# An Electromagnetic Compatibility Problem via Unscented Transform and Stochastic Collocation Methods

**S. Lalléchère [1,2], P. Bonnet [1,2], I. El Baba [1,2], and F. Paladian [1,2]**

[1] Clermont University
Blaise Pascal University, LASMEA, Clermont-Ferrand, 63000, France

[2] CNRS
UMR6602, LASMEA, Aubière, 63177, France
sebastien.lallechere@lasmea.univ-bpclermont.fr, pierre.bonnet@lasmea.univ-bpclermont.fr,
ibrahim.elbaba@lasmea.univ-bpclermont.fr, francoise.paladian@lasmea.univ-bpclermont.fr

***Abstract*** **─** This paper aims to illustrate the current interest about the use of stochastic techniques for electromagnetic compatibility (EMC) issues. This problem may be handled from various methods. First, we may focus on the Monte Carlo (MC) formalism but other techniques have been implemented more recently (the unscented transform, UT, or stochastic collocation, SC, for instance). This work deals with solving a stochastic EMC problem (transmission line) with the UT and SC techniques and to compare them with the reference MC results.

***Index Terms*** **─** Sensibility analysis, stochastic collocation, stochastic electromagnetic compatibility, unscented transform.

## I. INTRODUCTION

Nowadays, although one may note the growing interest of the electromagnetic (EM) community for measurement techniques and numerical codes with increasing accuracy, the trend is to improve their efficiency by optimizing them. Most of the computational works in electromagnetics remain deterministic (i.e., one single result per set of exact input data). Although the uncertainties are intrinsic in EM analysis (slight variations from large production, environmental factors, reproducibility drifts), very few studies are achieved to efficiently take them into account. One of the most spread techniques relies on Monte Carlo (MC) simulations [1]. The aim of this work is to focus on two additional methods which present the simplicity of MC method with faster convergence rates. Thus, the unscented transform (UT) method [2, 3] and the stochastic collocation (SC) technique [4] will be detailed from their foundations to an electromagnetic compatibility (EMC) application.

## II. THEORETICAL BASIS

Many methods are used to take uncertainties into account in EM simulations; one of the simplest ways to do so may remain as the use of MC developments. Some probabilistic techniques are also available: we can mention, without exhaustiveness, the polynomial chaos [5] or the kriging technique [6]. In this article, we focus on the interest of the UT [2] and SC [4] methods to solve EMC problems. They are two non-intrusive methods whose main advantages are simplicity and efficiency. Contrary to a classical MC simulation, well known for its slow convergence rate, the collocation methods are computationally very interesting since they need a limited number of well-chosen points related to the distribution of the random variables (RV). Thus, the UT and SC techniques share many similarities; they only differ regarding two or more random parameters. In our model, a stochastic parameter $Z$ will be defined according to a RV $\hat{u}$ following

$$Z = Z^0 + \hat{u}, \qquad (1)$$

with $Z^0$ the initial value (mean) and without any loss of generality $\hat{u}$ follows a certain distribution

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law (zero-mean and a given variance). The random value $Z$ may stand for different parameters: material characteristics (dielectric), geometry (for instance size or location of a target) or the source parameters (magnitude, frequency ...).

**A. UT principles**

As explained in [3], the UT method is similar to the MC technique. The main difference relies on the number of realizations needed to obtain the statistical moments of a given output. Instead of several thousands of repetitions, only a few selected ones are necessary.

**A single RV UT case.** Some conditions are required to compute UT for a single RV: we may know both the moments of the RV $\hat{u}$ and the nonlinear mapping of the random output. Its *n*th order moment may be expressed as follows:

$$E\{I(\hat{u})^n\} = \int I(u)^n pdf(u) du, \qquad (2)$$

where *pdf(u)* is the probability density function of the RV $\hat{u}$. A discrete equivalent of the relation (2) is used for the integration

$$\int I(u)^n pdf(u) du \approx \sum_i \omega_i I(S_i)^n, \qquad (3)$$

where $S_i$ are the so-called sigma points (for the integration). If the nonlinear mapping $I(\hat{u})$ is well behaved, it could be expressed from Taylor polynomial series ($g_j$ coefficients) as

$$\int I(u)^n pdf(u) du = \sum_{j=0}^{\infty} g_j \int u^j pdf(u) du. \qquad (4)$$

