Input Impedance Sensitivity Analysis of Patch Antenna with Discrete Perturbations on Method-of-Moment Grids

Yifan Zhang, Natalia K. Nikolova, and Mohamed H. Bakr

Department of Electrical and Computer Engineering
McMaster University, Hamilton, ON L8S 4L8, CANADA
zhangyf98@grads.ece.mcmaster.ca, nikolova@ieee.org, mbakr@mail.ece.mcmaster.ca

Abstract — A new practical approach is proposed to the response sensitivity analysis of high-frequency structures modeled with the method of moments. The response sensitivities are calculated with the self-adjoint approach and discrete shape perturbations on the method of moments grids. The approach requires certain computational effort as a pre-process. This effort is due to building a global system matrix that covers all possible geometrical variations which may arise during design optimization. The technique is illustrated through the sensitivity analysis of the input impedance of a Yagi-Uda array and a printed patch antenna. The computed sensitivities are validated by comparing with the central finite-difference estimates at the response level.

Index Terms — Antenna analysis, frequency-domain, response sensitivity analysis, self-adjoint.

I. INTRODUCTION

The purpose of response sensitivity analysis is to evaluate the sensitivity of the responses of a system to its design parameters. The response sensitivity is represented by the response gradient in the design-parameter space. In high-frequency structure analysis, the design parameters typically describe the geometry and the electromagnetic (EM) properties of the involved materials.

The system response may be defined as: 1) a distributed response represented by the state variables such as current or field distributions; 2) a set of engineering parameters describing the structure's performance such as *S*- or *Z*-parameters; and 3) a single scalar function, which represents a global performance measure, such as the objective function in an optimization problem. The sensitivity information is crucial in gradient based optimization. In this paper, we propose a new technique using method of moments (MoM) grids and the respective current solutions to perform the sensitivity analysis and to carry out gradient optimization.

Our technique is based on the adjoint variable method (AVM). The AVM offers an efficient approach to the design sensitivity analysis of problems of high complexity where the number of state variables is much greater than the number of the required response derivatives [1-10].

Approaches to the sensitivity analysis with the MoM have been developed before [11-13]. There, the analytical formulation of the system matrix derivatives is abandoned as it is impractical for a general sensitivity solver. Instead, the derivatives of the system matrix are estimated with finite differences or the Broyden update. With these approaches, however, the computational speed is still limited due to the following factors: (a) the need to actually compute the perturbed system matrices, and/or (b) the need to perform an adjoint-system analysis, which means one additional full-wave simulation.

A general self-adjoint approach to the sensitivity analysis of network parameters was formulated in [15, 16]. It requires neither an adjoint problem nor analytical system matrix derivatives. An application to the sensitivity analysis of *S*-parameters with the MoM is considered. However, the approach has three drawbacks. First, the computational overhead of the sensitivity is still significant due to the n additional matrix fills for the n perturbed structures. These n matrices are needed to carry out the forward finite differencing of the system matrices. Second with commercial software, users can only have access to the system matrices after they are written on the disk. The time needed to export n+1 large dense system matrices in every iteration (one system matrix for the nominal structure and n system matrices for the perturbed structures) may be significant. Finally, a special mesh control has to be enforced when perturbing the design variables. This is difficult to implement with most existing commercial MoM solvers.

Here, we propose a new self-adjoint sensitivity analysis (SASA) technique with the MoM solutions. This technique uses discrete perturbations of the optimizable shape parameters on a pre-determined MoM grid. The purpose is to aid gradient-based optimization of antenna structures. A global system matrix is calculated only once at the beginning of the analysis. This system matrix covers the whole range of structures (in MoM, these are metallic surfaces), which could be considered during the design optimization. The system matrix of any particular structure arising during optimization is assembled by disabling the elements of the global system matrix corresponding to segments or surfaces which are not metalized.

Take a planar patch antenna as an example. The global system matrix is built for a sufficiently large area, which is discretized into a predetermined number of rectangular subsections. Every structure that is smaller than this large area can be represented by a proper selection of subsections. The patches of the small structure are simply a sub-set of the patches of the large area.

The advantage of the technique is that it accelerates not only the response sensitivity analysis but also the optimization procedure. This is due to the fact that the global matrix is used to assemble quickly not only the perturbed-structure system matrices needed in sensitivity analysis but also the system matrices for all iterates during the optimization.

In Section II, we state the basics of the self-adjoint sensitivity analysis. In Section III, we introduce the discrete perturbation technique on MoM grids. The application of the approach to the sensitivity analysis of a wire array and a printed patch are presented in Section IV. An example of optimizing a printed patch is given in Section V together with comparisons with conventional optimization. The implications and significance of this work are briefly discussed in the conclusions.

II. SELF-ADJOINT SENSITIVITY ANALYSIS

Using the MoM notations, a linear EM system is represented by

$$\boldsymbol{Z}(\boldsymbol{x})\boldsymbol{I} = \boldsymbol{V} \ . \tag{1}$$

Here, $\boldsymbol{x} = [x_1 \ldots x_n]^T$ is the vector of design parameters; $\boldsymbol{Z}$ is the system matrix whose complex coefficients depend on the geometry and the materials; $\boldsymbol{I} = [I_1 \ldots I_m]^T$ is the solution provided by the MoM solver at the nominal design; and $\boldsymbol{V}$ is the excitation vector.

