Advanced Techniques for Efficient Modeling of Electrically Large Structures on Desktop PCs

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Abstract ― The method of moments (MoM), applied to surface integral equations (SIEs) in the frequency domain, enables very accurate analysis of composite metallic and dielectric structures. Particularly, on a desktop PC, the electrical size of solvable problems is very limited by memory and time resources. This limit can be significantly extended by using advanced techniques, which are shortly revealed. The focus of the paper is to present results that illustrate current possibilities of MoM/SIEs solution on a desktop PC: (1) monostatic RCS of a cube of side 80 λ, (2) beam steering of an array of 30 by 30 microstrip patch antennas at 9.2 GHz, and (3) beam steering of 4 by 4 patch antennas at 5 GHz, placed on a 19 m long helicopter.

Index Terms ― Higher order basis functions, method of moments, surface integral equations.

I. INTRODUCTION

Electromagnetic modelling of composite metallic and dielectric structures in the frequency domain can be performed very accurately by solving surface integral equations (SIEs) using the method of moments (MoM). Using the MoM theory [1, 2], induced currents over metallic surfaces and equivalent currents over material boundary surfaces are approximated by a series of known basis functions multiplied by unknown coefficients. The SIE is transformed into a system of linear equations, which is solved for the unknown coefficients. However, the size of the solvable problem in terms of the number of unknowns, N, is limited by memory and time resources of the computer used for the simulation. By increasing N, the memory occupation and matrix-fill time increase as N^2 and the matrix-solution time (in case of direct methods as Gaussian elimination or LU decomposition) increases as N^3. Typical size of operative memory (RAM) of modern PCs used for number crunching is 8 GB. In that case, systems of linear equations in the complex domain of up to about $N_{max} = 30{,}000$ unknowns can be solved "incore", which means keeping the whole matrix of the system in RAM during solving. On the other hand, the electrical size of the solvable problem (in λ^2 of surface area) is dependent on the choice of basis functions. For the Rao-Wilton-Glisson (RWG) basis functions defined over triangles, typical edge length of triangles is $\lambda/10$, resulting in about 300 unknowns per λ^2 for metallic surfaces [3]. Particularly, for $N_{max} = 30{,}000$, the electrical size of the surface area of a structure is limited to about 100 λ^2.

Many techniques have been developed to increase the electrical size of the solvable structure within the limits of PC computer resources. In what follows, we shall focus on techniques implemented in the commercial software package WIPL-D Pro v9.0 [4]. Generally, these techniques can be grouped into three classes.

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In the first class, there are techniques that decrease the number of unknowns:

(a) application of higher order basis functions [5, 6],
(b) exploiting symmetry of the problem [4],
(c) "smart reduction" of expansion order [7],
(d) construction of macro-basis functions by physical optics (PO) driven MoM [8], and
(e) hybrid MoM-PO methods [9].

In the second class, there are techniques that decrease the memory resources and matrix-fill/solution time for given number of unknowns:

(a) iterative techniques [10-13],
(b) fast multipole method (FMM), and
(c) multilevel fast multipole algorithm (MLFMA) [14-18].

Finally, in the third class, there are techniques that enable efficient usage of modern hardware resources:

(a) out-of-core solution of matrix equation [19],
(b) parallelization on CPU based on OpenMP [20], and
(c) parallelization on GPU based on CUDA [21-23].

The goals of the paper are: (1) to reveal various techniques for increasing the electrical size of the solvable structure within the limits of PC computer resources, (2) to compare these techniques and discuss optimal usage of these techniques, and (3) to show numerical results for some typical electrically large structures.

II. TECHNIQUES THAT REDUCE NUMBER OF UNKNOWNS

A. Higher-order basis functions

The basic way to decrease the number of unknowns is to apply higher-order basis functions (HOBFs). For interpolatory HOBFs defined over triangles [6], the maximum edge length of triangles can be extended to 0.5~1λ, resulting in 40–70 unknowns per λ^2 [15]. In case of polynomial HOBFs defined over quadrilaterals [2], [5], the maximum edge can be extended to 1~2λ, resulting in 20–35 unknowns per λ^2 [5]. In both cases, the expansion orders are chosen according to the electrical size of patches. Thus, the electrical size of a solvable structure within the limit of computer resources is increased by an order of magnitude when compared with RWG basis functions.