From the discrete sum (3), each integration term of (4) may be expressed from $k$+1 ($k$=0, 1 ...) equations as

$$\int u^k pdf(u) du \approx \sum_i \omega_i S_i^k = E(\hat{u}^k). \qquad (5)$$

The nonlinear system depicted in (5) allows the computation of the sigma points $S_i$ and weights $\omega_i$ from the moments of the RV $\hat{u}$. As detailed in [2], the minimum number of $S_i$ points for a given order of the UT technique may be derived using the Gauss quadrature schemes. Indeed, considering (5), the solution is not unique and different sets of $(S_i, \omega_i)$ may be obtained as illustrated in the following.

**UT for multi-RV.** Based upon the results obtained for a single RV, the Taylor polynomial representation is still suitable for two RV. In the two variables case, the system (5) may be written using statistical moments cross terms [2] following

$$\sum_i \omega_i \left(S_i^1\right)^m \left(S_i^2\right)^n = E\{\hat{u}_1^m \hat{u}_2^n\}. \qquad (6)$$

The sigma points and weights are computed for the two RV $\hat{u}_1$ and $\hat{u}_2$ and derived from the possible power combinations of $m$ and $n$ (natural numbers) with $0 \leq m + n \leq 4$. Once more there are several possible solutions: in the following we will give different sets of sigma points and weights solving the system (6).

**B. SC foundations**

This section is dedicated to the presentation of the SC technique [4].

**SC basis for a single random parameter.** The idea of the technique is to find a polynomial approximation of a given output $I$ depending on a random parameter $Z$ (1). In a first time, the function $S \to I(Z^0; S)$ is split up a Lagrangian basis with $n$ the approximation order

$$I(Z^0; S) \approx \sum_{i=0}^{n} I_i(Z^0) L_i(S), \qquad (7)$$

with $L_i(S) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{S - S_j}{S_i - S_j}$. One of the most interesting properties of the Lagrangian basis relies on its reducing characteristic: $L_i(S_j) = \delta_{ij}$ (Kronecker $\delta$) and we may write $I_i(Z^0) = I(Z^0; S_i)$. Then, the integration computation is based upon the Gauss quadrature with identical points $S_i$ than the ones previously needed by the SC method

$$\int_D pdf(u) f(u) du \approx \sum_{i=0}^{n} \omega_i f(S_i). \qquad (8)$$

Similarly to the UT case, the real numbers $\omega_i$ are called integration weights. From (7), we may detail $I$ with its polynomial approximation

$$E\{I(Z^0; S)\} = \sum_{i=0}^{n} I_i(Z^0) \int_D L_i(s) pdf(s) ds. \qquad (9)$$

From (9), we may straightforward compute weights following

$$\omega_i = \int_D L_i(s) pdf(s)\, ds. \qquad (10)$$

We will detail in Section II-D the statistical moments computation enabled by the pair ($S_i$, $\omega_i$).

**Multi-RV SC case.** The previous theoretical elements may be generalized to the multivariate case. Therefore, by considering a two-variable random problem, for instance involving two RV $\hat{u}_1$ and $\hat{u}_2$ standing for two random parameters, respectively $Y$ and $Z$, we may write $Y$ and $Z$ from the relation (1) including two initial values ($Y^0$ and $Z^0$) and potentially two random distributions. From the same theoretical foundations, we may project the function $(s, t) \to I(Y^0, Z^0; s_i, t_j)$ on a Lagrangian basis

$$I(Y^0, Z^0; s_i, t_j) \approx \sum_{i=0}^{n} \sum_{j=0}^{n} I_{ij}(Y^0, Z^0) L_i(s) L_j(t), \qquad (11)$$

with $I_{ij}(Y^0, Z^0) = I(Y^0, Z^0; s_i, t_j)$. It is rather simple from (11) to compute the moments of the output $I$ through a tensor product in each direction (i.e., for each RV) based upon the generalization of (8). Comparatively to MC, the technique may appear limited when the number of RV increases. Other methods exist [7, 8] to ensure efficiency with a good level of accuracy but for few RV, UT, and SC reveal particularly precise.