We define a general response function $f(\boldsymbol{x}, \bar{\boldsymbol{I}}(\boldsymbol{x}))$ at the current solution $\bar{\boldsymbol{I}}$ of (1) with respect to the design parameter $\boldsymbol{x}$. The objective of the sensitivity analysis is to obtain the gradient of the system response, i.e.,

$$\nabla_{\boldsymbol{x}} f, \text{subject to } \boldsymbol{ZI} = \boldsymbol{V} \ , \tag{2}$$

where $\nabla_{\boldsymbol{x}} f$ is the row operator

$$\nabla_{\boldsymbol{x}} = [\partial/\partial x_1, \partial/\partial x_2, \cdots, \partial/\partial x_n] \ . \tag{3}$$

Assuming that the $\boldsymbol{Z}$ matrix is not singular, $\nabla_{\boldsymbol{x}} \boldsymbol{I}$ is obtained from (1) as

$$\nabla_{\boldsymbol{x}} \boldsymbol{I} = \boldsymbol{Z}^{-1} \left[\nabla_{\boldsymbol{x}} \boldsymbol{V} - \nabla_{\boldsymbol{x}} (\boldsymbol{Z} \bar{\boldsymbol{I}}) \right] . \tag{4}$$

The response gradient $\nabla_{\boldsymbol{x}} f$ of (2) can be written as

$$\nabla_{\boldsymbol{x}} f = \nabla_{\boldsymbol{x}}^{e} f + \nabla_{\boldsymbol{I}} f \cdot \nabla_{\boldsymbol{x}} \boldsymbol{I} \ , \tag{5}$$

where $\nabla_{\boldsymbol{I}}$ is a row operator analogous to $\nabla_{\boldsymbol{x}}$ in (3). The gradient $\nabla_{\boldsymbol{x}}^{e} f$ represents the explicit dependence of $f(\boldsymbol{x}, \bar{\boldsymbol{I}}(\boldsymbol{x}))$ on $\boldsymbol{x}$. Substituting (4) into (5) leads to

$$\nabla_{\boldsymbol{x}} f = \nabla_{\boldsymbol{x}}^{e} f + \hat{\boldsymbol{I}}^{T} \left[\nabla_{\boldsymbol{x}} \boldsymbol{V} - \nabla_{\boldsymbol{x}} \left(\boldsymbol{Z} \bar{\boldsymbol{I}} \right) \right] . \tag{6}$$

The adjoint vector $\hat{\boldsymbol{I}}$ is the solution to

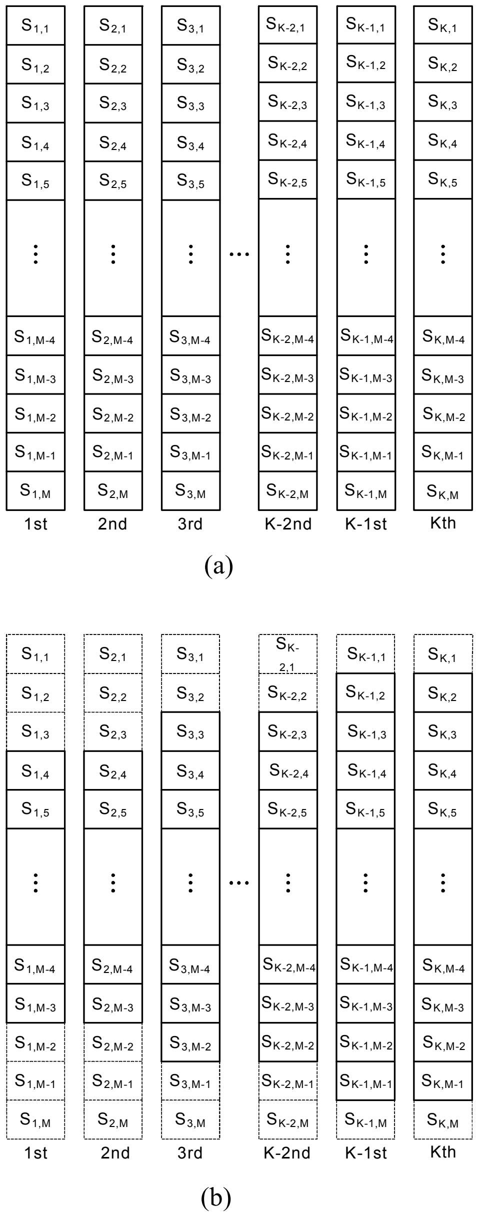


Fig. 1. Discretization of a wire antenna array: (a) the large library structure; (b) the new structure as a sub-set of the library structure. Segments shown with thick line correspond to metal and those shown with dash line correspond to air.

$$\boldsymbol{Z}^T \hat{\boldsymbol{I}} = \hat{\boldsymbol{V}} \, , \tag{7}$$

where

$$\hat{\boldsymbol{V}} = [\nabla_{\boldsymbol{I}} f]^T \tag{8}$$

is the adjoint excitation [11].