B. Exploitation of geometrical symmetry of the problem

If the geometry of a structure is symmetrical with respect to one plane, and the excitation is either symmetrical or anti-symmetrical with respect to this plane, the unknown coefficients on one side of the plane are equal to the coefficients from the other side of the plane multiplied by ±1, so the original number of unknowns is halved. The memory requirements are decreased four times and the matrix solution time is decreased eight times. In some cases, there are two or even three mutually orthogonal planes of symmetry, so that the number of unknowns can be decreased four or eight times, respectively.

The symmetry of geometry can be exploited to decrease the number of unknowns even if the excitation is not (anti) symmetrical [4]. In that case, the excitation can be decomposed into a set of symmetrical and anti-symmetrical excitations. Consequently, the problem is decomposed into set of sub-problems, for which not only the geometry is symmetrical, but also the excitations are symmetrical and/or anti-symmetrical. In that case, the number of unknowns is halved and the number of sub-problems to be solved is doubled for each symmetry plane. Once all sub-problems are solved, the final results are obtained by superposition. For each symmetry plane, the memory requirements are reduced four times, the matrix fill time is unchanged, while the matrix solution time is shortened by a factor of four.

C. Smart reduction of expansion order

In the case of an antenna placed at some platform (e.g., airplane fuselage), the currents, which are induced over the fuselage, are of the largest magnitude in the vicinity of the antenna, and decrease going away from the antenna. Since the currents of the lower magnitude have a smaller impact on global quantities, such as antenna input impedance or gain, their distribution can be determined with lower accuracy. With this in mind, the expansion orders of currents can be linearly reduced going from the antenna to the most distant part of the fuselage, for which the maximum reduction is specified (e.g., in %). For a maximum reduction of 100%, the expansion order is reduced to order one along each side of the patch, i.e., approximation of currents over relatively large

patches (up to 2 by 2 λ) is performed using rooftop basis functions.

In the case of two antennas placed at some platform, for mutual coupling between them, the most important currents are in the 1st Fresnel region. In that sense, the space around two antennas can be subdivided into the 1st and higher-order Fresnel regions. Then expansion orders for currents can be reduced gradually, from the 1st Fresnel region to the highest Fresnel region that still contains parts of the fuselage.

Finally, we know that, in the "shadow region" of a fuselage (i.e., in the region without optical visibility from the antennas), the induced currents are also much smaller than those in the lit region. In particular, if these currents are farther away from the border of the lit region, we consider them to be deeper in the shadow. In that sense, different levels of reduction in expansion orders can be specified for a given depth of the shadow.

Generally, by increasing the level of reduction of expansion orders, the number of unknowns is decreased and the solution error is decreased. It is shown that, by proper choice of reduction techniques and levels, the number of unknowns can be significantly reduced with negligible loss of accuracy.

As an example in [7], a half-wavelength dipole at 2 GHz is placed above a payload fairing of length 8.9 m and largest diameter 2.9 m. Basically, with one symmetry plane applied, the model requires 44,614 unknowns. By combining the reduction techniques explained above the number of unknowns can be reduced almost ten times, down to 4,983 unknowns.

D. PO driven MoM

The main goal of the PO driven MoM method is to solve electrically large problems using a small numbers of unknowns, and thus, reduce memory and time resources [8]. In case of scatterers, the method starts from the PO solution and improves it iteratively. In each iteration, the structure is excited by the solution from the previous iteration and correctional PO currents are determined and grouped into a small number of macro-basis functions (MBFs). Unknown coefficients multiplying all macro basis functions are obtained by minimizing the residuum of the original MoM solution. Thus, the solution after the 1st iteration is better than the PO solution, and in each next iteration, it approaches the original MoM solution. The number of MBFs added per iteration is adopted to be comparable with the square root of the number of unknowns. An acceptable solution is obtained even after the 1st iteration with a reduction in the number of unknowns of two orders of magnitude.

In the case of an antenna placement problem, the only difference is that the starting solution is obtained as the MoM solution of the antenna isolated from the fuselage.

