Optimal Location of Multi-Antenna Systems - Influence of Noise-Corrupted Data

M. Zoppi [1], C. Dedeban [2], Ch. Pichot [1], S. Selleri [3], and G. Pelosi [3]

[1] Laboratoire d'Electronique, Antennes et Télécommunications (LEAT)
University of Nice Sophia-Antipolis, CNRS, Valbonne, 06560, France
mzoppi@unice.fr, christian.pichot@unice.fr

[2] Antenna R&D Unit
Orange Labs, La Turbie, 06320, France
claude.dedeban@orange-ftgroup.com

[3] Dipartimento di Elettronica e Telecomunicazioni
University of Florence, Florence, 50134, Italy
stefano.selleri@unifi.it, giuseppe.pelosi@unifi.it

Abstract **─** This paper describes the development of an antenna synthesis procedure for determining the optimal location of 2D array configurations. An inverse scattering algorithm based on a conjugate gradient method is used. The influence of noise-corrupted data on the results is investigated.

Index Terms **─** MIMO, multi-antennas, position optimization.

I. INTRODUCTION

Research on multi-antennas systems has received a growing interest in the past few years. MIMO systems have demonstrated the potential for increasing capacity but many other applications are also using multi-antenna systems such as radar applications with inverse scattering including microwave imaging. However, the problem of determining their optimal location for each application, inside a noisy environment, remains of great interest. Previous works related to this subject are reported in [1] and [3]. Moreover, in [1], we have considered the reconstruction of the optimal location of multi-antenna system and obtained preliminary results using noiseless data. Here, we are studying the performance of the synthesis procedure versus noise in order to show the robustness of the algorithm. We are considering two kinds of noise affecting the data, an uniform white noise (UWN) and an average white Gaussian noise (AWGN). Most of the inverse scattering algorithms based on gradient optimization are using forward and adjoint problems for calculating the cost function derivative. Here, in order to save computing time, we are calculating directly the derivative of the cost function. In this way, we have direct access to the sensitivity of the cost function versus the parameters we are interested in. The inverse problem is formulated in terms of an inverse scattering problem. We are interested in retrieving the location of N antenna elements modeled by monopoles located on a planar surface, illuminated successively by a certain number of plane waves.

II. THEORY

We are considering a 2D array of small antenna elements modeled by elementary sources non-regularly distributed on the surface of the xy plane (Fig.1) and described with an element factor EF which varies such as:

$$EF(\theta) = \cos(\theta). \qquad (1)$$

The inverse problem consists in retrieving the location of the sources from the knowledge of their radiation patterns when they are illuminated with L successive incident plane waves. The

Submitted On: Oct. 12, 2011
Accepted On: Jan. 26, 2012

1054-4887 © 2012 ACES

position of a single antenna element n in the rectangular coordinate system is:

$$\vec{x}_n = (\vec{x}_{n1}, \vec{x}_{n2}, 0). \quad (2)$$

Considering antenna elements as sources defined in the xy plane presenting a simple cosineθ dependance, the coupling effect between elements is supposed to be negligeable.

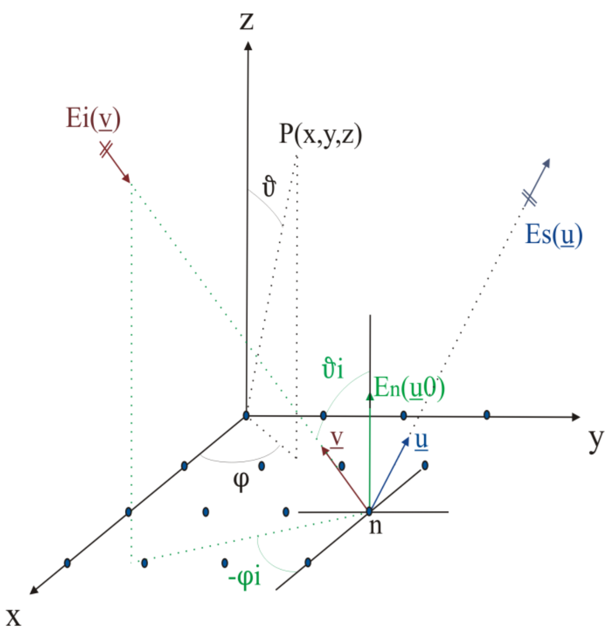

Fig. 1. Geometry of the problem.

