A Memory-Efficient Hybrid Implicit–Explicit FDTD Method for Electromagnetic Simulation

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Abstract **–** As the explicit finite-difference time-domain (FDTD) method is restricted by the Courant−Friedrich−Levy (CFL) stability condition and inefficient for simulation in some situations, implicit methods are developed. The hybrid implicit−explicit (HIE) FDTD method is one popular method among them. In this paper, a memory-efficient HIE FDTD method is designed for electromagnetic simulation. The proposed HIE-FDTD method is based upon the divergence relationship of electric fields, nearly reduces one field component, and realizes a memory reduction rate of 33% approximately. Two numerical experiments are carried out to validate the proposed method and the results indicate that the proposed memory-efficient HIE-FDTD method can work well.

Index Terms **–** Finite-difference time-domain (FDTD), hybrid implicit−explicit FDTD (HIE-FDTD), memory-efficient.

I. INTRODUCTION

Solving electromagnetic (EM) field is a necessary part in device design and analysis of EM phenomena, and many methods such as ray tracing method [1], scatter matrix method (SMM) [2–4], and Wentzel−Kramers−Brillouin (WKB) method [5] have been proposed. Methods with analytical approximation are relatively accurate and full of physical information, while their scopes are limited. As a result, numerical methods are also developed. The finite-difference time-domain (FDTD) method [6] has been regarded as one of the most effective and versatile methodologies [7, 8] mainly due to its direct temporal computing and simplicity. However, the explicit FDTD method is constrained by the Courant−Friedrich−Levy (CFL) stability condition [9]. As a result, when there are fine structures in the computation task, the FDTD method has to employ relatively small cell sizes and, thus, unavoidably takes a relatively small time step and consequently is confronted with heavy burden of long running time. In order to solve the issue, researchers resort to implicit schemes and have proposed a series of methods such as alternating-direction implicit (ADI) FDTD method [10, 11], locally one-dimensional (LOD) FDTD method [12, 13], Crank−Nicolson (CN) FDTD method [14, 15], weighted Laguerre polynomial (WLP) FDTD method [16, 17], and hybrid implicit−explicit (HIE) FDTD method [18, 19]. Among those methods, the ADI-FDTD method and the LOD-FDTD method both employ time split schemes and seem somewhat complex. The CN-FDTD method and the WLP-FDTD method both adopt fully implicit schemes and result in a huge sparse matrix which is expensive to handle. Whereas, the HIE-FDTD method only executes implicit difference schemes for the spatial partial derivatives in the direction along which there are fine structures and takes general explicit difference schemes for the remaining spatial partial derivatives in the directions along which there are no fine elements. In such an arrangement, the HIE-FDTD method finishes the restriction of the fine spatial cell sizes on time step size, acquires the ability to improve computational efficiency, and has drawn much attention in recent years [19–22]. Compared with the conventional FDTD method and even some other absolutely stable FDTD methods such as the ADI-FDTD method in some situations [23], the HIE-FDTD method showed higher efficiency, and a lot of work including but not limited to simulations of designing devices, implementations of PML, and reducing numerical dispersion error [19–22, 24] have been put forward.

Compared with the FDTD method, the HIE-FDTD method becomes more complex and needs more memory to implement, and, recently, the authors in [25] also point out that the character exists in some previous algorithms that employ implicit schemes. As a result, a form of HIE-FDTD method that is both free of strict CFL stability condition and is also of low memory requirements without bringing much complexity may be of value. In the paper [26], the authors proposed a scheme based on divergence relationship and achieved a memory reduction rate near to 33%. Then the thought of memory saving was adopted into some other situations [27–29] in different ways.

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In this paper, based on the work proposed in [18] and [26], a memory-efficient HIE-FDTD method is developed based on the divergence relationship of electric fields. It will be seen that the proposed HIE-FDTD method nearly only stores two field components in computation, reduces 33.33% of memory approximately, and maintains the accuracy and efficiency of the original method.

