Macro-Modeling of Electromagnetic Domains Exhibiting Geometric and Material Uncertainty

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Abstract — A methodology is presented for the development of stochastic electromagnetic macro-models for domains exhibiting geometric and material uncertainty. Focusing on the case of domains exhibiting geometric/material invariance along one of the axes of the reference coordinate system, the methodology makes use of the theory of polynomial chaos expansion and the concept of a global impedance/admittance matrix relationship defined over a circular surface enclosing the cross-sectional geometry of the domain of interest. The result is a stochastic global impedance/admittance matrix, defined on the enclosing circular surface, whose elements are truncated polynomial chaos expansions over the random space defined by the independent random variables that parameterize the geometric and material uncertainty inside the domain. Use is made of sparse Smolyak grids to reduce the computational cost of constructing the stochastic macro-model. Numerical examples are used to demonstrate some of the attributes of the proposed stochastic macro-models to the numerical solution of electromagnetic scattering problems by an ensemble of cylindrical targets exhibiting uncertainty in their shape and relative positioning.

Index Terms — Finite elements, global impedance matrix, macro-modeling, polynomial chaos expansion, random geometry, scattering/RCS.

I. INTRODUCTION

Over the years the term macro-modeling has been used in scientific and engineering modeling and simulation to describe a variety of things. In the context of electromagnetic (EM) modeling and simulation, macro-modeling is widely understood to mean the process through which a compact physical or mathematical model is defined to describe the EM attributes of a portion of the system, the detailed description of which requires a large number of degrees of freedom (state variables) for its modeling. In this context, low-order, EM macro-models have been used extensively for a variety of applications. These include, expedient calculation of the broadband response of passive EM devices; use of domain decomposition techniques for the EM field modeling of electrically-large structures of high complexity; and the abstraction of distributed portions of composite systems that include both lumped circuit components and distributed electromagnetic structures.

For the purposes of this paper, we are interested in the macro-modeling of portions of linear, passive EM structures that exhibit geometric and/or material uncertainty. In particular, we are interested in structures formed as an ensemble of multiple domains, with the aforementioned material and geometric uncertainty occurring in some or all of these domains but not in the medium in which these domains are immersed. Figure 1 depicts a representative example of such a composite structure.

The problem of electromagnetic modeling of structures that exhibit randomness is one of significant interest to the electromagnetics community because of its relevance to several application domains such as remote sensing, EMI/EMC in electronic systems, and EM wave propagation in random media. For the case of EM wave scattering by composite random structures like the one depicted in Fig. 1, the complexity of a Monte Carlo numerical solution is compounded by the need to

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generate a discrete numerical model for each one of the geometries resulting from the sampling of the multi-dimensional random space defining the randomness of the structure. For example, in the context of the finite element solution of the EM boundary value problem (BVP), a new finite element grid needs to be generated for each one of the realizations of the geometry during the Monte Carlo sampling of the random space. An alternative approach, presented by Miller [1], makes use of an adaptive methodology capable of constraining an estimated radiation or scattering pattern to satisfy an uncertainty specification by employing appropriate fitting models to minimize the required number of samples.

In this paper, a macro-modeling methodology is proposed as a means of alleviate the repeated discretization of the computational domain in the numerical solution of the stochastic EM BVP. The proposed methodology makes use of the mathematical framework of polynomial chaos expansions and stochastic collocation [2– 6], which has been applied recently to the numerical solution of a variety of EM BVPs (see [7–11] for representative examples). These are combined with the concept of network matrix representation of passive EM structures to develop a compact stochastic impedance (or admittance) matrix macro-model on a fixed boundary enclosing each one of the domains that exhibits randomness. In this manner, only a single numerical grid is needed for the Monte Carlo solution of the EM scattering by the ensemble of the random domains.

The proposed approach is described in Section II. Section III presents examples from the application of the method to the two-dimensional scattering by arrays of conducting cylinders exhibiting geometric randomness. The paper concludes with some remarks on future extensions of the method.