To perform the computations in (6), we need the current solution $\bar{\boldsymbol{I}}$ of (1), the solution $\hat{\boldsymbol{I}}$ of (7), and the system matrix derivatives $\partial \boldsymbol{Z}/\partial x_i$ $(i = 1,\ldots,n)$.

The matrix derivatives can be obtained by central finite differences (CFD) as

$$\frac{\partial \boldsymbol{Z}}{\partial x_i} \approx \frac{\boldsymbol{Z}(x_i + \Delta x_i) - \boldsymbol{Z}(x_i - \Delta x_i)}{2\Delta x_i}, \; i = 1,\ldots,n \, . \tag{9}$$

The shape parameter perturbations Δx_i $(i = 1,\ldots,n)$ are equal to the respective segment size in the MoM discrete grid. This is explained in detail in Section III.

The adjoint current $\hat{\boldsymbol{I}}$ is the solution to (7). With the self-adjoint approach, we do not need to solve (7), which would be as computationally demanding as solving (1). From (1) and (7), we see that if the system matrix is symmetric, $\boldsymbol{Z} = \boldsymbol{Z}^T$,[1] and the excitation vectors $\boldsymbol{V}$ and $\hat{\boldsymbol{V}}$ fulfill

$$\hat{\boldsymbol{V}} = c\boldsymbol{V} \, , \tag{10}$$

therefore

$$\hat{\boldsymbol{I}} = c\boldsymbol{I} \, . \tag{11}$$

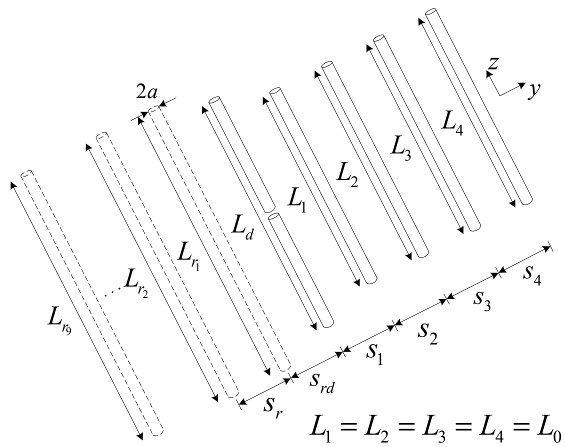
Here, c is a complex number called the self-adjoint constant. The adjoint simulation is thus avoided. In the case of an antenna input impedance (a response of interest here), the self adjoint constant is [15]

$$c = -I_{in}^{-2} \tag{12}$$

where I_{in} is the current at the feed point of the antenna.

III. DISCRETE PERTURBATION WITH THE METHOD OF MOMENTS

Here, we propose a method for system analysis, which is particularly suitable for design optimization. It is based on deriving a complete set of mutual-coupling coefficients $\boldsymbol{Z}_{i,j}$ (i, j = 1,…, $N_{\max}$), for all possible metallic segments of the antenna structure. This approach reduces the computational load associated with building the system matrices of the optimized structures during the iterative process. It is particularly advantageous in response sensitivity analysis as discussed next.



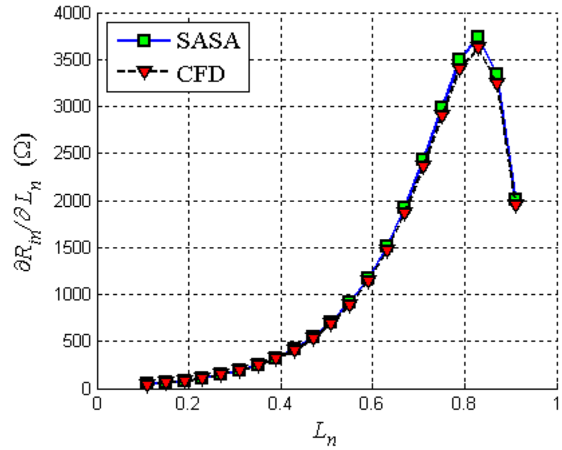
$$L_1 = L_2 = L_3 = L_4 = L_0;$$
$$L_{r_1} = L_{r_2} = \cdots = L_{r_9} = L_r;$$
$$s_1 = s_2 = s_3 = s_4 = s_d.$$

Fig. 2. Six-element Yagi-Uda antenna.