III. TECHNIQUES THAT REDUCE MEMORY REQUIREMENTS AND NUMBER OF OPERATIONS

A. Iterative methods

The number of operations needed to solve a matrix equation can significantly be decreased by using iterative methods, generally speaking from $N^3/3$ to MN^2, where M is number of iterations. The number of iterations generally depends on: (a) the maximum allowed mean square value of residuum of matrix equation (e.g., $R = 0.001$), (b) the type of problem to be solved, (c) the type of MoM/SIE method used to obtain the solution, and (d) the type of iterative procedure itself [10]. For example, scattering problems (distributed excitation) require fewer iterations than antenna problems (localised excitation), which require fewer iterations than closed problems (e.g., resonant cavities). The convergence for different types of problems can be improved by using various preconditioners, or other techniques that decrease the matrix condition number [11].

In particular, HOBFs cannot be efficiently used for iterative solutions if they are not orthogonalized in some way [12, 13]. The best results are obtained by using maximally orthogonalized HOBFs [13], which enable almost the same convergence of the matrix solution for higher-order basis functions as for the rooftop basis functions.

Fully developed iterative techniques can solve the matrix equation in a relatively small number of iterations, which is much smaller than the number of unknowns, i.e., $M << N$, and thus significantly reduce the time needed for matrix solution. However, in many cases, the iterative solution cannot outperform the direct solver (e.g., LU

decomposition), not only because of slow convergence. Namely, in case of a set of independent excitations (e.g., multiple excitation), the iterative procedure is performed from the beginning for each excitation. On the other hand, in case of a direct solver, once the LU decomposition is performed, the solution for each excitation is obtained by the so-called forward and backward substitution in N^2 operations, which corresponds to a single iteration in an iterative procedure.

B. Multilevel fast multipole algorithm

Both direct solvers and iterative procedures for the solution of matrix equations are limited by the memory required to store the matrix. In case of iterative solvers, the memory requirements, as well as solution time, can significantly be reduced by using FMM and MLFMA [14-16]. In both cases, the method is based on the acceleration of matrix-vector multiply operations performed in each iteration of an iterative solver, by taking into account interactions between widely separated groups of basis functions instead of interactions between individual basis functions belonging to these groups. (Particularly, in case of MLFMA the grouping is performed in few levels.) Reduction of interaction between individual basis functions to interaction between their widely separated groups is enabled by the multipole expansion of the free-space Greens function. The most efficient reduction is obtained for groups, which are far enough away so that the multipole expansion can be represented by a single term, which corresponds to a far-field approximation (FFA).

Generally, the application of FMM and MLFMA introduces an additional error in the MoM solution. This error can be decreased (e.g., by increasing the number of terms in the multipole expansion, or by increasing the relative distance at which FFA is applied), which results in an increase of memory resources and simulation time. Particularly, the error issues are critical in the case of application of HOBFs [15, 17]. In order to enable the efficient application of HOBFs, an improved far-field approximation is proposed in [18]. Even with this improvement, efficient MLFMA solutions are only possible for maximally 2nd and 3rd orders of HOBFs.

As an example in [24], the fighter scatterer, 12 m long, is analyzed at 4 GHz (160 λ). The problem requires 307,170 unknowns and 754 GB of RAM for storage of the full matrix. The MLFMA solution is obtained in 3.2 hours using a standard quad core PC with 8 GB of RAM. In this case, the application of MLFMA reduces the memory needs to 7.2 GB.

IV. TECHNIQUES THAT ENABLE EFFICIENT USAGE OF HARDWARE RESOURCES

A. Out-of-core solver

In the case when the MoM matrix is too large to be stored in RAM, it must be split into blocks, which are stored on a hard disk. In particular, if there are many excitation columns (multiple excitation) they can be also stored in the disk. The solution of such stored matrix equation is obtained using an appropriate out-of-core solver.

In the case of out-of-core direct solution, Gaussian elimination or LU decomposition is performed using two by two blocks from the disk. The size of the blocks is limited so that two of them can be stored in RAM. Since the total number of readings of the full matrix is equal to the half of the total number of blocks, it is optimal that these two blocks have size slightly smaller than the size of RAM. As an example, consider a problem of 120,000 unknowns that should be solved using 8 GB of RAM. In that case, the matrix size is about 107.3 GB and each block can have a size of 3.8 GB, so that for the full solution the matrix is read 15 times.