### C. Sigma points/weights computation

Although the UT and SC appear very similar considering the computation of their respective sigma points/weights, they differ from their basis. From the different solutions proposed in one-variable case, the minimum number of ($S_i$, $\omega_i$) pairs (for a given order $n$) is straightforward available by the Gauss quadrature scheme (identical to the SC case [4, 9]). Therefore, the expression of the integration points/weights is similar for the UT [2] and SC [10], and the results will be identical. The Table 1 gives an overview of the points/weights in single RV case following a standard normal distribution. We may construct similarly the multi-RV set of points/weights for UT and SC. Based upon [2], it is possible to extend the previous set of points (Table 1) to the numerical examples presented in the following (one or two RV) including the distribution law variance. As depicted in [2], the UT solution is not unique when solving the system (6) and we may obtain (for a same order) different sets of sigma/weights points.

Table 1: Sigma/weights points (one RV) for a standard normal law (UT/SC)

| *n* | | Pt1 | Pt2 | Pt3 | Pt4 | Pt5 |
|---|---|---|---|---|---|---|
| 2 | $S_i$ | $-\sqrt{3}$ | 0 | $\sqrt{3}$ | | |
| | $\omega_i$ | 1/6 | 2/3 | 1/6 | | |
| 4 | $S_i$ | -2.9 | -1.4 | 0 | 1.4 | 2.9 |
| | $\omega_i$ | 0.01 | 0.22 | 0.53 | 0.22 | 0.01 |

### D. Calculation of the statistical moments

The collocation technique gives the collocation points ($S_i$) and weights ($\omega_i$) necessary to entirely compute the $n$th-order ($n$=1, 2 ...) $Z$ statistical moments [10]. From previous notations ($E\{Z\}$ for instance), in order to simplify the discussion, the brackets $\langle Z \rangle$ and $\sigma_Z$ symbol will stand respectively for the mean and standard deviation of the $Z$ output.

Table 2: Statistical moments computation with the UT/SC for one-dimensional RV case

| Moment | *n* | Computation |
|---|---|---|
| Mean | 1 | $\langle \boldsymbol{Z} \rangle = \sum_{i=0}^{n} \boldsymbol{\omega_i S_i}$ |
| Variance | 2 | $\sigma_Z^2 = \sum_{i=0}^{n} \omega_i S_i^2 - \langle Z \rangle^2$ |

The Table 2 gives the computation of the first and second statistical moments from UT/SC (one RV). The computation of the sigma points and weights (multi-RV) may be found respectively in [2, 9]. In the following, we will present the numerical differences existing between the UT and SC.

## III. A STOCHASTIC EMC PROBLEM

The case of a simple transmission line of diameter $d$, at frequency $f$, length $L$, placed at a height $h$ above an infinite ground plane and illuminated by a uniform linearly polarized plane wave is considered (Fig. 1). An analytical formulation can be obtained for the current $I(L)$ at load $Z_L$=1$k\Omega$ [6], $Z_0$ is set to 50Ω.

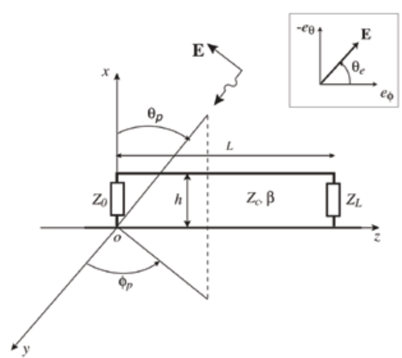


Fig. 1. Transmission line striken by a plane wave.

### A. Accuracy of the UT and SC methods (single RV case)

Both for the UT and SC, the accuracy of the stochastic techniques presented in this paper depends on the order $n$ of the approximation as presented in (8). Thus, in the following, we will talk about UT and SC “order $n$” which means a polynomial approximation with $n$+1 pairs ($S_i$, $\omega_i$).