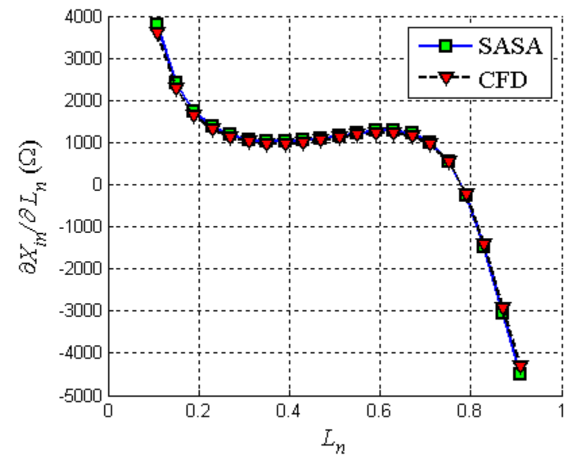
Table 1: Nominal design parameters of the Yagi-Uda antenna

Design parameter	Value
L_0 / λ	0.386
L_d / λ	0.426
L_r / λ	0.604
s_d / λ	0.340
s_{rd} / λ	0.320
s_r / λ	0.050
a / λ	0.003

A sufficiently large structure is built which covers all the possible metallic segments that may be used in the structures arising during the optimization or sensitivity analysis. Its system matrix is referred to as library matrix $\boldsymbol{Z}_{lib}$. Any new structure can be viewed as a sub-set of the library structure. Also, each new system matrix $\boldsymbol{Z}$ can be obtained by switching off the corresponding elements of $\boldsymbol{Z}_{lib}$ and filling the respective rows and columns with zeros. Take a K-element Yagi-Uda array, as an example. We first choose a suitable segment length δ and discretize each wire into M segments as shown in Fig. 1 (a). Thus, $\boldsymbol{Z}_{lib}$ has the dimension of $N_{\max} \times N_{\max}$ ($N_{\max} = K \cdot M$). The new structure has its segments with indices i_1, i_2 ,..., i_k de-metalized. These segments are shown with a dash line in Fig. 1 (b). The new system matrix is obtained by switching off all the



(a)

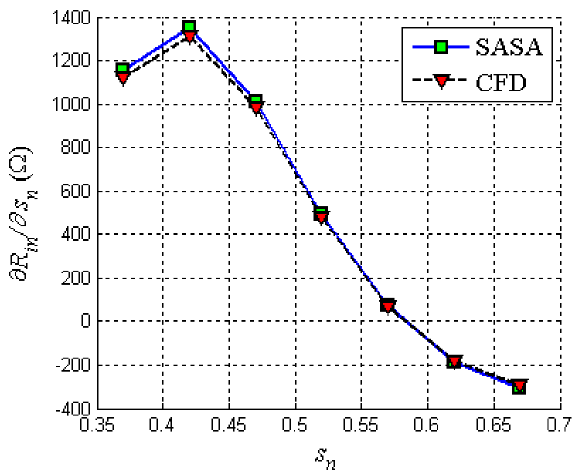


(b)

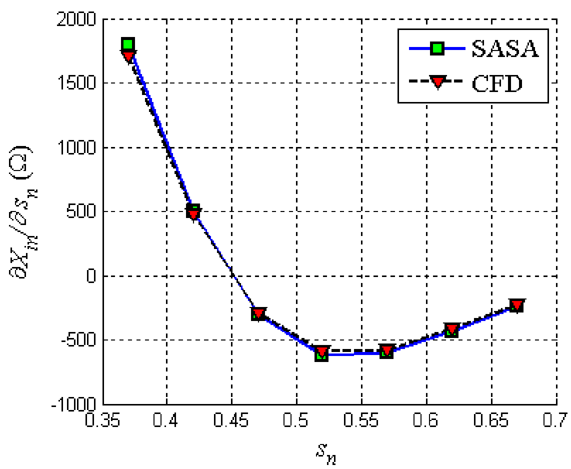
Fig. 3. Sensitivity of the input impedance of the Yagi-Uda antenna with respect to the normalized length of the driving element: (a) resistance derivative; (b) reactance derivative.

corresponding matrix elements in $\boldsymbol{Z}_{lib}$, i.e., all matrix elements with subscripts containing $i_1, i_2, \ldots, i_k$ are set to zero.

In order to perform the sensitivity calculation in (9), we need to obtain $\boldsymbol{Z}(x_i + \Delta x_i)$ and $\boldsymbol{Z}(x_i - \Delta x_i)$. These are the system matrices of the perturbed antenna structures where the ith parameter is perturbed in the forward and backward directions. Each one of these perturbed-structure $\boldsymbol{Z}$ matrices is obtained from $\boldsymbol{Z}_{lib}$ by switching off "air" segments. Note that all shape parameters are thus constrained to a large but finite set of segment combinations. A typical perturbation Δx_i is equal to one segment length δ (on a uniform MoM grid).



(a)



(b)

Fig. 4. Sensitivity of the input impedance of the Yagi-Uda antenna with respect to the distance between the driving element and the reflector: (a) resistance derivative; (b) reactance derivative.

IV. VALIDATION EXAMPLES FOR THE SENSITIVITY ANALYSIS WITH THE MOM

We use a six-element Yagi-Uda array and a printed patch to illustrate the SASA of the input impedance of the two antennas.