Therefore, we can consider the total Field E as the sum of antenna elements. The scattered field for a single element n is:

$$E^S(M, \vec{x}_n) = EF(\theta) m(\vec{x}_n, \vec{u}_0). \quad (3)$$

We can define the term m corresponding to the direction of incidence $\vec{u}_0$ of a plane wave illuminating an antenna element:

$$m(\vec{x}_n, \vec{u}_0) = a e^{jk(\vec{x}_n, \vec{u}_0)}, \quad (4)$$

where $a \in \mathbb{R}$ is the plane wave amplitude. The dependance of scattered field with respect to the incident one is explicitly expressed as follows:

$$E^S(M, \vec{x}_n) = a\cos(\theta) e^{jk(\vec{x}_n, \vec{u}_0)}, \quad (5)$$

For N antenna elements, we have:

$$E^S(\vec{u}, \vec{u}_0) = a\cos(\theta) \sum_{n=1}^{N} e^{jk\langle \vec{x}_n, (\vec{u} - \vec{u}_0)\rangle}, \quad (6)$$

Then we define:

$$\langle \vec{x}_n, \vec{v}_0 \rangle = \langle \vec{x}_n, (\vec{u} - \vec{u}_0) \rangle, \quad (7)$$

where $\vec{v}_0 = \vec{u} - \vec{u}_0$.

The derivative of E^S with respect to an antenna element position is:

$$\frac{\partial E^S}{\partial x_n} = a\cos(\theta) jk e^{jk\langle \vec{x}_n, \vec{v}_0 \rangle} \frac{\partial \langle \vec{x}_n, \vec{v}_0 \rangle}{\partial x_n}. \quad (8)$$

We can now write taking into account the three Cartesian coordinates:

$$\frac{\partial \langle \vec{x}_n, \vec{v}_0 \rangle}{\partial x_n} = \frac{\partial}{\partial x_n} \sum_{k=1}^{3} (x_n)_k (v_0)_k = \frac{\partial v_0}{\partial x_n}. \quad (9)$$

Then the final expression of the derivative becomes:

$$\frac{\partial E^S}{\partial x_n} = a\cos(\theta) jk e^{jk\langle \vec{x}_n, \vec{v}_0 \rangle} \frac{\partial v_0}{\partial x_n}. \quad (10)$$

E_{meas} is the given scattered field value obtained from numerical or experimental measurements, the cost function is defined such as:

$$J(\vec{x}_n) = \sum_{l=1}^{L} \left| E^S(\vec{v}_l) - E_{meas} \right|^2. \quad (11)$$

Its derivative with respect to the unitary element position is given by:

$$\frac{\partial J(\vec{x}_n)}{\partial x_n} = 2 \sum_{l=1}^{L} \Re\left\{ E^S(\vec{v}_l) \bar{-} E_{meas} \right\} \frac{\partial E^S}{\partial x_n}. \quad (12)$$

Using the derivative of the cost function, an inverse algorithm is developed using a Polak-Ribière conjugate gradient method.

III. NOISE MODEL

Our previous [1] work was based on the study of a noise-free cost function defined in (11). Here, in order to study the robustness of the inversion procedure, the measured scattered field E_{meas} have been corrupted by an additive noise. As noise, we consider an ergodic stationary random process.

For simulating corrupted measured data, a signal noise has been added to synthetic data:

$$E_{meas} \approx E_{synt} + n, \quad (13)$$

where E_{synt} is the noise-free field, n the noise signal.

In order to compute the simulated measurement data, we assume to use an I-Q measurement model; it is well known that both I and Q measurements are afflicted by a noise signal. If we also assume to have additive noise we can easily write the I and Q signals expressions:

$$I = E_{Isynt} + n_I, Q = E_{Qsynt} + n_Q, \quad (14)$$

where n_I, n_Q are two random processes, with the same distribution [4]. So the simulated measurement data become:

$$E_{synt} = [E_{Isynt} + n_I] + j[E_{Qsynt} + n_Q]. \quad (15)$$

Once E_{synt} is calculated it is possible to obtain the noise n expression when fixing a desired SNR

level. In order to do this, the signal-to-noise ratio SNR equation is needed:

$$N = \frac{S}{SNR} = [| E_{synt} |^2/2]/[10^{SNR/10}], \quad (16)$$

where N is the noise power and S the signal power.

