Finite Difference Time Domain Diakoptic Strategies

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Abstract — In many applications, it may be advisable to "tear apart" the computational domain into several sub-domains separated by "seams," each one treated separately. The sub-domains are then sewn back together at appropriate stages of the computation. Three main diakoptic strategies have been developed in the recent past. Out of these, the diakoptics on-the-fly strategy can serve the purpose of parallelizing a FDTD process over several processors, each of which being responsible for the treatment of a certain sub-domain, as presented below.

Keywords: FDTD, diakoptics, domain decomposition, absorbing boundary conditions.

I. INTRODUCTION

Many electromagnetic scattering problems are best solved by a *diakoptic* approach; that is, by tearing them into smaller sub-problems, exciting each sub-problem with a variety of different excitations, and combining the results. The approach is facilitated by situating each sub-problem in its own sub-domain, and characterizing the "seams" between domains as boundary conditions (BCs). The diakoptic approach is most useful in two very different types of problems: problems involving tightly packed inhomogeneous sub-domains with strong interactions between them, and problems involving sub-domains separated by oceans of "white space."

Diakoptic schemes already have been successfully incorporated into frequency domain methods, e.g., [1–3] (the fast multiple method (FMM), when applied to large dense matrices [4], may also be considered as a frequency domain diakoptic method). They also hold great promise in the time domain (TD), where any sub-domain or a given group of sub-domains can be characterized by a grid-compatible Green's function over their bounding surface, as a basis for repeated computations of changes in the complementary sub-domains, e.g., for layered media [5]. In this context, a strategy presented in Section II has been recently proposed. Two more strategies, namely FDTD coupled with stabilized TD integral equations and diakoptics-on-the-fly, are presented Sections III and IV, respectively. Conclusions are drawn in Section V.

II. GREEN'S FUNCTION TYPE DIAKOPTICS

This strategy is useful in an iterative design process. The computational domain is decomposed into a fixed "basic scatterer" and a reduced computational domain to be optimized in the course of the design process (see Fig. 1). A Green's function is used to connect the sub-domains via a surface representation over the interface. A salient characteristic of this strategy is the usage of differential equation approach to generate grid-compatible time domain Green's functions that serve to represent sub-domains over the interface.

Subsets of this strategy are as follows.

A. The green's function method (GFM)

The GFM [6,7] (see Fig. 2(a)) leads to a field, or spatial domain procedure, analogous to cascaded

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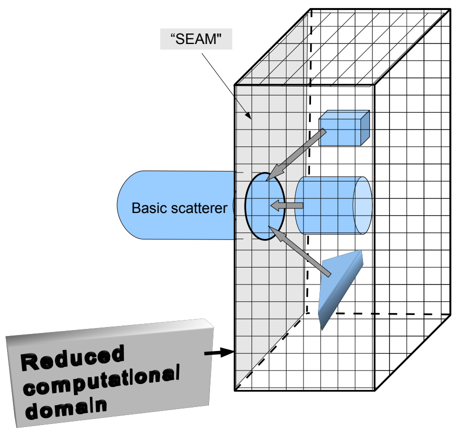


Fig. 1. A basic scatterer with different attachment, suitable for usage in a diakoptic scheme with a grid-based Green's function (see Section II-A).

impedance matrix representation in microwave circuit theory.

B. Spectral representation

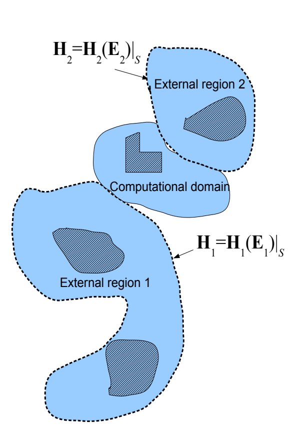
This decomposition (Fig. 2(b)) is analogous to cascaded scattering matrices. Here the field is resolved into outgoing and incoming harmonics across the seam. An issue with this formulation is the restriction to convex and separable seams that limits the control of white space.