II. ALGORITHM FORMULATION

In simple, isotropic, and lossless media, according to [18], the HIE FDTD method is

$$E_x^{n+1} = E_x^n + \frac{\Delta t}{2\varepsilon}\frac{\partial\left(H_z^{n+1}+H_z^n\right)}{\partial y} \tag{1}$$

$$E_y^{n+1/2} = E_y^{n-1/2} - \frac{\Delta t}{\varepsilon}\frac{\partial H_z^n}{\partial x} \tag{2}$$

$$H_z^{n+1} = H_z^n + \frac{\Delta t}{2\mu}\frac{\partial\left(E_x^{n+1}+E_x^n\right)}{\partial y} - \frac{\Delta t}{\mu}\frac{\partial E_y^{n+1/2}}{\partial x}. \tag{3}$$

In order to acquire the solutions, a user either replaces H_z^{n+1} in eqn (1) with H_z^{n+1} in eqn (3), solves matrix equations, and acquires E_x^{n+1} or inserts eqn (1) into eqn (3), handles matrix equations, and solves H_z^{n+1} and, in the end, calculates the remaining field variables explicitly. We term them as HIE-E-FDTD method and HIE-H-FDTD method, respectively.

The divergence relationship of electric fields can be directly written as

$$\begin{aligned}&\frac{\partial E_x^{n+1}}{\partial x} + \frac{\partial E_y^{n+1/2}}{\partial y}\\ &= \frac{\partial E_x^n}{\partial x} + \frac{\partial E_y^{n-1/2}}{\partial y} + \frac{\Delta t}{2\varepsilon}\frac{\partial^2\left(H_z^{n+1}-H_z^n\right)}{\partial x\partial y}.\end{aligned} \tag{4}$$

Eqn (4) can also be rewritten as

$$\begin{aligned}&\frac{\partial E_x^{n+1}}{\partial x} + \frac{\partial E_y^{n+1/2}}{\partial y} - \frac{\Delta t}{2\varepsilon}\frac{\partial H_z^{n+1}}{\partial x\partial y}\\ &= \frac{\partial E_x^n}{\partial x} + \frac{\partial E_y^{n-1/2}}{\partial y} - \frac{\Delta t}{2\varepsilon}\frac{\partial^2 H_z^n}{\partial x\partial y}.\end{aligned} \tag{5}$$

Clearly, the divergence relationship is time invariant. The initial condition is 0; so eqn (5) can be further rewritten as

$$\frac{\partial E_x^n}{\partial x} + \frac{\partial E_y^{n-1/2}}{\partial y} - \frac{\Delta t}{2\varepsilon}\frac{\partial^2 H_z^n}{\partial x\partial y} = 0. \tag{6}$$

In fact, eqn (6) is just the result of linear combination of the two curl equations of electric field components in the HIE-FDTD method. In the region containing source, eqn (6) may fail; so one can directly apply the HIE-FDTD method without any change and construct linear equations to solve the fields or get the discrete divergence relationship by directly adding the numerical expressions of the two field components in the conventional HIE-FDTD method according to the regular form of divergence relationship.

Conductor (PEC) will also destroy eqn (6). On the surface of conductor, tangential E is 0; so eqn (6) is omitted. As to normal E, we recommend using the HIE-FDTD method on surface of conductor directly.