Two different kinds of noise are taken into account, i.e., an uniform white noise (UWN) and an average white Gaussian noise (AWGN). It is possible to find the noise signal contribution within n_I, n_Q (15) by simply using the variance value σ^2 of (16) and defining the appropriate expressions with respect to the chosen statistic process [4]. So for an UWN process, the expression for n_I and n_Q is:

$$n_{UNIF} = (x_{RAND} - 0.5)2\sqrt{3\sigma^2}, \quad (17)$$

while, for an AWGN process, the n expression is defined as:

$$n_{AWGN} = x_{RAND}\sqrt{\sigma^2}, \quad (18)$$

where $x_{RAND} \in [0,1]$ in (17) and (18) is a pseudo-random number generated by the numerical algorithm.

Finally, the cost function taking into account of computed noisy data is:

$$JN(\vec{x}_n) = \sum_{l=1}^{L} | E^S(\vec{v}_l) - E_{meas} |^2, \quad (19)$$

and for the derivative form of (19):

$$\frac{\partial J(\vec{x}_n)}{\partial x_n} = 2\sum_{l=1}^{L} \Re\{E^S(\vec{v}_l) \bar{-} E_{meas}\}\frac{\partial E^S}{\partial x_n}. \quad (20)$$

We use these last two expressions inside the optimization algorithm.

IV. NUMERICAL EXPERIMENTS

Different numerical experiments have been carried out in order to test the robustness of the inverse scattering algorithm with corrupted data. The working frequency is 2.45 GHz (λ_0= 12.24 cm). All the tests considered here start from a regular array 7x7 ($\lambda_0/3$ step on both x-axis and y-axis) for initial guess to retrieve an irregular array ($\lambda_0/6$ step on x-axis and $\lambda_0/9$ on y-axis). The knowledge of the radiation is related to a semi-hemisphere i.e. known over the upper semi-hemisphere ($\theta \in [0,90°]$ and $\varphi \in [0,180°]$). The tests have been done using a parametric sweep respectively over: the signal-to-noise ratio, the number of measurement points in θ and ϕ and the number of the incident plane waves. A comparison between the two types of noise have also been done.

A. First test case: noiseless data

We consider a N = 7x7 array illuminated with L = 4 incident plane waves. The radiation pattern is known over the entire upper hemisphere. For this case, the considered noise level is zero in order to have a reference case for the corrupted-data tests. The value of the initial cost function is equal to 37.20 dB. After 2648 iteration steps, the final value of the cost function is -20.70 dB.

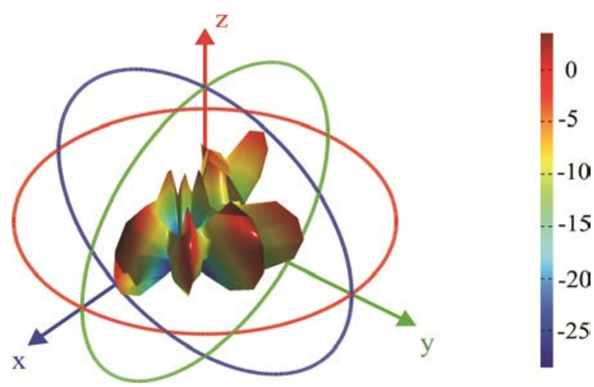

Fig. 2. Radiation pattern at initial guess (regular distribution): noiseless model.

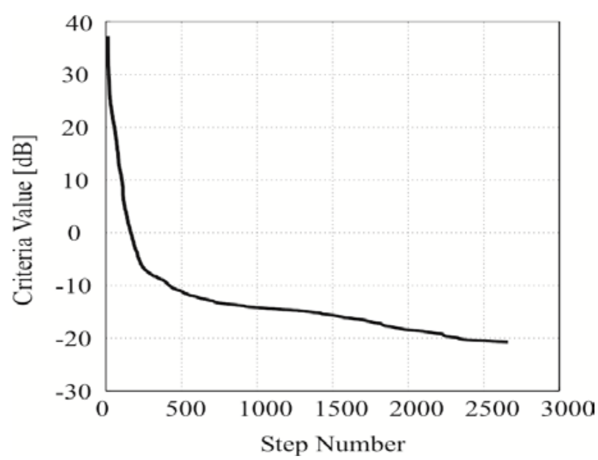

Fig. 3. Cost function (criteria values) with respect to the iteration step: noiseless model.