A. Sensitivity analysis of a Yagi-Uda array

The size of the library structure in this example is determined by two factors. First, the length of each wire element in the library structure needs to be sufficiently long in order to cover the whole range of lengths allowed in the optimization

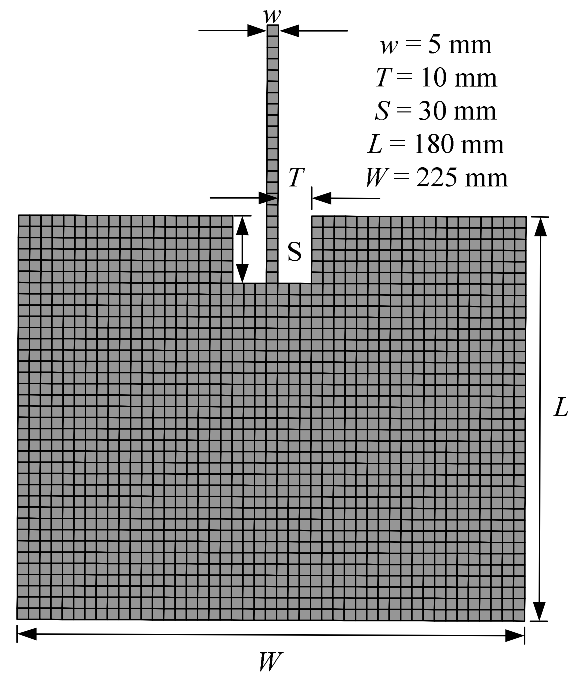
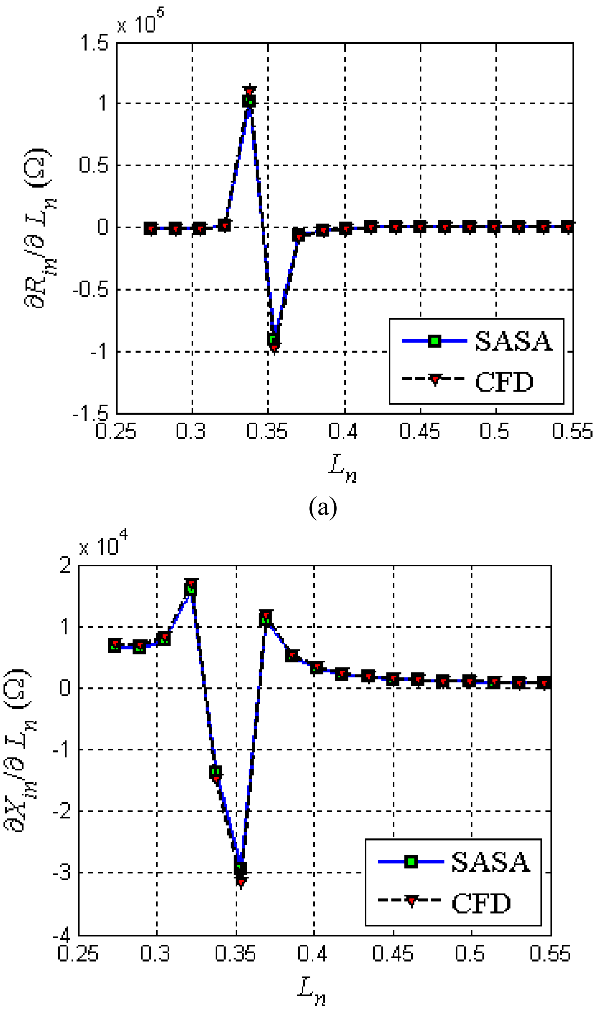


Fig. 5. Library structure of a planar patch.

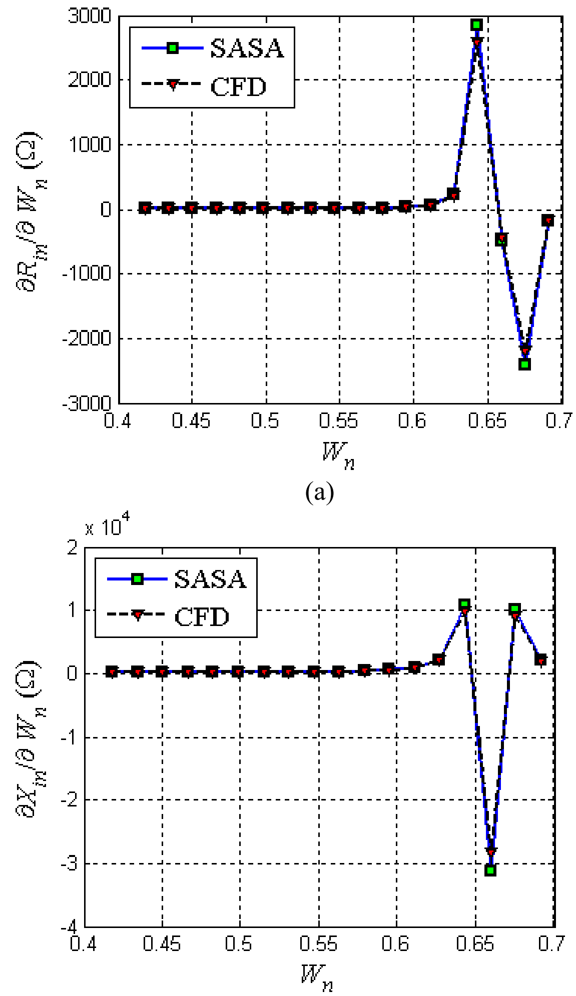
and needed by the sensitivity analysis. We fix the lengths of all six wire elements to $L=\lambda$, with a radius $a = 0.003\lambda$. Each wire element is discretized into $M=101$ segments. The segment length is thus $\delta=\lambda/M=\lambda/101$. Second in order to perform the sensitivity analysis and optimize with respect to the separation distance between the driving element and the reflector, we assign $K_p=9$ positions at which the reflector can be positioned. These are shown in Fig. 2 with dash lines. The total number of wire elements in the library structure is thus $K_{lib}=K+K_p-1=14$. Therefore, the size of $\boldsymbol{Z}_{lib}$ is $N_{lib} \times N_{lib}$ where $N_{lib} = M \times K_{lib} = 101 \times 14 = 1414$.