After applying finite difference approximation to spatial derivatives, the full numerical form of eqn (6) can be written as

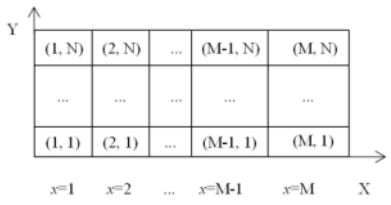
$$\begin{aligned}E_x^n(i,j) &= E_x^n(i-1,j) - \frac{\Delta x}{\Delta y}\left(E_y^{n-1/2}(i,j) - E_y^{n-1/2}(i,j-1)\right)\\ &+ \frac{\Delta t}{2\varepsilon}\left(\begin{array}{c}\dfrac{H_z^n(i,j)-H_z^n(i,j-1)}{\Delta y}\\ -\dfrac{H_z^n(i-1,j)-H_z^n(i-1,j-1)}{\Delta y}\end{array}\right).\end{aligned} \tag{7}$$

In the computation domain, cells with the same x coordinate are defined as a sub-region registered as $x = i$. In this paper, each sub-region is rectangle, and the domain consists of $M \times N$ cells and is grouped into many rectangles like $x = i - 1$, $x = i$, and $x = i + 1$, which is shown in Figure 1.

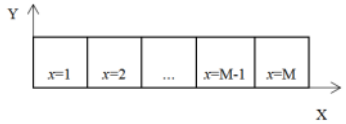
Fields in these regions are expressed as vectors and registered as $E_y(i)$, $E_x(i)$, and $H_z(i)$. From eqn (7), it can be seen that E_x^n in $x = i$ only requires E_x^n in $x = i - 1$ except $E_y^{n-1/2}$ in $x = i$ and H_z^n in $x = i$ and in $x = i - 1$. In this paper, E_y fields are at sides of cells in y direction, E_x fields are at sides of grids in x direction, and H_z fields are at the centers of each cell. As a result, eqn (7) can also be rewritten as a matrix form

$$E_x^n(i) = E_x^n(i-1) + A\,E_y^{n-1/2}(i) + B\,H_z^n(i) + C\,H_z^n(i-1) \tag{8}$$

where A, B, and C are matrices determined by coefficients in front of fields in eqn (7).



(a)



(b)

Fig. 1. (a) The computation domain with cells and sub-regions. (b) The computation domain with sub-regions.

Suppose $x = i - 1$ is the initial position where our calculation begins. We define $E_y(k)$ and $H_z(k)$ where k can be each rectangle in the whole domain. As to E_x, only two vectors e_{x1} that can store E_x^n in $x = i - 1$ and e_{x2} that has the memory sufficient to store E_x in $x = i$ are defined.

First, as e_{x1} is equal to E_x^n in $x = i - 1$, e_{x2} can be valued through eqn (8) and is equal to E_x^n in $x = i$. As E_x^n, $E_y^{n-1/2}$, and H_z^n are known in $x = i - 1$, $E_y^{n+1/2}$ and H_z^{n+1} can be solved by the conventional HIE-FDTD procedure in $x = i - 1$. Then e_{x1}=e_{x2} which is equal toE_x^n in $x = i$, and E_x^n in $x = i + 1$ can be valued by eqn (8) and can reuse the memory occupied by e_{x2}. And now as E_x^n, $E_y^{n-1/2}$ and H_z^n are known in $x = i$, and $E_y^{n+1/2}$ and H_z^{n+1} can be solved by the conventional HIE-FDTD procedure in $x = i$ again. Repeating this process to the last rectangle, it will be seen that all $E_y^{n+1/2}$ and H_z^{n+1} have been solved, and only two vectors that store E_x^n in each sub-region temporarily and are alternatively used are sufficient to finish the calculation, while in the conventional HIE-FDTD method, all field components are required to store. As a result, in the proposed method, one electric field component is nearly eliminated and the aim of memory reduction is realized in such a way.

A short pseudo-code is written below. Some terms in this content will be explained. In this part, $T_{\max}$ is the last time step, each value of x presents the rectangle corresponding to cells with the same coordinate of x in the computation domain, and $X_{\max}$ presents the last rectangle. e_{x1} and $e_{x2}$are both vectors, and E_x and H_z are matrices. t presents the current time step in iteration.