In the case of an out-of-core iterative solution, the matrix must be read at least once during each iteration. The number of iterations needed to obtain a sufficiently accurate solution is usually much larger than the number of blocks into which the matrix is split. Thus, the total time needed for an out-of-core iterative solution is much larger than for a direct solution. Hence, the out-of-core solver is used only for direct solution.

B. Parallelization on CPU based on OpenMP

Modern CPUs contain more than one core, usually 4 cores. Using OpenMP, the calculation can be performed in parallel at an arbitrary number of threads, but an efficient parallelization uses a number of threads approximately equal to the number of cores. Since the memory used for calculation is shared by all threads, for an efficient parallelization, it is very important not to access the same variable with more than one thread. Generally,

the code that was optimal for serial performance should be reorganized to support efficient parallel performance. For example, the efficient matrix fill in serial mode is organized so that mutual coupling between two patches is calculated one at a time. However, the basis functions that provide continuity between patches (doublets in general case, roof top basis functions in special case) belong to both patches. So, it can happen that if such couplings are calculated in parallel two or more threads approach to the same element in the matrix. This can be avoided if the couplings are calculated for groups of test and basis function patches, such that all patches in a group have no common basis functions. Using such grouping, the efficiency of parallelization of 90% (70%) is achieved for matrix fill at 4 (8) cores.

In the case of the matrix solution, the MKL [25] library enables almost 100% efficiency for 4 (8) cores.

C. Parallelization on GPU based on CUDA

In cases when the calculation consists of many repetitions of the same operation, graphical processing units (GPU) can perform the calculation up to 10 times faster than a quad-core CPU. However, the code that is executed in parallel mode using OpenMP on CPU cannot be simply transferred into CUDA on the GPU. Again, the algorithms should be reorganized to support parallel execution on thousands of threads. In particular, it is desirable that tasks performed in parallel at all threads use the same sequence of operations the same number of times. Since not all parts of the MoM code are suitable for GPU parallelization, the optimal code is one that combines parallelization on the CPU and the GPU.

For example, in the case of an out-of-core solver on a CPU, there are two critical operations: (1) storing/reading the matrix to/from disk, and (2) calculations performed in LU decomposition. So, for significant acceleration, it is not enough to parallelize the calculations on the GPU. It is also necessary to perform storing and reading of the matrix blocks in parallel on the CPU and in parallel with the calculations on the GPU [22].

V. OPTIMAL COMBINATION OF TECHNIQUES FOR EFFICIENT AND ACCURATE EM MODELING

The basic combination for efficient and accurate modelling on PCs is that based on higher-order basis functions, incore/out-of-core direct solvers and combined CPU/GPU parallelization. In particular, the simulation can be accelerated by exploiting geometrical symmetry and by using the "smart reduction" for antenna placement problems. In the case of electrically large structures with a single excitation (e.g., bistatic RCS for scatterers or radiation pattern for antenna placement), the simulation can be further accelerated by using iterative methods, e.g., MLFMA or PO driven MoM.

VI. NUMERICAL RESULTS

Figure 1 shows the monostatic RCS of a cube, of side 80 λ, illuminated by a vertically polarized plane wave in the *xOy*-plane from 3,552 directions. Using expansions of 5th order and one symmetry plane, as shown in the left inset, the problem is reduced to 655,380 unknowns. The results are obtained by using all three symmetry planes, as shown in right inset. The problem is decomposed into four sub-problems, each containing 163,845 unknowns. Total simulation time for all 4 sub-problems and post-processing is 33 hours.

The second problem considered is a phased array consisting of n by n probe-fed microstrip patch antennas, where $n = 10$, 20, 30, and 40. (The array for $n = 10$ is shown in Fig. 2.) The task is to determine the gain for a set of directions of the main beam, with angle θ going from 20 to 90 degrees and angle φ going from 0 to 90 degrees, both with a step of 5 degrees, so that the total number of excitations is 285.

It is also required that the finite size and finite thickness of the substrate, as well as the finite thickness of the metallization, are taken into account, as shown in Fig. 3. The size of a square metallic patch is 10 mm by 10 mm, while the distance between antenna centers is 15 mm. The thickness and relative permittivity of the teflon substrate are 0.5 mm and 2.1, respectively. The thickness of the metallization is 34 μm. The operating frequency is 9.2 GHz.