II. STOCHASTIC MACRO-MODELING

As suggested in the introduction, the electromagnetic structures of interest to this discussion are composite structures comprising several subdomains, with a good number of them exhibiting uncertainty in their material and/or geometric composition. The EM analysis of such a structure using, for example, a Monte Carlo (MC) process, requires the development of as many FE/FD models (including the generation of an FE/FD mesh for each model) as the samples in the random space used in the MC process. One way to reduce the associated computational cost is by removing the need for the repeated mesh generation. The way stochastic macro-modeling makes this possible is demonstrated in this section.

To fix ideas, we will consider the case of electromagnetic wave scattering by a collection of targets embedded in an unbounded linear host medium. While the geometric and material attributes of each one of the targets exhibit a statistically defined randomness, the host medium does not. Figure 1 serves as a representative example of such a structure. Any randomness in geometric attributes or material properties occurs only inside the N regions $V_1, V_2, \cdots V_N$ bounded by surfaces $S_1, S_2, \cdots S_N$, respectively. The exterior medium, including the volume bounded by surface S_0, is assumed to be fixed in terms of its geometric attributes and its electromagnetic properties. In view of this, it is immediate apparent that, under the assumption that the N surfaces $S_1, S_2, \cdots S_N$ are fixed, the domain V_E bounded by these N surfaces and the surface at infinity is a fixed domain free from any geometric or material uncertainty. The way the randomness of the interiors of the domains $V_n, n = 1, 2, \cdots, N$, manifests itself in the solution of the exterior BVP in V_E is through the boundary conditions on the surfaces. This, then, suggests the idea of a stochastic macro-model for each one of the N subdomains $V_n, n = 1, 2, \cdots, N$, in terms of a global surface impedance relationship on $S_n, n = 1, 2, \cdots, N$. The way this is done is described next.

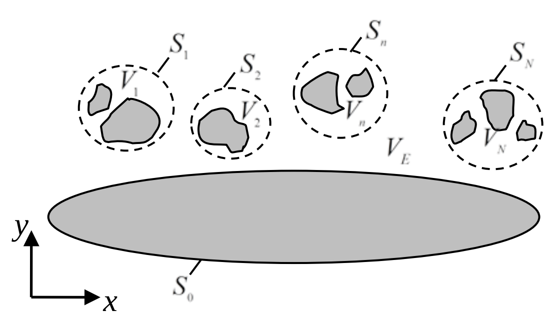

Fig. 1. Reference geometry for the discussion of the concept of stochastic macro-modeling.

A. Global impedance matrix

For the purposes of this discussion, let the geometry of Fig. 1 be the cross-sectional geometry of an infinitely long cylindrical scatterer. Under the assumption that both the cross-sectional geometry of the scatterer and the exciting electromagnetic field are z invariant, the pertinent electromagnetic boundary value problem is a two-dimensional one. Focusing on the case of TE_z polarization, where the magnetic field is linearly polarized in the z direction, we assume, without loss of generality, that S_n is a circle. A local reference coordinate system is introduced, with origin the center of the circular boundary S_n. Fourier series expansions in the polar angle ϕ are used to represent the tangential electric and tangential magnetic fields on S_n. For each Fourier mode in the expansion of the tangential magnetic field on S_n the solution of the interior BVP in V_n yields a tangential electric field on S_n. In this manner, a global impedance condition is established on S_n, defined in terms of the matrix relationship,

$$\xi_k = \sum_{m=-M}^{M} Z_{km}^{(n)} h_m ,\; k = 0, \pm 1, ..., \pm M \; , \tag{1}$$

where ξ_m, h_m are, respectively, the coefficients in the Fourier series expansions of the tangential electric field and the tangential magnetic field on S_n,

$$E_\phi \approx \sum_{m=-M}^{M} \xi_m e^{jm\phi},\; H_z \approx \sum_{m=-M}^{M} h_m e^{jm\phi}. \tag{2}$$