Pseudo-code of the proposed method:

Vector v stores $E_x^1(1)$(=0); $H_z^1 = 0$; $E_y^{1/2} = 0$.

for $t = 2{:}T_{\max}$

  $e_{x1} = v$;

  e_{x1} works as $E_x^{t-1}(1)$;

  $e_{x2} = e_{x1} + A E_y^{t-1-1/2}(2) + B H_z^{t-1}(2) + C H_z^{t-1}(1)$;

  e_{x2} works as $E_x^{t-1}(2)$;

  $E_y^{t-1/2}(1)$, $H_z^t(1)$, and $E_x^t(1)$ are solved by regular HIE FDTD procedure;

  $v = E_x^t(1)$;

  for i=2: $X_{\max}$-1

    $e_{x1} = e_{x2}$;

    e_{x1} works as $E_x^{t-1}(i)$;

    $e_{x2} = e_{x1} + A E_y^{t-1-1/2}(i+1) + B H_z^{t-1}(i+1) + C H_z^{t-1}(i)$;

    e_{x2} works as $E_x^{t-1}(i+1)$;

    $E_y^{t-1/2}(i)$ and $H_z^t(i)$ are solved by regular HIE-FDTD procedure;

  end

  $i = X_{\max}$;

  $E_y^{t-1/2}(i)$ and $H_z^t(i)$ are solved by regular HIE-FDTD procedure.

end

It must be pointed out that E_x and it with identifications of different time steps and space positions both in the description above and in the proposed method only indicate electric fields of x component and does not mean a matrix that is defined, occupies memory and stores fields covering the whole calculation domain.

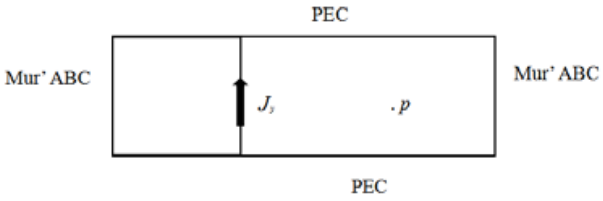
From the description above, it can be stated that in an arbitrary iteration at $(n + 1)$th time step, the proposed method does not need to store all the E_x^nfields beforehand; only two vectors e_{x1} and e_{x2} that are repeatedly used are sufficient for the run of the algorithm, and, in such a way, the memory reduction is realized. In this description, E_x are chosen as unknown variables in linear equations. For the case that H_z works as unknown variables, the algorithm is implemented in a similar way.

In fact, in the proposed method, at each time step, E_x appear in each sub-region by eqn (7) but do not need to store after fields in this region are solved. Supposing E_x in some regions are required, one only needs to store those solutions when the run goes through the local regions.

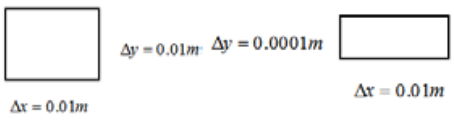
III. NUMERICAL VALIDATION

In order to validate the proposed method, two numerical experiments that both use 2D parallel plate waveguides [15] are carried out. The structures used in the two examples are shown in Figure 2.

The excitation for the two numerical tests taken from [11] is



(a)



(b) (c)

Fig. 2. (a) The structures in the two numerical examples, (b) one cell in the first example, and (c) one cell in the second example.

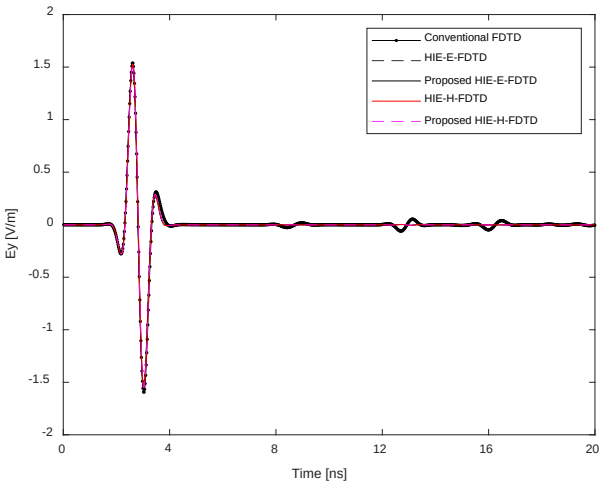


Fig. 3. Transient E_y at p point.