Using the Gauss quadrature to compute weights and sigma points leads to a similar ($S_i$, $\omega_i$) set both for the UT and SC. The single variable uncertainty is first introduced considering $L$. The line length is associated to a RV $\hat{u}$ following a normal distribution (zero-mean, variance=0.053); different $L^0$ mean values are regarded in the set $D_L$=[1.2m; 4.5m]. In this example, the values of parameters are set following: $d$=1mm, $f$=50MHz, $h$=20mm, with initial electric field amplitude $E_0$=1$k$V/m, a normal striking is considered here.

**Statistical moments from stochastic study.** The Fig. 2 clearly shows the convergence of the stochastic techniques (UT/SC). Indeed, from the 3/5/7-points discretization (i.e., respectively $n$=2/4/6), the $\langle I \rangle$ curves almost overlap. For $n$=2, the different ($S_i$; $\omega_i$) points may be summarized following (-0.4; 1/6), (0; 2/3) and (0.4; 1/6). As expected from Table 2, the 2nd order statistical moment is also computed from UT/SC. The current $I$ standard deviation ($\sigma_I$) also appears through the numerical dispersion around the mean value $\langle I \rangle$. The 2nd order results ($\sigma_I$) agree well regarding the UT/SC 5 and 7-points accuracy and check the 2nd order convergence. Obviously, it may have been expected that the 2nd order convergence requires a higher accuracy from stochastic treatments. Moreover, one may put the focus on the interest of these stochastic formalisms. A faster approach may have been to only consider the mean value (central value $L^0 \in D_L$): it is given on Fig. 3 by the pink curve. It may be noticed that the central data gives a trend of the uncertainty impact but does not fit well to the converged stochastic (UT/SC) behavior. The differences appearing (for a relatively weak randomness uncertainty) between a single deterministic simulation (mean value) and UT/SC results ($\langle I \rangle \pm \sigma_i$ overruns) confirm the importance of the stochastic modeling.

Figure 3 gives an overview of the convergence of the UT/SC methods comparatively with a MC reference. In order to compute the relative error due to stochastic computing, we consider a converged set of MC data (here with 100,000 realizations). Then, a criterion is defined standing for this relative error

$$err_i = 100 \times \frac{\left|z_i^{UT/SC} - z_i^{MC}\right|}{\left|z_i^{MC}\right|}, \qquad (12)$$

where $i$ corresponds to the considered random length $L_i$ ($L_i \in D_L$) and $z^{UT/SC}$, $z^{MC}$ stand for a given statistical moment, for instance mean or variance (standard deviation), of the random output (current $I$) obtained respectively from UT/SC and MC. The results from Fig. 3 show the convergence of the stochastic formulations since respectively less than 0.25% and 2% errors appear from the current mean and standard deviation computations.

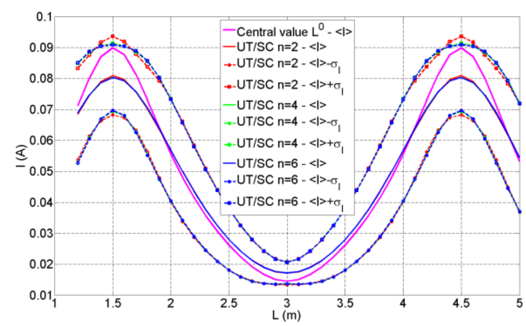


Fig. 2. $\langle I \rangle \pm \sigma_I$ from UT / SC convergence (n=2, 4, 6) for normally distributed randomness ($\sigma_{\hat{u}} = 0.231$).

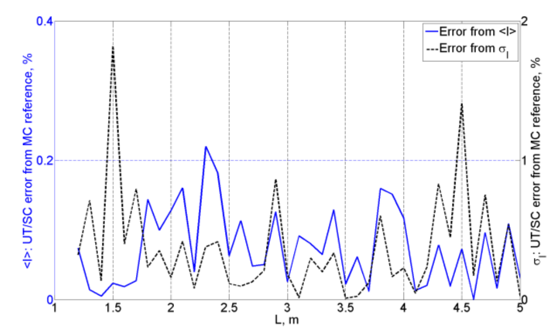


Fig. 3. Relative error between SC and 100,000 MC realizations (reference) with $\sigma_{\hat{u}} = 0.231$.