The truncation of the expansions in (2) is necessary for the numerical implementation of (1). The important observation here is that the global impedance matrix, $\mathbf{Z}^{(n)}$, defined through (1), (2), serves as an electromagnetic macro-model for the region V_n. Once the impedance matrices for all domains $V_n, n = 1, 2, \cdots, N,$ are available, the solution to the exterior electromagnetic BVP in V_E due to an arbitrary excitation at the frequency of interest is computed in a straightforward fashion. In the presence of geometric and/or material uncertainty in V_n, the elements of the global impedance matrix $\mathbf{Z}^{(n)}$ can be used to account for the impact of the randomness of the region to the electromagnetic response of the overall structure. By abstracting the randomness of the interior region on the global impedance matrix defined on a fixed boundary, a single numerical grid is necessary for the solution of the exterior BVP. In the next subsection, a process is described for abstracting the randomness in the geometric and/or material properties of the region V_n to the elements of the global impedance matrix $\mathbf{Z}^{(n)}$ on the fixed boundary S_n.

B. Stochastic global impedance matrix

Let $\boldsymbol{\chi} = (\chi_1, \chi_2, \cdots, \chi_D)$ denote the set of independent random variables necessary for describing the uncertainty in V_n. Furthermore, let $\rho(\boldsymbol{\chi})$ denote their joint probability density function. The objective is to develop a systematic and expedient process for obtaining a global impedance matrix $\mathbf{Z}^{(n)}$ that serves as an accurate macro-model of the electromagnetic attributes of V_n for any point in the D-dimensional probability space Ω defined by $\boldsymbol{\chi} = (\chi_1, \chi_2, \cdots, \chi_D)$.

Toward this objective, use is made of the machinery of polynomial chaos expansion of random functions in Ω. Following the ideas in [4], a truncated polynomial chaos expansion of $Z_{km}^{(n)}$ is of the form

$$Z_{km}^{(n)} \approx \sum_{i=0}^{Q} c_i \Psi_i (\boldsymbol{\chi}), \tag{3}$$

where $\Psi_i(\boldsymbol{\chi})$ are multidimensional orthogonal polynomials with regard to the inner product,

$$\langle \Psi_i, \Psi_j \rangle \equiv \int_\Omega \Psi_i(\boldsymbol{\chi}) \Psi_j(\boldsymbol{\chi}) \rho(\boldsymbol{\chi}) d\boldsymbol{\chi} = \delta_{ij} \left\| \Psi_i \right\|^2 . \tag{4}$$

The type of random variables dictates the family of the polynomials to be used [6]. For example, for the case of Gaussian random variables, Hermite polynomials are used. The number of terms, Q, included in the truncated polynomial chaos expansion depends on the dimensionality D of the random space and the highest order p of the multi-dimensional polynomials used, and is given by

$$Q + 1 = \frac{(D+p)!}{D!\,p!}. \tag{5}$$

In view of (3), the coefficients in the polynomial chaos approximation of $Z_{km}^{(n)}$ are computed using the orthogonality relation (4),

$$c_i = \frac{1}{\left\| \Psi_i \right\|^2} \int_\Omega \rho(\boldsymbol{\chi}) \Psi_i(\boldsymbol{\chi}) Z_{km}^{(n)}(\boldsymbol{\chi}) d\boldsymbol{\chi} . \tag{6}$$

Clearly, the expedient calculation of the integral in (6) calls for an efficient multivariate quadrature rule on Ω. For example, use of the Smolyak algorithm [12] leads to the approximation of (6) through the summation,

$$c_i \approx \frac{1}{\left\|\Psi_i\right\|^2}\sum_{r=1}^{R}\rho(\boldsymbol{\chi}_r)\Psi_i(\boldsymbol{\chi}_r)Z_{km}(\boldsymbol{\chi}_r)w_r, \qquad (7)$$

where the number of nodes, R, is significantly less than the one required by a tensor product rule. The selection of the quadrature points, their weights w_r, and the level of accuracy that dictates the sparsity of the Smolyak grid are well documented in the literature and will not be repeated here (see [12–14] for details).

Equations (3) and (7) define the desired stochastic global impedance matrix macro-model of the random domain V_n. The process for its construction is summarized in terms of the following algorithm.