C. Source decomposition method (SDM)

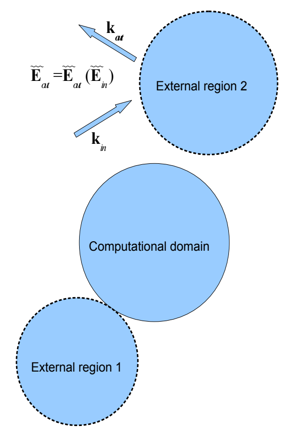
The SDM [8] (Fig. 2(c)) combines advantages of the other two strategies. In one dimension, it coincides with the spectral approach, however in two or three dimensions it avoids the need for using separable geometries.

III. FDTD COUPLED WITH STABILIZED TD INTEGRAL EQUATIONS

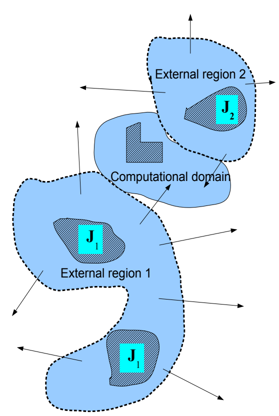
This work is combined with the stabilizing method of D. Weile [9], who identifed the source of instabilities in the manner of the spatial integration employed, in particular, in the way numerical integration rules approximated the integrals of functions that suddenly vanished on the boundary between illuminated and shadow regions. Stabilization is achieved through an approximated $\mathcal{Z}$-domain formulation that maps the left half of the s-plane into the unit circle in the z-plane. This recursive marching-on-in-time scheme can be merged with the FDTD, providing a stable integral equation interface at the seam.



(a) GFM



(b) Spectral representation



(c) Source decomposition

Fig. 2. Three grid-compatible Green's function approaches (see Introduction).

IV. DIAKOPTICS ON THE FLY

This strategy is motivated by parallel FDTD analysis. In this strategy, we tear apart and recombine

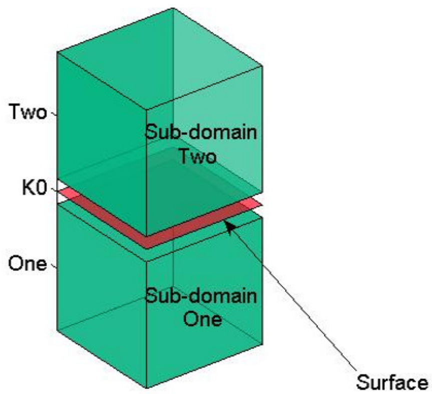


Fig. 3. Diakoptics-on-the-fly: two sub-domains with a planar seam.

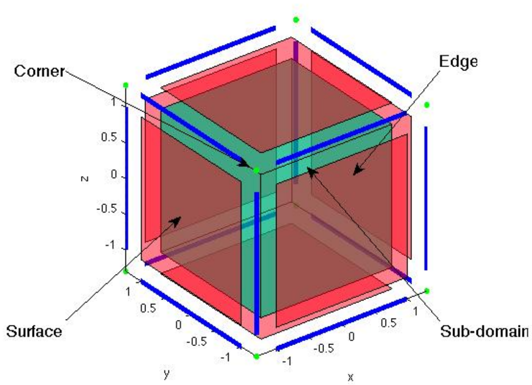
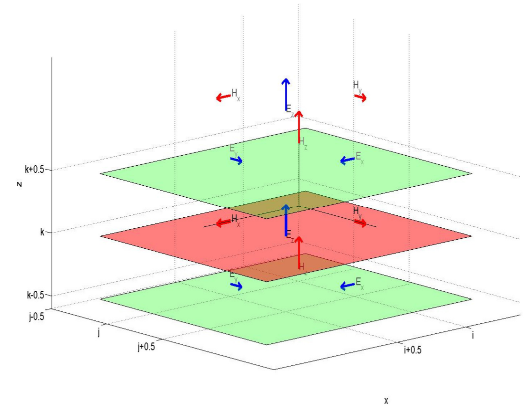


Fig. 4. The making of a seam: surfaces, edges, corners.