**High-order statistical moments.** As detailed in Table 2, both the UT and SC techniques allow computing straightforward high-order moments. The Fig. 4 shows the kurtosis convergence of the UT/SC techniques comparing to MC data. Therefore, although the results from 5-points UT/SC are in a good agreement with reference (MC), we may need an accuracy level from 7-points ($n$=6) UT/SC to fit with reference. The same kind of conclusion could be obtained from the skewness (3rd order) computation. Of course, for higher statistical moments needs, the UT/SC will require higher precision.

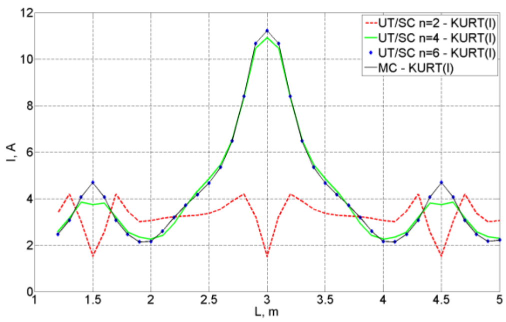


Fig. 4. Stochastic convergence from UT/SC and 100,000 MC realizations ($\sigma_{\hat{u}} = 0.231$).

**B. Stochastic EMC expectations**

The aim of this part is to illustrate the ability of the SC method to handle various randomness patterns (distribution, intensity ...) needed for EMC problems.

**Variations around statistical distributions.** In this case, we focus on two additional random distributions: uniform and log-normal ones (Figs. 5 and 6). The mean and variance of the example from Section III-A remain unchanged. For instance, comparatively to previous case, the pairs ($S_i$; $\omega_i$) are (-0.31; 0.28), (0; 0.44) and (0.31; 0.28) for a uniform law and $n$=2. The example of the log-normal law needs ($S_i$; $\omega_i$) following (-0.39; 1/6), (-0.01; 2/3) and (0.41; 1/6).

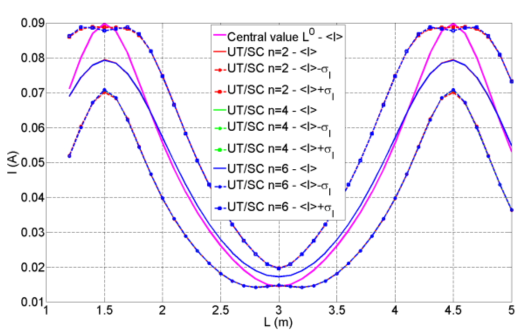


Fig. 5. Convergence (n=2, 4, 6) with a uniform random distribution, $\langle\hat{u}\rangle = 0$ and $\sigma_{\hat{u}} = 0.231$.

The SC convergence is obtained from 5-points ($n$=4). Even if some slight differences appear both on the levels and the convergence rate, the SC method allows modeling different kind of uncertainties. The randomness distributed from a log-normal law seems to require a different SC order to converge (Fig. 6) comparatively to normal or uniform distributions.

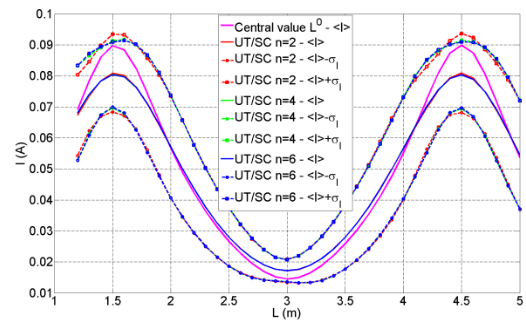


Fig. 6. Convergence (n=2, 4, 6) with a log-normal random law, $\langle\hat{u}\rangle = 0$ and $\sigma_{\hat{u}} = 0.231$.