C. Algorithm

The stochastic global impedance matrix macro-modeling of domain V_n bounded by S_n is summarized next.

1. Choose the dimension of the global impedance matrix and, hence, the number of Fourier modes used in the expansion of the tangential electric and magnetic fields on S_n.
2. Represent geometric/material randomness in terms of D independent random variables $\boldsymbol{\chi} = (\chi_1, \chi_2, \cdots, \chi_D)$.
3. Choose polynomial family and order for truncated polynomial chaos expansion.
4. Generate Smolyak grid on probability space Ω defined by $\boldsymbol{\chi} = (\chi_1, \chi_2, \cdots, \chi_D)$.
5. For each point $\boldsymbol{\chi}_r, r = 1, 2, \cdots, R$, on the Smolyak grid, solve the deterministic interior BVP to obtain $\mathbf{Z}^{(n)}(\boldsymbol{\chi}_r)$.
6. Using the matrices obtained in Step 5, calculate the coefficients in the polynomial chaos expansion of $\mathbf{Z}^{(n)}$ using (7).

D. Solution of the exterior stochastic BVP

Once the stochastic global impedance matrices on the fixed circular boundaries $S_n, n = 1, 2, \cdots, N$, have been constructed, the numerical solution of the electromagnetic scattering problem by the union of the N+1 targets, $V_n, n = 0, 1, 2, \cdots, N$, amounts to solving an exterior electromagnetic BVP in V_E. As already stated, since the circular boundaries are fixed, the finite element solution of this exterior BVP requires a single numerical grid. The randomness of each one of the N regions manifests itself in terms of the polynomial chaos expansions of the elements of its stochastic impedance matrix. With D_n denoting the number of independent random variables used to parameterize the uncertainty in V_n, the dimension of the random space Ω_E for the exterior stochastic BVP is $D_E = \sum_{n=1}^{N} D_n$. Irrespective of the process used for the solution of the exterior stochastic BVP, the global impedance matrix on each one of the circular boundaries is readily computed Ω_E from its polynomial chaos expansion for each sample in the random space.

III. NUMERICAL VALIDATION AND DEMONSTRATION STUDIES

In this section, several numerical examples involving electromagnetic wave scattering by arrays of infinitely long cylinders are used to validate the proposed methodology and demonstrate its key attributes.

We begin with the problem of TE_z time-harmonic uniform plane wave scattering by a perfect electric cylindrical conductor of circular cross section and of random radius, $a = 0.80\ (1+\chi)$ m, where χ is a Gaussian random variable with zero mean and standard variation of 0.06 m. The cylinder is immersed in free space and its axis coincides with the z axis of the reference coordinate system. The amplitude of the incident magnetic field is 1 A/m and its angular frequency is 9×10^8 rad/s. The availability of an analytic solution for this problem makes possible the use of a standard Monte Carlo analysis to calculate the reference solution for the statistics of the scattered magnetic field on a circle of radius 1.2 m centered at the origin. Use of 10^4 sampling points in the Monte Carlo process yielded an accuracy of 10^{-5} in the calculation of the mean value of the magnitude of the scattered magnetic field.

Next, the problem was solved making use of the stochastic global impedance condition defined over the circle of radius 1.2 m. The polynomial chaos approximation of the elements of the impedance matrix is in terms of Hermite polynomials up to third order. The dimension of the impedance matrix is 11. Since the dimension of the random space is 1, the Smolyak grid reduces to a simple Gaussian quadrature rule. For accuracy level of 5, a Smolyak involving 5 grid points is required for the calculation of the integrals in (7). Since the random space for the exterior stochastic BVP is the same with that for the interior, the same Smolyak grid used for the construction of the global impedance matrix is used for solving the exterior stochastic BVP. The mean and variance of the magnitude of the scattered magnetic field thus computed are compared with those obtained analytically in Fig. 2. The error bars represent a ±3σ deviation with respect to the mean. Very good agreement is observed. More specifically, the average error in the mean value of the magnitude of the scattered magnetic field between the analytical and the numerical solution, averaged over all angles, is 1.20%.