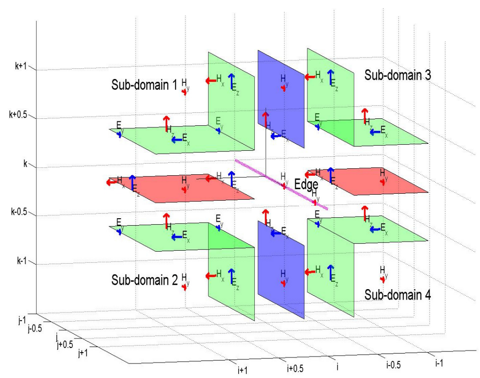
the sub-domains at each time step or after each small group of time steps (see Sections IV-A and IV-B, respectively). This strategy opens the door for optimization of the use of different clusters of processors, both CPUs and GPUs. The individual groups can be treated in parallel by separate processors, or sequentially by the same processor.

A. Update process: sewing at each time step

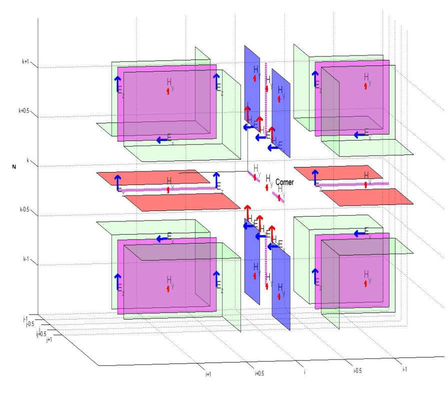
Each FDTD update sequence is divided into the following two phases: In the first phase, that can



(a) Surface.



(b) Edge.



(c) Corner.

Fig. 5. The surface, edges and corners interact with two, four and eight neighboring sub-domains, respectively.

be run concurrently or sequentially for all sub-domains, the field at the the seam is considered known, acting as a boundary condition for the update of the field within each sub-domain. The second phase is the update at the seam using the known fields at the sub-domain boundaries. This second phase also includes separate updates for the edges and corners of the seam, involving four and eight neighboring sub-domains, respectively.

The seam is an appropriate Yee grid plane. If, e.g., the sub-domains "1" and "2" are aligned along the x axis (see Fig. 3), and the seam is at an E_z-field surface at $i+\frac{1}{2}$, then adjacent $\mathbf{H}$-field surfaces are the bounding surfaces of sub-domains 1 and 2 at i and $i+1$, respectively. Divide now the FDTD update sequence into the following two phases: separate updates for sub-domains 1 and 2, followed by a "sewing" update phase.

1) The first phase: updates within the sub-domains: This phase can be run concurrently or sequentially for the two sub-domains. The field at the seam is considered a BC for the update of the $\mathbf{H}$-field at the surfaces for i, $i+1$:

$$\left.\begin{array}{l}\frac{\gamma_x}{\eta}E^n_{z(i+\frac{1}{2},j+\frac{1}{2},k)} \Rightarrow H^{n+\frac{1}{2}}_{y(i,j+\frac{1}{2},k)}\\ \frac{\gamma_x}{\eta}E^n_{y(i+\frac{1}{2},j,k+\frac{1}{2})} \Rightarrow H^{n+\frac{1}{2}}_{z(i,j,k+\frac{1}{2})}\end{array}\right\}\text{sub-domain 1,}$$

$$\left.\begin{array}{l}\frac{\gamma_x}{\eta}E^n_{z(i+\frac{1}{2},j+\frac{1}{2},k)} \Rightarrow H^{n+\frac{1}{2}}_{y(i+1,j+\frac{1}{2},k)}\\ \frac{\gamma_x}{\eta}E^n_{y(i+\frac{1}{2},j,k+\frac{1}{2})} \Rightarrow H^{n+\frac{1}{2}}_{z(i+1,j,k+\frac{1}{2})}\end{array}\right\}\text{sub-domain 2.} \tag{1}$$

2) The second phase: Update at the Seam: Update the field at the seam using the known $\mathbf{H}$-fields at i and $i+1$:

$$\begin{aligned}&\eta\gamma_x\left(H^{n+\frac{1}{2}}_{z(i,j,k+\frac{1}{2})} - H^{n+\frac{1}{2}}_{z(i+1,j,k+\frac{1}{2})}\right)\\ &\qquad\Rightarrow E^{n+1}_{y(i+\frac{1}{2},j,k+\frac{1}{2})}\\ &\eta\gamma_x\left(H^{n+\frac{1}{2}}_{y(i+1,j+\frac{1}{2},k)} - H^{n+\frac{1}{2}}_{y(i,j+\frac{1}{2},k)}\right)\\ &\qquad\Rightarrow E^{n+1}_{z(i+\frac{1}{2},j+\frac{1}{2},k)}.\end{aligned} \tag{2}$$

This phase also includes separate updates for the edges and corners of the seam, see Fig. 4. These are shown in Fig. 4. The update process for the surfaces, edges and corners that involve interaction with two, four and eight neighboring sub-domain, as can be seen in Figs. 5(a)–5(c), respectively.