After a nominal structure is built and analyzed, its sensitivity analysis is carried out. Its forward and backward perturbed structures with respect to the length of a wire are obtained by adding and subtracting one segment at each wire end. The forward and the backward perturbed structures with respect to the separation are obtained by selecting the neighbouring positions to that of the nominal reflector position. The nominal parameters of the Yagi-Uda array are shown in Table 1. Note that s_r is the fixed distance between



(b)

Fig. 6. Sensitivity of the input impedance with respect to L_n at W_n = 0.66: (a) resistance derivative; (b) reactance derivative.



(b)

Fig. 7. Sensitivity of the input impedance with respect to W_n at L_n = 0.48: (a) resistance derivative; (b) reactance derivative.

neighbouring reflector positions [see Fig. 2].

The derivatives of the antenna input impedance are calculated with the proposed approach for a sweep of the length of the driving element. The length of the driving element L_d is swept from 0.1λ to 0.9λ while the separation distance between the driving element and the reflector is fixed at 0.32λ. The results for the derivatives with respect to the normalized lengths of the driver $L_n = L_d/\lambda$ are plotted in Fig. 3. There, two derivative curves are shown. The curves marked with "SASA" are obtained using our approach. The curves marked with "CFD" (center finite difference) are obtained with the perturbation approach where finite differences at the response level are used.

The results plotted in Fig. 4 present the sensitivities with respect to the normalized separation distance $s_n = s_{rd}/\lambda$ between the reflector and the driving element. This distance is swept from 0.37λ to 0.67λ with a step of 0.05λ. The length of the driving element is fixed at $L_d = 0.426\lambda$. The agreement between the adjoint sensitivities and those obtained with finite differences at the response level is excellent as shown in both Fig. 3 and Fig. 4.

B. Sensitivity analysis of a printed patch

The library structure is shown in Fig. 5. Here, we set the edge length of the square subsection to be δ = 5.0 mm. The length of the library structure is L = 36δ = 180.0 mm and its width is W = 45δ = 225.0 mm. Sensitivity analysis is carried out with respect to the length and width of the patch antenna. The forward and backward perturbed structures with respect to the length of the patch are obtained by adding and subtracting one line of subsections at the patch edge opposite to the feeding-point edge. The forward and the backward perturbed structures with respect to the width of the patch are obtained by adding and subtracting one line of subsections at both sides of the patch. The nominal design parameters of the patch are shown in Fig. 5.

The analysis is carried out at the frequency f_0 = 0.97 GHz. The derivatives of the antenna input impedance are calculated with the proposed approach for a sweep of the length of the patch. The length L is swept from 0.27λ to 0.55λ while the width of the patch is fixed at W_n = W/λ = 0.66. Here, λ is the wavelength in air. The results for the derivative with respect to the normalized length of the patch L_n = L/λ are plotted in Fig. 6. The results shown in Fig. 7 present the sensitivities with respect to the normalized width of the patch W_n = W/λ which is swept from 0.42λ to 0.69λ while the length is fixed at L_n = L/λ = 0.48. Again, excellent agreement is observed between the sensitivities calculated with the proposed approaches and those calculated with response-level finite differences.

Note that in calculating the CFD sensitivity, two additional EM simulations have to be performed per parameter. In obtaining the adjoint sensitivity, the calculation involves only two matrix subtractions and a vector-matrix-vector multiplication. Thus, our approach is much faster than the CFD method.