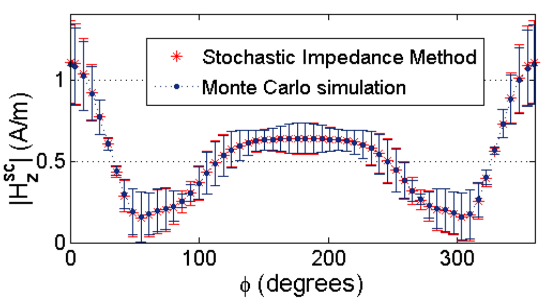

Fig. 2. Mean value of the magnitude of the scattered magnetic field. Error bars represent ±3σ deviation from the mean.

Next, the case of TE_z wave scattering by an array of four elliptical cylinders is considered. All cylinders are perfect electric conductors, and the background medium is free space. The angular frequency of the excitation is 9×10^8 rad/s. In the absence of any statistical variability, the centers of the four cylinders coincide with the vertices of a square of side 2.4 m (see Fig. 3). The randomness in the cross-sectional geometry is introduced through a set of four independent random variables for each cylinder. Two of them, χ_1, χ_2, are associated with the lengths $2a$ and $2b$ of the major axis (along the x axis) and minor axis (along the y axis), respectively, of the elliptical cylinder. More specifically, with the two random variables taken to be Gaussian of mean value of 0 and standard deviation of 0.025, the lengths of the two axes (in meters) are given by

$$2a(\chi_1) = 1.4(1+\chi_1), \quad 2b(\chi_2) = 1.4(1+\chi_2). \quad (8)$$

The other two random variables, χ_x, χ_y, control the random displacement of the center of the cylinder from the vertex of the reference square. The position of the center is given by

$$\vec{r}_c = \vec{r}_o + \hat{x}\chi_x + \hat{y}\chi_y, \quad (9)$$

where $\vec{r}_o$ denotes the corresponding position of the vertex of the square. Variables χ_x and χ_y are Gaussian random variables of zero mean value and standard deviation of 0.025 m. Even though the same four variables are being used to quantify the geometric uncertainty for each one of the four cylinders, when considering the four-cylinder array, the four sets of random variables are assumed to be independent. Thus, the randomness of the cross-sectional geometry is parameterized in terms of 16 independent random variables.

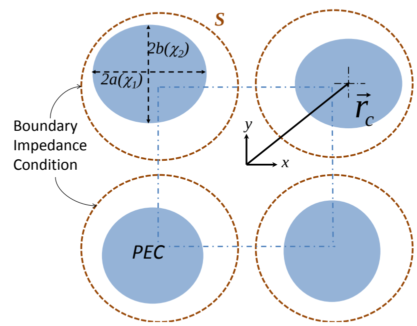

Fig. 3. Array of four PEC elliptical cylinders with random axes lengths and random positions.

Depicted in Fig. 3 are the four fixed circular boundaries on which stochastic global impedance boundary conditions will be defined, one for each one of the four cylinders. The center for each circle coincides with corresponding vertex of the reference square formed by the unperturbed centers of the four cylinders in the array. The radius of each circle is such that the cylinder associated with it remains enclosed by it for all points in the four-dimensional domain in the random space defined by the random variables χ_1, χ_2, χ_x, χ_y. For this specific example, this radius was taken to be 1 m. Given that the random variables are Gaussian distributions, orthogonal Hermite polynomials are used for the polynomial chaos expansion of the elements of the stochastic impedance matrix. The calculation of the coefficients in the polynomial chaos expansion using polynomials of up to second order is carried out efficiently through the use of a Smolyak sparse grid on the four-dimensional random space. More specifically, for the case considered here, a Kronrod–Patterson rule [13], [14] of accuracy level 5 was used, resulting in 201 points on the four-dimensional random space. For each point on the Smolyak grid a finite element solution of the interior BVP was used to calculate a global impedance matrix of dimension 21. The computed matrices were subsequently used for the calculation of the coefficients in the polynomial chaos expansion of the stochastic impedance matrix making use of (7).