B. Update process - sewing after several time steps

One disadvantage of the procedure in Section IV-A is the need to halt the volume calculations after each time step and allocate "overhead" time for data transfer into the separate processes of seam updates and back. This problem can be avoided, albeit at the cost of increasing the computational size at each sub-domain, by performing seam updates after each group of N time steps, as follows.

Consider the two sub-domains in Fig. 6(a). Extend each sub-domain into the region of its neighbor, as depicted by the blue boxes in Fig. 6(b). This box protrudes into the adjacent sub-domain, as seen in the red boxes in Fig. 6(c). Fill this extension with zeros, and compute the field of each sub-domain separately over N time steps, where N is chosen such that the signal would not traverse more then the length of the extension. While the main body of the sub-domain (green boxes) includes the full solution at the N^{th} time step, the protrusions (red and blue boxes) have a partial solution for the given sub-domain . A superposition of the partial solutions from adjacent sub-domain then produces the full solution. The entire process is then repeated for the next N time steps.

V. CONCLUSIONS

The diakoptic approach has the potential to reduce computer requirements for a large class of problems. An advantage is seen in situations including the following:

(a) Splitting computational effort between processes by using diakoptics-on-the-fly;

(b) Design processes, where the computational domain undergoes several modifications while the external boundary remains unchanged, as depicted in Fig. 3;

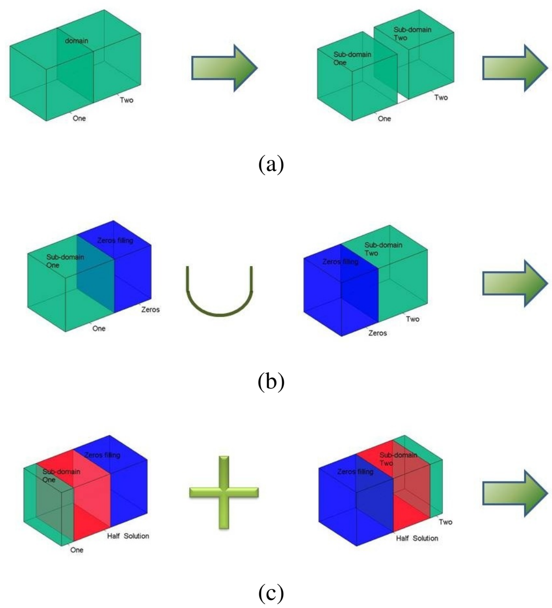


Fig. 6. Parallel FDTD computation with update after N time steps. (a) Two adjacent sub-domains, analyzed in parallel, (b) Zero padded extensions (blue boxes) of each sub-domain. The extensions protrude into the adjacent sub-domain, (c) Superposition of the overlapping solutions over each sub-domain, that includes the green and red boxes.

(c) White space elimination, in particular with non-convex scatterers. Typical absorbing boundary conditions (ABCs) require sometimes excessively large "white space" between the scatterer and the ABC boundary to ensure accuracy because standard ABCs must be applied on convex boundaries. ABCs that track the shape of the scatterer can minimize the size of the white space and allow for the inclusion of reflective external domains;

(d) Multiple-scatterer scenarios for cases where the scatterers are distinct and separated by a substantial "white space." The proposed scheme will allow the sub-problems to interact analytically over homogeneous domains;

(e) Problems best analyzed using sub-problems of different grid sizes and are solvable with methods other than FDTD, or involve moving objects.

Challenges, however, involve

(a) efficiency, since direct computations of Green's functions sometimes involve series with huge terms of alternating signs and

(b) instabilities that may occur when combining methods for boundary conditions.

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