} of impedance curves shifts, the strain can be inferred occur along the *0*-deg direction. Linear models between experimented Z_{nor} and multi-directional strain shown in Fig. 9 indicate that strain sensitivities of 93.05, 48.33, -51.25 and -91.03 ppm/με obtained by experiment also shows high sensitivities and good linearity verified by R^2 (all>0.96).

Experimented linear models compared with simulated ones are shown in Table. 1. Slight discrepancies were observed between experimented and simulated results, which may be attributed to the ideal boundary conditions or nominal values of material property parameters in simulation.

Table 1: Simulated linear models compared with measured ones

α (°)	Simulated Models	Experimental Models
0	Z_{nor}=101.5ε-9944	Z_{nor}=93.05ε-7007
30	Z_{nor}=56.11ε-13754	Z_{nor}=48.33ε-4199
60	Z_{nor}=-46.18ε-10400	Z_{nor}=-51.25ε-10640
90	Z_{nor}=-100.5ε-6021	Z_{nor}=-91.03ε-2346
120	Z_{nor}=-62.15ε+165	/
150	Z_{nor}=40.63ε+2296	/
180	Z_{nor}=98.88ε-13354	/

The fitting curve of relationship between sensitivity data and sensor orientation α (in unit of degree) is shown in Fig. 10. It can be clearly drawn from Fig. 10 that sensitivity data is almost symmetric with the sensor orientation of 90°, which is promising for multi-directional strain sensing of metallic structure. The relationship between sensitivity K and sensor orientation α can be expressed in a cosine function:

$$K(\alpha)=C_1\cos(C_2\alpha), \quad (1)$$

where C_1 is equal to 99.4 which represents the upper and lower bounds of the cosine function are ±99.4 ppm/με, C_2 is obtained by regression as 0.03409, which indicates the period T of cosine function is $T=\frac{2\pi}{0.03409} \cong 184^{\circ}$.

The coefficient C_1 of 99.4 indicates a maximum sensitivity of 99.4 ppm/με at 0° orientation whereas a minimum sensitivity of -99.4 ppm/με at orientation 90°. These are consistent with the maximum sensitivity of 93.05 ppm/με and minimum sensitivity of -91.03 ppm/με obtained from the measurement (Fig. 9). The coefficient C_2 represents the period of 184° which can be verified by the function displayed in Fig. 10. That's to say, strain orientation can be inferred by the magnitude and polarity of sensitivity using cosine function. When strain occurs in an unknown direction, the input impedance curve shifts and Z_{nor} can be obtained. Then, sensitivity calculated by linear model between Z_{nor} and multi-directional strain is substituted into (1) to obtain the sensor orientation (i.e., the direction of strain arisen). Unfortunately, sensitivity of around "0" ppm/με is inferred by orientation of 45° and 135° as observed from Fig. 10. Other strain measurand(s) and methods have to be used in order to obtain the strain sensitivities occurred at 45° and 135° orientation. These will be the objectives in future studies.

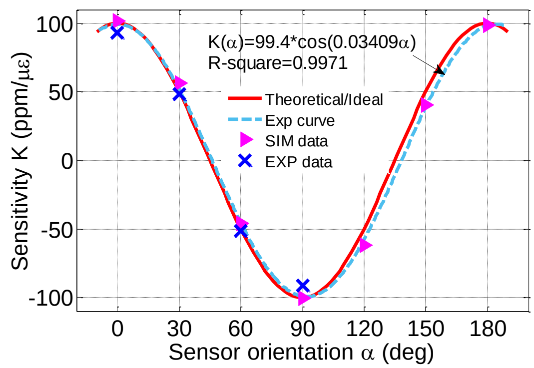


Fig. 10. Variation of sensitivity as a function of sensor orientation, α.

V. CONCLUSION

In this paper, we propose a near-circular-patch-based strain sensor with a novel 3D feeding mechanism for multidirectional strain sensing of metallic structures. The 3D feeding is realized by a 90-deg bended microstrip line, which allows the sensor to be placed in any position on the structure surface when the strain direction is not known beforehand. High sensitivity of about 100 ppm/με and linearity have been achieved simultaneously for multi-directional strain sensing. The relationship between strain sensitivity and sensor orientation is derived as a cosine function, where the strain direction can be estimated based the polarity and magnitude of the strain sensitivity. Symmetric sensitivity magnitude is obtained with respect to the 90° orientation attributed to the cosine function. Therefore, a single sensing element of the proposed strain sensor can detect unknown directional strain which is very viable yet cost-saving in practical application.

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