**Randomness intensity and convergence.** Obviously, increasing the level of uncertainty of the studied random parameter will lead the analysis close to a "working" threshold. For a given problem, the SC method will operate well until a certain boundary. This may be easily understood since we are talking about stochastic and random parameters far from a complete parametric study. After all, we may wonder if the SC method is robust. In the following, we consider that $\hat{u}$ follows a uniform law U[-1;1] ($\langle\hat{u}\rangle = 0$ and $\sigma_{\hat{u}} = 0.577$). For $n$=2, the different ($S_i$; $\omega_i$) points may be summarized following (-0.78; 0.28), (0; 0.44) and (0.78; 0.28). In comparison with Fig. 5, the results from Fig. 7 show the convergence of the SC technique. Obviously, it is more difficult since the magnitude of variations is huge ($\pm$ 1m over the line length for each initial value $L^0$). As expected, the central data (Fig. 7, pink curve) does not fit at all with the SC mean computation. Even if the variations are great due to randomness, the Fig. 8 shows the well accuracy of SC results in comparison with 100,000 MC realizations. Indeed, $\langle I\rangle$ and $\sigma_I$ are close to reference data: the SC relative error from (12) is respectively lower than 0.45% and 1.5% for the current mean and standard deviation (Fig. 8). The lengths $L_1$=1.5m and $L_2$=4.5m are particular points (resonances). As depicted in Fig. 3 and Fig. 8, the highest error levels are obtained for $L_1$ and $L_2$ (both around 1.5%) according to the current standard deviation.

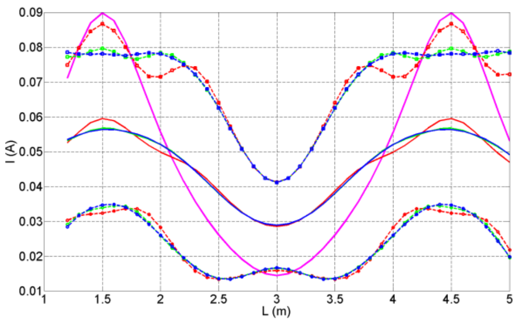


Fig. 7. Convergence (n=2, 4, 6) with a uniform random distribution ($\sigma_{\hat{u}} = 0.577$, legend Fig. 5).

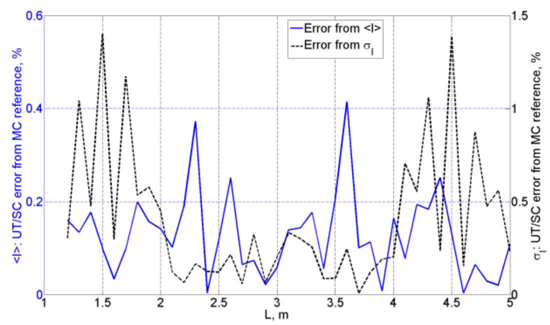


Fig. 8. Relative error between SC and 100,000 MC realizations for $\langle I \rangle$ and $\sigma_I$ (randomness following a uniform distribution U[-1;1]).

### C. A multivariate random problem

Since the Taylor polynomial expansion is still usable for two RV, the UT may be used to achieve multivariate stochastic problem (see Section II-A). Moreover, as illustrated in [10], the single-variable SC technique may be generalized to multi-RV problems. Based upon their distinct foundations, for the multivariate case, the different two RV ($S_i$, $\omega_i$) sets jointly with the different moments computation involve variations around the numerical results. In this section, we will add the source frequency *f* to the line length *L* to achieve a stochastic treatment of the EMC problem (Fig. 1). Both *L* and *f* will be given by two independent RV ($\hat{u}_1$ and $\hat{u}_2$) following a normal distribution (zero-mean). The variances are respectively given by $\sigma^2_{\hat{u}_1} = 2.083.10^{-2}$ and $\sigma^2_{\hat{u}_2} = 2.083.10^{10}$. Based upon the relation (1), we may write

$$L = L^0 + \hat{u}_1$$
$$f = f^0 + \hat{u}_2, \quad (13)$$

where $f^0$ and $L^0$ stand respectively for the *f* and *L* initial values. As depicted in Fig. 9, the current variance *var(I)* is calculated in a straightforward manner for a large set of points. Thus, each element of the set $\{L_i^0 \epsilon D_L = [1.2; 4.5]\text{m}; f_i^0 \epsilon D_f = [1; 35]MHz\}$ is subjected to the previous random variations. The SC convergence and sensibility of the model appears in Fig. 9. The results depicted show the convergence of the SC method (from the *I* variance). Considering the SC accuracy for $3^2$ and $5^2$ points (respectively SC3 and SC5), the two data sets almost overlap. Convergence is obtained considering the current mean and the SC technique approximates well the random behaviour of the system.