V. DESIGN OPTIMIZATION EXAMPLE

We use the technique described above to optimize the input impedance of the planar patch antenna shown in Fig. 5. The objective function is defined as

$$f(\boldsymbol{x}) = \left| \frac{Z_{in} - \bar{Z}}{\bar{Z}} \right|, \qquad (13)$$

where $\bar{Z} = 50\,\Omega$ and Z_{in} is the input impedance of the antenna. The vector of design parameters is $\boldsymbol{x} = [L, W]^T$. The values of the rest of the design parameters are fixed at those given in Fig. 5. The objective function (13) depends on a single complex-valued current I_f at the feed-point. The input impedance is then calculated with $Z_{in} = V_f / I_f$, where $V_f = 1$ V. The relation between $\nabla_{\boldsymbol{x}} f$ and $\nabla_{\boldsymbol{x}} Z_{in}$ is given by

$$\nabla_{\boldsymbol{x}} f = \mathrm{Re}\left[\frac{1}{\bar{Z}} \cdot \frac{(Z_{in} - \bar{Z})^*}{|Z_{in} - \bar{Z}|} \cdot \nabla_{\boldsymbol{x}} Z_{in} \right]. \qquad (14)$$

The sensitivity $\nabla_{\boldsymbol{x}} Z_{in}$ is calculated by our proposed approach. The optimization is implemented by using the Matlab function *fmincon*, whose algorithm is based on the line-search method with sequential quadratic programming (SQP). At each iteration, the SQP sub-problem is solved and its solution is used to define a search direction.

For comparison, the optimization is carried out in two separate procedures using two different methodologies: a) optimization with the sensitivity information offered by our self-adjoint method and using the library matrix $\boldsymbol{Z}_{lib}$; b) optimization without the sensitivity information and without using the $\boldsymbol{Z}_{lib}$ matrix.

A. Design optimization with sensitivity information and pre-calculated library matrix

The frequency of interest is f_0 = 0.97 GHz. The initial design is set to $\boldsymbol{x} = [165, 195]^T$ mm. The EM solver [17] is used to compute the library matrix $\boldsymbol{Z}_{lib}$. It is, also, called by the optimization algorithm to compute the input impedances of the antenna design iterates. The system matrices of these iterates are obtained by switching on and off the corresponding elements of the library matrix according to the geometry information provided by the optimizer. The optimization process converges after 4 iterations with an optimal design $\boldsymbol{x}^* = [175, 205]^T$ mm and objective function $f(\boldsymbol{x}^*) = 0.098$. The progress of the optimization is shown in Fig. 8. Only four optimization iterations are needed. During these iterations, the EM solver is called 14 times. The values of the design parameters as well as the values of the input impedance, and the objective function are listed in Table 2.

Table 2: Design parameters, input impedance, and objective function in the optimization with sensitivity information and using the $\mathbf{Z}_{lib}$ matrix

	L	W	R_{in}	X_{in}	f
0	165	195	9.96	411.09	8.261
1	175	210	7.24	190.88	3.912
2	170	205	53.6	-16.60	0.340
3	175	205	50.1	4.90	0.098
4	175	205	50.1	4.90	0.098

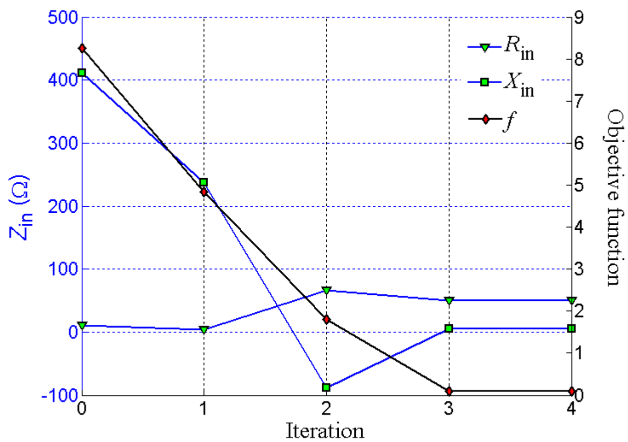


Fig. 8. Progress of the objective function and the input impedance during the optimization with sensitivity information.

B. Design optimization without sensitivity information and pre-calculated library matrix

In order to illustrate the efficiency of the gradient-based design optimization with sensitivity information, we present a conventional optimization without the sensitivity information and without the use of $\mathbf{Z}_{lib}$. The conventional approach has the same settings, except that the gradient is not provided to the optimizer. The system matrices are built by the EM solver for each particular structure. Optimization starts with the same initial values. After 23 calls to the EM solver (6 iterations), the result converges to an optimal design $\mathbf{x}^* = [175, 205]$ mm and objective function $f(\mathbf{x}^*) = 0.098$. The progress is shown in Fig. 9 and the design parameters are given in Table 3.

C. Comparison between the two optimization procedures

For the optimization with the proposed approach, the computational overhead is due to two calculations: 1) filling the system matrix $\mathbf{Z}_{lib}$ at the beginning of the optimization; and 2) solving

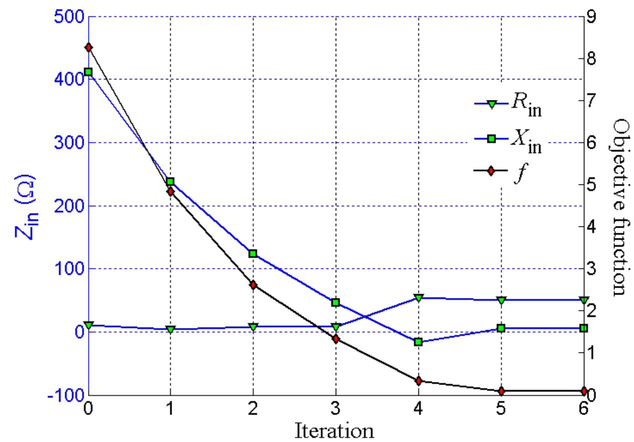


Fig. 9. Progress of the objective function and the input impedance during the optimization without sensitivity information.