With the stochastic global impedance matrix available on each one of the four circular boundaries, a finite element model was used for the solution of the exterior stochastic BVP with excitation by a uniform, time-harmonic plane wave propagating in the $+x$ direction with magnetic field amplitude of 1 A/m. Since the circular stochastic impedance boundaries are fixed, a single finite element mesh is needed for the discretization of the geometry. The only changes to the finite element matrix are those associated with the specific values of the stochastic impedance matrices on the four boundaries for each sample realization in the 16-dimensional random space. Rather than a standard Monte Carlo process, a Smolyak sparse grid of accuracy level 3 was used to extract the statistics of the scattered fields and the radar cross section. The number of points in the sparse Smolyak grid is 513. The mean and variance of the output parameters are computed by performing the integration over the random space. These integrals are approximated by weighted summations of the scattered field, computed at each one of the 513 nodes as previously described,

$$\left\langle \left| H_z^{sc}(\rho,\varphi) \right| \right\rangle \approx \sum_{r=1}^{513} \left| H_z^{sc}(\rho,\varphi,\boldsymbol{\chi}_r) \right| \rho(\boldsymbol{\chi}_r) w_r \quad (10)$$

$$\operatorname{var}\left(\left| H_z^{sc}(\rho,\varphi) \right| \right) \approx \sum_{r=1}^{513} \left| H_z^{sc}(\rho,\varphi,\boldsymbol{\chi}_r) \right|^2 \rho(\boldsymbol{\chi}_r) w_r - \left\langle \left| H_z^{sc}(\rho,\varphi) \right| \right\rangle^2 . \quad (11)$$

The calculated scattered magnetic field, sampled on a circle of radius 3.4 m centered at the center of the reference square defined by the unperturbed centers of the four cylinders is depicted in Fig. 4. More specifically, shown in the figure is the mean value of the magnitude of the scattered magnetic field along with error bars that indicate a ±3σ deviation from the mean value. The radar cross section (RCS) is depicted in Fig. 5. Again, the mean value is plotted, along with error bars that denote ±3σ deviation from the mean.

The final numerical study considers the case where the polarization of the excitation is TM_z with the electric field linearly polarized along the z axis. For this case and in view of the fact that for the two-dimensional BVP considered the governing equation is the scalar Helmholtz equation for the z component of the magnetic field, a global admittance matrix is used instead of a global impedance matrix. The global admittance matrix relates the Fourier coefficients in the expansion of the tangential magnetic field on each one of the circular boundaries S_n to the Fourier coefficients in the expansion of the tangential electric field.

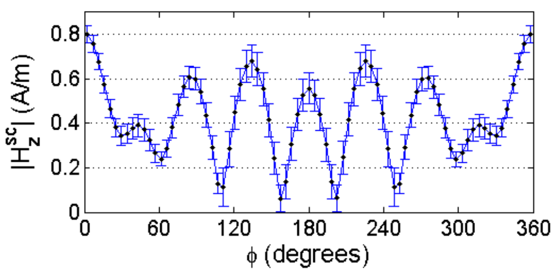

Fig. 4. Mean value of the magnitude of the scattered magnetic field on a circle of radium 3.8 m enclosing the four cylinders. Error bars represent ±3σ deviation from the mean.

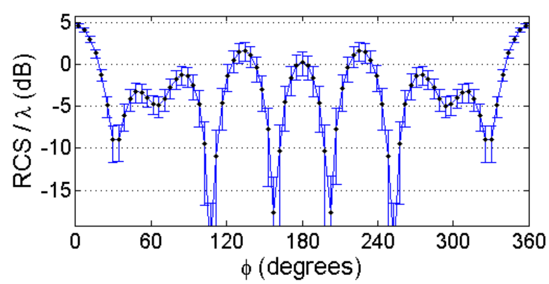

Fig. 5. Mean value of the radiation cross section (RCS) of the magnetic field. Error bars represent ±3σ deviation from the mean.