$$J_y(t) = \exp\left(-\left(\frac{t-T_c}{T_d}\right)^2\right)\sin\left(2\pi f_c(t-T_c)\right), \quad (9)$$

where

$$T_d = \frac{1}{2f_c}$$
$$T_c = 3T_d. \quad (10)$$

In the structure, p is the probe point recording fields at different time step, and $_{J_y}$ is the excitation current.

In the first example, there are 200 and 100 cells with the sizes set as $\Delta x = 0.01$ m and $\Delta y = 0.01$ m along x and y directions, respectively. The source is placed on line of x = 10 and the point p(50,5) is chosen as observation point recording E_y at each time step and the two sides of the extended waveguide are both truncated by first-order Mur' absorbing boundary condition. In this situation, according to the CFL stability condition, the largest time step sizes for the conventional FDTD method and the HIE-FDTD method are 23.57 and 33.33 ps, respectively. For the convenience of comparative analysis, the time steps for conventional FDTD and HIE-FDTD methods in this case are both simply set as 20 ps.

Figure 3 shows E_y at p point and Table 1 records the running time consumed by the FDTD method, HIE-FDTD method, and the proposed memory-efficient version of the later method with the uniform time step size. From Figure 3, we can see that the numerical results supplied by several algorithms are all in good agreement and, thus, validate the correctness of the new memory-efficient version of HIE-FDTD method. It can be seen that different implementations of the HIE-FDTD methods show a little difference in computation time. As the HIE-FDTD method is an implicit method, it costs more time than the conventional FDTD method when a uniform time step is used.

Table 1: Comparison of computation time of different methods with uniform time step size

FDTD methods	Time step	Iteration numbers	CPU time (s)
Conventional FDTD	20 ps	1000	1.23
Original HIE-E-FDTD	20 ps	1000	33.35
Proposed HIE-E-FDTD	20 ps	1000	33.76
Original HIE-H-FDTD	20 ps	1000	35.58
Proposed HIE-H-FDTD	20 ps	1000	35.71

In order to measure the accuracy of the proposed method, error $= \sqrt{\sum_{i=1}^{T}\left(E_y^i - E_{y,Ref}^i\right)^2 / \sum_{i=1}^{T}\left(E_{y,Ref}^i\right)^2}$ is defined as error function. In this function, T stands for the total number of time step iterations and equals 1000. The solutions of the original HIE-E-FDTD method and original HIE-H-FDTD method are set as reference solutions when the errors of two implementations of the proposed memory-efficient method are discussed. Setting the solution from the original HIE-E-FDTD method as standard, the error between the original HIE-E-FDTD method and the proposed HIE-E-FDTD method is 0. Setting the solution from the original HIE-H-FDTD method as standard, the error between the original HIE-H-FDTD method and the proposed original HIE-H-FDTD method is 3.59×10^{-15}. It can be seen that the errors are very small and show almost no difference. So the accuracy of the proposed method can be seen as the same as that of the original HIE-FDTD method.

In the second numerical example, we still employ the extended plate waveguide as test model mentioned above but fine the mesh size along y direction to 0.0001 m, and all the other conditions stay the same. The relevant results and time cost are shown in Figure 4 and Table 2.

Figure 4 still shows that the results calculated by the conventional FDTD method, HIE-FDTD method, and the proposed memory-efficient HIE-FDTD method are all in good agreement.