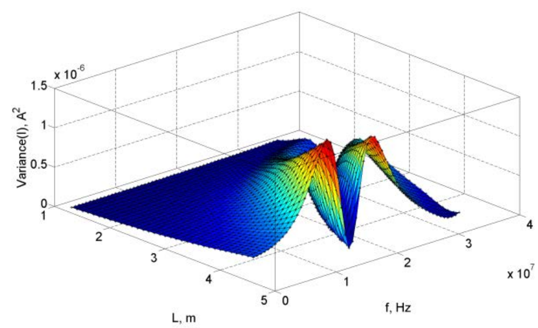


Fig. 9. Variance(I) from SC3 and SC5 (respectively colored slice and black asterisks).

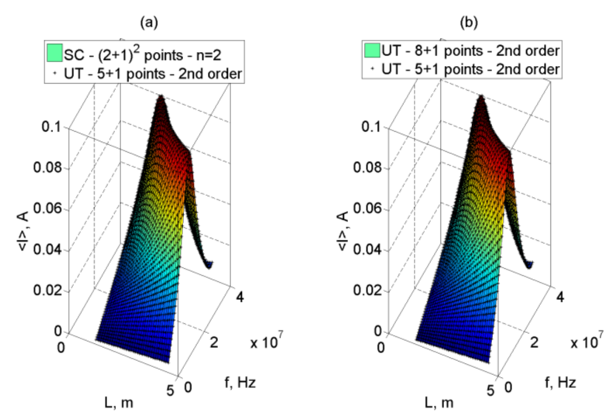


Fig. 10. Mean(I) from SC3, UT5 and UT8.

In Fig. 10, the results from UT fit very well with the "converged" data from SC. Thus, Fig. 10-a shows the agreement for $\langle I \rangle$ between SC3 and the UT approximation (2nd order) involving 6 (=5+1) points (UT5). A great agreement appears also from Fig. 10-b considering the slight differences existing between a same UT accuracy (2nd order) involving 5+1 or 8+1 points (respectively UT5 or UT8). The UT5 and UT8 differences rely on the non-uniqueness of the solution in (6). In order to properly define the accuracy of each stochastic formalism (UT/SC), one may refer to MC simulations. First, it is necessary to determine a reference set of $\langle I \rangle$ values: empirically, 100,000 MC realizations are necessary. Relying on the relation (12) obtained for a single RV, we may define for each pair ($f_k$, $L_k$) ($k$=1... $N_p$ with $N_p$ the total number of frequency/length points) a similar parameter in bidimensional RV case.

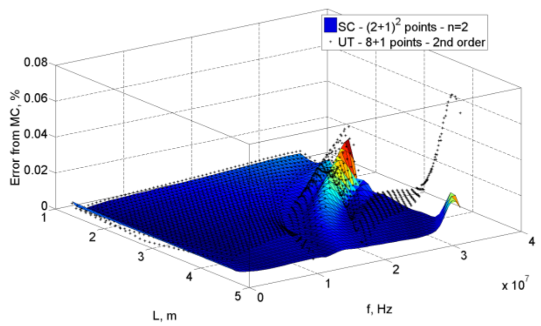


Fig. 11. Mean(I) error from MC data for SC3/UT8.

Figure 11 shows the gap between SC3 and UT8 accuracies. Even if the SC/UT numerical precision is high (less than 0.08% from MC reference), the precision remains widely better for SC (3 times) comparatively to UT. The time and memory saves appear clearly from previous examples since UT and SC need less than 10 realizations compared to the MC technique which requires about 100,000 simulations. Furthermore, it would be possible to improve the SC efficiency using techniques from [11] to reduce the number of SC realizations needed; it could be particularly interesting for multivariate stochastic problems involving many RV [12]. Another solution may be to reduce this number to a minimum regarding their relative influence. From [2], the comparison of results from 1-RV simulation with those involving a set of RV provides information on significant parameters.