Table 3: Design parameters, input impedance, and objective function in the optimization without sensitivity information and without using the $\mathbf{Z}_{lib}$ matrix

	L	W	R_{in}	X_{in}	f
0	165	195	9.96	411.09	8.261
1	175	215	3.33	237.69	4.845
2	165	205	7.48	123.62	2.615
3	165	210	7.71	45.94	1.324
4	170	205	53.61	-16.60	0.340
5	175	205	50.1	4.90	0.098
6	175	205	50.1	4.90	0.098

Table 4: Comparison between the computational overhead of the gradient-based optimization with and without sensitivity information / library matrix

	Proposed approach	Conventional approach
Iterations	4	6
Call for solver	14	23
Matrix fill time (s)	3.9	66.8
Solve system time (s)	52.0	80.6
Total CPU time (s)	55.9	147.4

the linear system of the nominal structure at each simulation call. In the second approach, the computational overhead at each call to the simulator is due to: 1) matrix fill, and 2) solving the system of equations. The comparison between the two approaches is shown in Table 4 in terms of: 1) the number of iterations, 2) the number of calls for EM simulations, 3) the CPU time for matrix fill, 4) the CPU overhead for solving the system, and 5) the total CPU overhead. It is evident that the optimization process with our approach converges faster and takes shorter time.

VI. CONCLUSION

A new approach to self-adjoint sensitivity analysis with discrete perturbations on MoM grids is proposed. The technique aims at computationally efficient gradient-based optimization of antenna structures analyzed by the MoM. A large system matrix (the "library matrix") is computed only once at the beginning. This matrix is then used for rapid sensitivity calculations as well as for quick matrix-building for the structures arising during the optimization.

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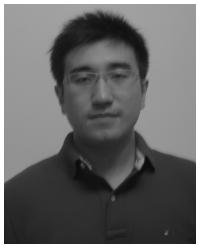

Yifan Zhang received the B. Eng. degree (with honors) in electrical engineering from Harbin Institute of Technology, Harbin, China, in 2008, and is currently working toward the Ph.D. Degree at McMaster University, Hamilton, Canada.

In September 2008, he joined the Computational Electromagnetics Research Laboratory, Department of Electrical and Computer Engineering, McMaster University, where he is currently a Teaching Assistant. His research interests include computational EM, computer-aided design for high frequency structures and antennas, EM optimization, and microwave image.

Natalia K. Nikolova received the Dipl. Eng. degree from the Technical University of Varna, Varna, Bulgaria, in 1989, and the Ph.D. degree from the University of Electro-Communications, Tokyo, Japan, in 1997.

From 1998 to 1999, she was with the Natural Sciences and Engineering Research Council of Canada (NSERC), during which time she was initially with the Microwave and Electromagnetics Laboratory, DalTech, Dalhousie University, Halifax, NS, Canada, and then for a year with the Simulation Optimization Systems Research Laboratory, McMaster University, Hamilton, ON, Canada. In July 1999, she joined the Department of Electrical and Computer Engineering, McMaster University, where she is currently a Professor. Her research interests include theoretical and computational electromagnetism, microwave methods for imaging and detection, as well as the computer-aided design of high-frequency structures and antennas.

Dr. Nikolova is a member of the Applied Computational Electromagnetics Society. She is also an International Union of Radio Science (URSI) correspondent, Commissions B and D. She was the recipient of an NSERC Post-Doctoral Fellowship from 1998 to 1999. She held a University Faculty Award of the NSERC from 2000 to 2005 and is currently Canada Research Chair in High-frequency Electromagnetics.

Mohamed H. Bakr received the B.Sc. degree in electronics and communications engineering with distinction (honors) and the M.Sc. degree in engineering mathematics from Cairo University, Cairo, Egypt, in 1992 and 1996, respectively, and the Ph.D. degree from McMaster University, Hamilton, ON, Canada, in 2000.

In 1997, he was a student intern with Optimization Systems Associates Inc. (OSA), Dundas, ON, Canada. From 1998 to 2000, he was a Research Assistant with the Simulation Optimization Systems (SOS) Research Laboratory, McMaster University. In November 2000, he joined the Computational Electromagnetics Research Laboratory (CERL), University of Victoria, Victoria, BC, Canada, as a Natural Sciences and Engineering Research Council of Canada (NSERC) Post-Doctoral Fellow. He is currently an Associate Professor with the Department of Electrical and Computer Engineering, McMaster University. His research areas of interest include optimization methods, computer-aided design and modeling of microwave circuits, neural-network applications, smart analysis of microwave circuits, and efficient optimization using time-/frequency-domain methods.

Dr. Bakr was a recipient of the Premier's Research Excellence Award (PREA) presented by the Province of Ontario, Canada, in 2003.