Table 2 represents the CPU time for the simulations run by different methods. In this situation, it can be seen that different implementations of the HIE-FDTD methods show a little difference in computation time. The proposed method takes almost the same time as the original HIE-FDTD method, while it is still much faster than the conventional FDTD method. As a result, the

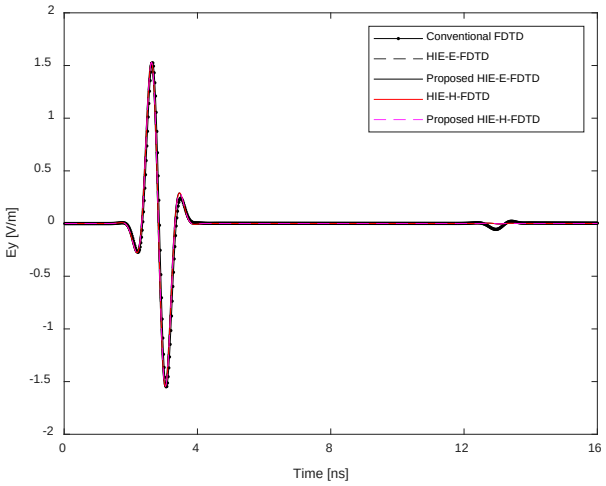


Fig. 4. Transient E_y at p point.

Table 2: Comparison of computation time of different methods with nonuniform time step sizes

FDTD methods	Time step	Iteration numbers	CPU time (s)
Conventional FDTD	0.2 ps	80,000	69.60
Original HIE-E-FDTD	20 ps	800	32.85
Proposed HIE-E-FDTD	20 ps	800	32.82
Original HIE-H-FDTD	20 ps	800	29.75
Proposed HIE-H-FDTD	20 ps	800	29.65

proposed memory-efficient HIE-FDTD method can both work well for problems with or without fine structures in one direction.

Using the same method of measuring the accuracy of the proposed method adopted in the first example, and setting the solution from the original HIE-E-FDTD method as standard, the error between the original HIE-E-FDTD method and the proposed HIE-E-FDTD method is 0. Setting the solution from the original HIE-H-FDTD method as standard, the error between the original HIE-H-FDTD method and the proposed HIE-H-FDTD method is 6.99×10^{-14}. In this function, T stands for the total number of time step iterations and equals 800. It can be seen that the errors both are very small and show almost no difference. So the accuracy of the proposed method is the same as that of the original HIE-FDTD method.

Table 3 shows the memory cost in different methods. As the numbers of cells in two numerical examples are

Table 3: Memory cost (KB) in different methods

	FDTD (KB)	HIE-E-FDTD	HIE-H-FDTD	Proposed HIE-E-FDTD	Proposed HIE-H-FDTD
A	468.75	468.75	468.75	314.06	314.06
B	468.75	468.75	468.75	314.06	314.06

equal, the memory of storing fields they require is also equal and listed in Table 3 where A presents the first numerical example and B presents the second one. It is clear that $1-312.50/468{,}075 \approx 33\%$; so the memory reduction rate is near to 33%.

IV. CONCLUSION

The analytic and semi-analytical methods are accurate and can also show physical aspects of a system explicitly. At the same time, numerical methods are also developed and are available in a wider range. Numerical methods require more memory than analytic methods in most situations. The HIE-FDTD method shows higher computation efficiency than the FDTD method in problems with fine elements in one direction. In this paper, efforts are made to reduce memory cost and a memory-efficient HIE FDTD method is proposed based on divergence relationship of electric fields. The proposed algorithm nearly eliminates one electric field component, saves nearly 33% of memory, and the implementation of the proposed method is not much more complex than the conventional HIE-FDTD method. Numerical experiments are carried out and validate that the proposed memory-efficient HIE-FDTD method can solve EM fields correctly and runs almost as fast as the original HIE-FDTD method, and in those situations, the computational efficiency can be interpreted as unchanged. The accuracy of the proposed memory-efficient HIE-FDTD method is also very close to that of the original HIE-FDTD method.

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