**D. A random sensibility analysis**

This part illustrates the SC ability to achieve sensibility analysis in a random EMC problem. Among all the variables depicted on Fig. 1, we will focus on the parameters $h$ and $f$. The other values are given by the previous example except the line length $L$ which is set to 1.65m. An influence criterion is defined in [2] to characterize the sensibility of one RV. Based upon the SC results, a similar parameter is

$$In_{Z_k} = -\log\left(\left|1 - \frac{var(I(Z_k))}{var(I(Z_1, Z_2, \ldots, Z_n))}\right|\right), \tag{14}$$

with $var(I(Z_k))$ and $var(I(Z_1, Z_2, \ldots, Z_n))$ the current $I$ variances given respectively from one RV $Z_k$ ($k=1, \ldots, n$) and $n$ RV. The two random outputs are given considering two RV (RV1 $\rightarrow h$ and RV2 $\rightarrow f$), respectively $\hat{u}_1$ and $\hat{u}_2$ both following a uniform distribution with $\sigma^2_{\hat{u}_1} = 2.083.10^{-10}$ and $\sigma^2_{\hat{u}_2} = 2.083.10^{10}$.

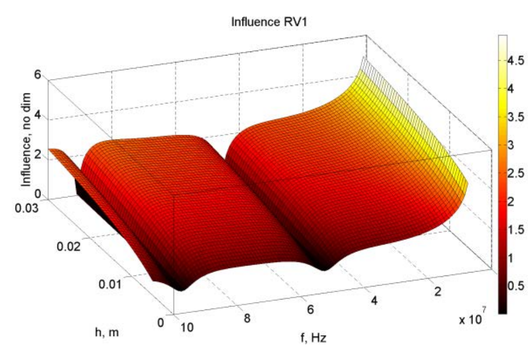


Fig. 12. Influence of RV1 ($h$).

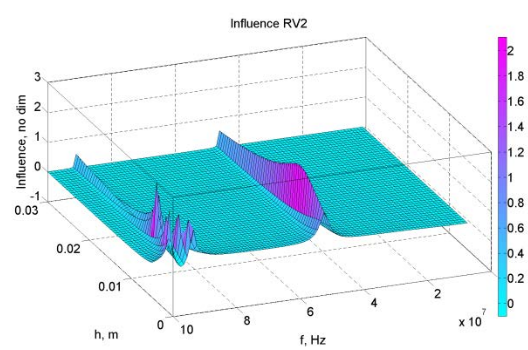


Fig. 13. Influence of RV2 ($f$).

The influence of each parameter is shown in Figs. 12 and 13. The computation of the influence from (14) lays emphasis on the dominant effect of height. As expected, the impact of the frequency is relatively smooth outside resonance frequencies. The *f*-effect should not be neglected around them with influence levels $In>1dB$. Comparatively to *f*-influence, the Fig. 12 shows the global *h*-impact ($1dB<In<5dB$ almost everywhere).

## VI. CONCLUSION

In this paper, both UT and SC techniques to solve stochastic EMC problems are presented. Uncertainties involving source parameters (frequency) and geometry of a transmission line (length, height) have been defined considering various RV following uniform, normal and log-normal distributions. The UT and SC methods appear similar to well chosen MC simulations: their main advantages rely on their effectiveness (minimizing CPU time more than 20,000 times) and non-intrusive characteristic. Directly linked to computational electromagnetics, we may perfectly apply these methods considering other EM simulation/experimental tools.

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**Sébastien Lalléchère** received his Ph.D. degree (2006) in Electromagnetism. Since 2007, he is an assistant professor at Clermont University and is interested in numerical methods for electromagnetism.

**Pierre Bonnet** received his Ph.D. degree in Electronics in 1998. Since 1999, he works as an assistant professor at Clermont University on computational electromagnetics.

**Ibrahim El Baba** received the M.S. degree in 2008. He is currently working towards the Ph.D. degree in Electronics at Clermont University.

**Françoise Paladian** received her Ph.D. degree (1985) in Electronics from Clermont University and is a professor in the Physics department. Her research area concerns numerical EMC and reverberating chambers.