The original problem for $n = 30$ has about 450,000 unknowns. Two symmetry planes are used

to facilitate the analysis, so that the original problem is decomposed into four sub-problems, each requiring about 112,500 unknowns. The simulation is performed using one generator at time, so that for each generator turned on, all others are turned off. For each such excitation, the radiation pattern of the array is determined. In addition, all these excitations are used to determine the matrices of mutual admittances and impedances.

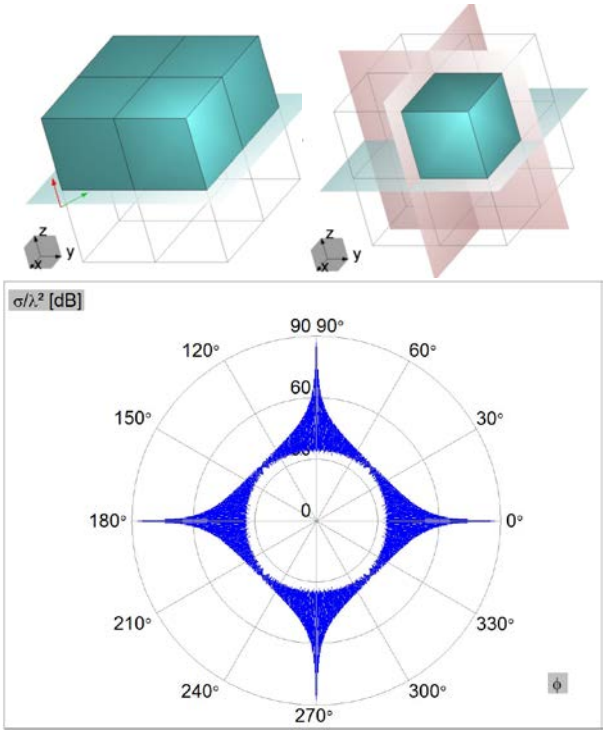

Fig. 1. Monostatic RCS of a cube of side 80 λ in *xOy*-plane versus angle φ.

Once the 3D EM simulation is performed, data are imported as an *n* by *n*-port device into the schematic feature of WIPL-D [26]. The current sources are attached to these ports. Once this simple circuit is solved for voltages at the ports, the total radiation pattern is easily obtained by superimposing radiation patterns due to each of these voltages. By properly adjusting the current sources, the main beam of the array can be positioned in a desired direction.

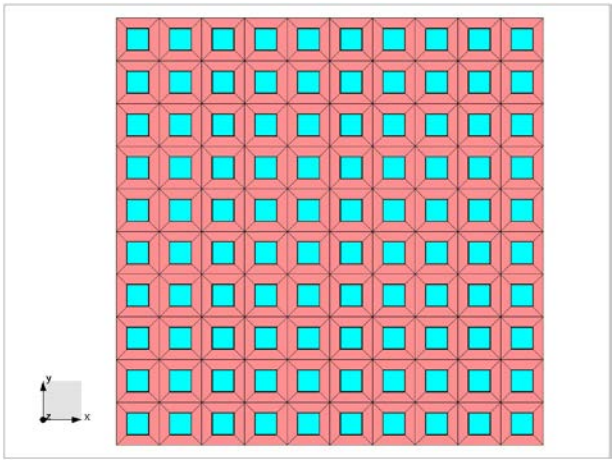

Fig. 2. Geometrical model of an array of 10 by 10 microstrip patch antennas.

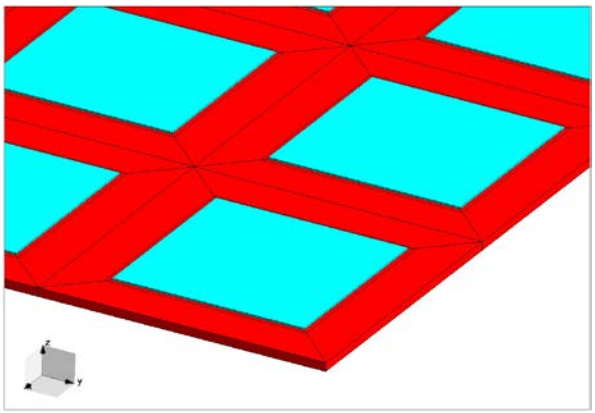

Fig. 3. Geometrical model of an array of 10 by 10 microstrip patch antennas.