For this case, the four-cylinder array depicted in Fig. 3 is illuminated by a time-harmonic, line current source of current phasor of 1 A, angular frequency 9×10^8 rad/s, and placed at position (3, 0) m with its axis parallel to the z axis. The development of the global stochastic admittance matrix, used on each one of the circular boundaries, was carried out following the same choices for Hermite polynomial chaos expansion order and Smolyak grid accuracy level as in the computation of the impedance matrix for the TE_z case.

Using expressions (10) and (11), the mean and the variance of the magnitude of the *z*-component of the scattered electric field were computed. Depicted in Fig. 6 is the mean value of the magnitude of the scattered electric field recorded on a circle of radius 3.4 m with its center at the center of the reference square of the four-cylinder array configuration. Also depicted in the figure are error bars indicating ±3σ deviation from the mean. The large deviation obtained in the forward scattering direction (on the side of the array where the line source is placed) is attributed to the close proximity of the source to the cylinders. Also depicted in Fig. 7 is the mean value of the magnitude of the total electric field for a distance in the far-field region. Again, the error bars represent ±3σ deviation from the mean.

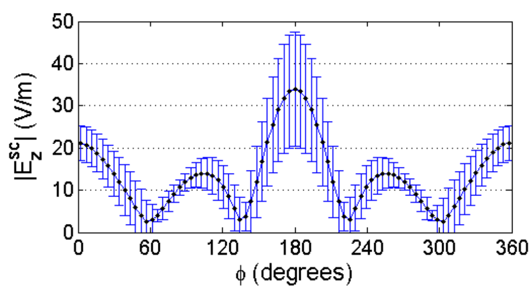

Fig. 6. Magnitude of the scattered electric field on a circle of radius 3.4 m. Error bars represent ±3σ deviation from the mean.

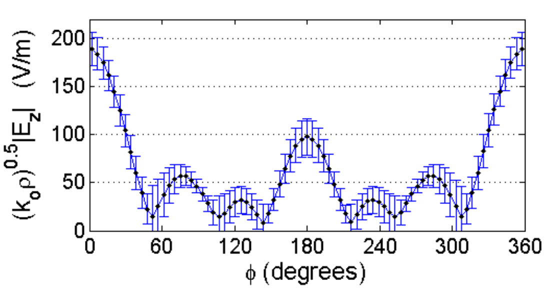

Fig. 7. Mean value of the magnitude of the far electric field at $\rho = 1000/k_o$. Error bars represent ±3σ deviation from the mean.

IV. CONCLUSION

In conclusion, a methodology was proposed and numerically demonstrated for the development of stochastic macro-models of sub-domains of a complex electromagnetic structure exhibiting geometric and/or material randomness. Under the assumption that the randomness inside the sub-domain is parameterized in terms of a set of independent random variables, the proposed methodology abstracts the randomness in the sub-domain in terms of a stochastic global impedance or admittance matrix defined on a fixed surface enclosing the sub-domain. The elements of the matrix are given in terms of truncated polynomial chaos expansions on the random space defined by the independent random variables.

As demonstrated through the numerical examples presented, use of such stochastic macro-models alleviates the computational complexity of the solution of the random scattering problem by eliminating the need for the repeated numerical discretization (e.g., the repeated mesh generation) for the entire structure for each sampling point in the Monte Carlo process.

While the proposed methodology was presented in the context of two-dimensional EM scattering, its extension to three dimensions is rather straightforward. For example, for the case of an ensemble of multiple three-dimensional objects, global stochastic impedance boundary conditions can be defined on spherical surfaces enclosing each object. This extension is currently under development.

Another extension of the proposed macro-modeling involves the case where the elements of the stochastic global impedance matrix are functions of frequency. As already demonstrated in [10, 11], this extension generalizes the concept of stochastic global impedance macro-modeling – in a manner consistent with the concept of network matrix representation of passive EM multi-ports – to provide for a broadband stochastic macro-model of a portion of a composite structure exhibiting material and/or geometric randomness. Results from on-going research on these topics will appear in forthcoming papers.

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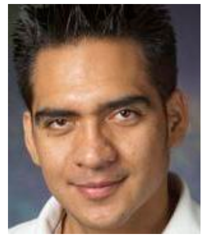

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