Figure 4 shows the antenna gain for various beam directions in the plane φ = 45°, which is 16 beam directions in total. Total simulation time including post-processing for 285 beam directions is 8 hours and 20 minutes.

The last problem considered is the radiation of a 4 by 4 microstrip patch antenna array placed at the bottom of a helicopter fuselage, as shown in Fig. 5. The array is adjusted to operate at a frequency of 5 GHz. Total length of helicopter is 19 m, so that its electrical length at this frequency is 316.7 λ. The original problem having 1,254,034 unknowns is solved using a GPU accelerated PO driven MoM method in 28.25 hours. The radiation pattern is shown in Fig. 6.

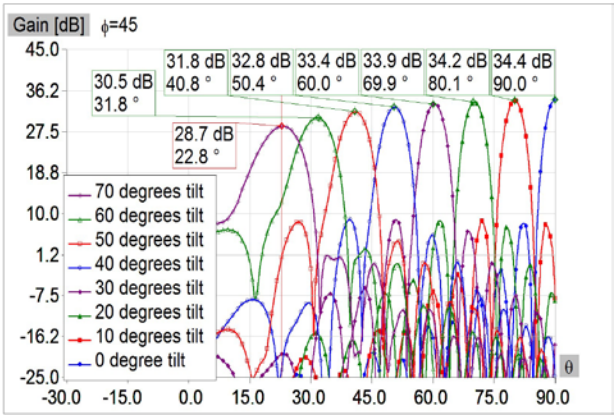

Fig. 4. Gain of array of 30 by 30 microstrip patch antennas for various directions of main beam in plane φ = 45°.

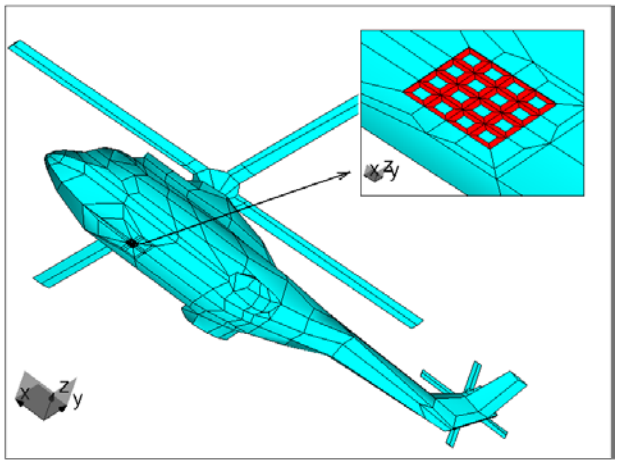

Fig. 5. Geometrical model of an array of 4 by 4 microstrip patch antennas placed on helicopter.

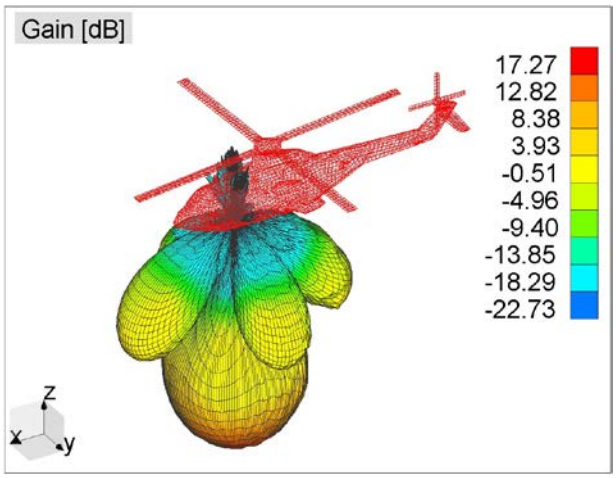

Fig. 6. Radiation pattern at 5 GHz of array of 4 by 4 microstrip patch antennas placed at helicopter.

VII. CONCLUSION

The basic combination for efficient and accurate EM modelling in the frequency domain on PCs is based on higher-order basis functions, incore/out-of-core direct solvers, and combined CPU/GPU parallelization. Where possible, the simulation can be accelerated by exploiting geometrical symmetry and the "smart reduction" of expansion orders. In particular, for single excitation problems (e.g., bistatic RCS and antenna placement), the analysis can be further accelerated using MLFMA and PO driven MoM.

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