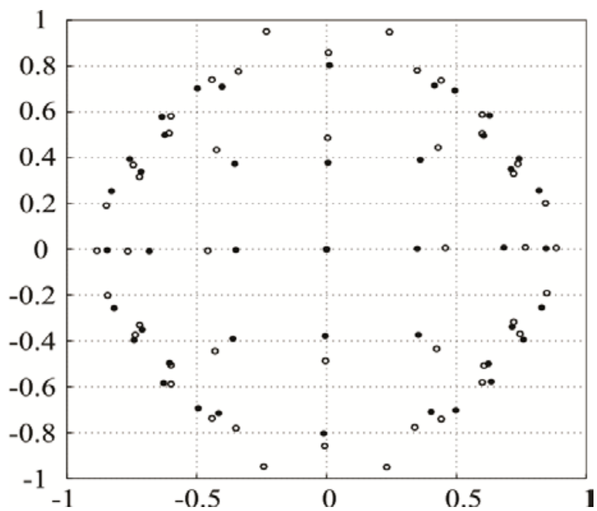

Fig. 4. Distribution of antenna elements at initial guess (white points) and final iteration (black points): noiseless model.

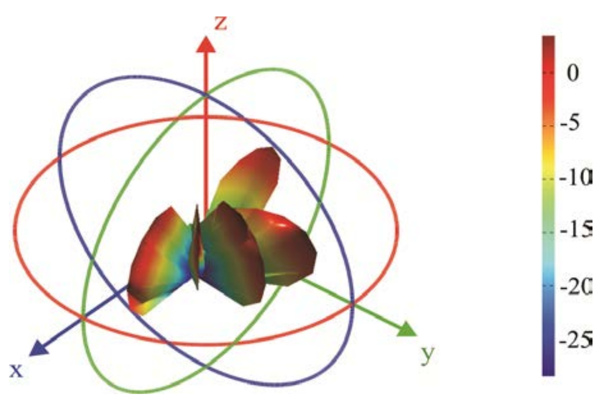

Fig. 5. Radiation pattern at final iteration (irregular distribution): noiseless model.

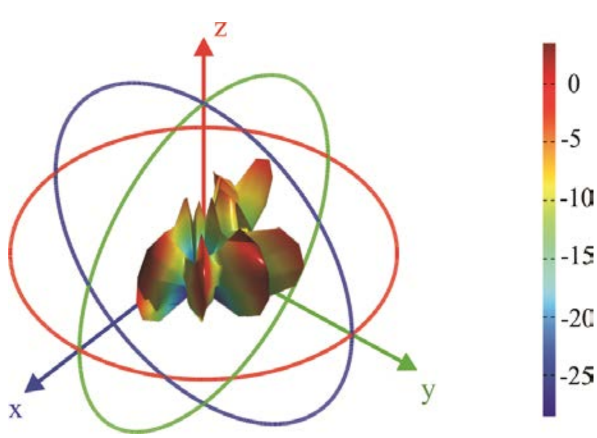

Fig. 6. Radiation pattern at initial guess (regular distribution): AWGN model with SNR 30dB.

B. Second test case: AWGN model with SNR=30dB

We consider a N = 7x7 array illuminated with L = 4 incident plane waves. The radiation pattern is known over the entire upper hemisphere. For this case, the considered noise level of an AWGN model corresponds to SNR=30dB. The value of the initial cost function is equal to 37.15 dB. After 13 iteration steps, the final value of the cost function is 23.51 dB.

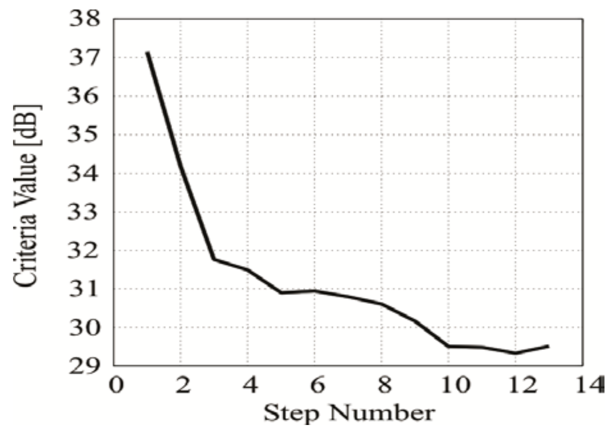

Fig. 7. Cost function with respect to the iteration step: AWGN model with SNR 30dB.

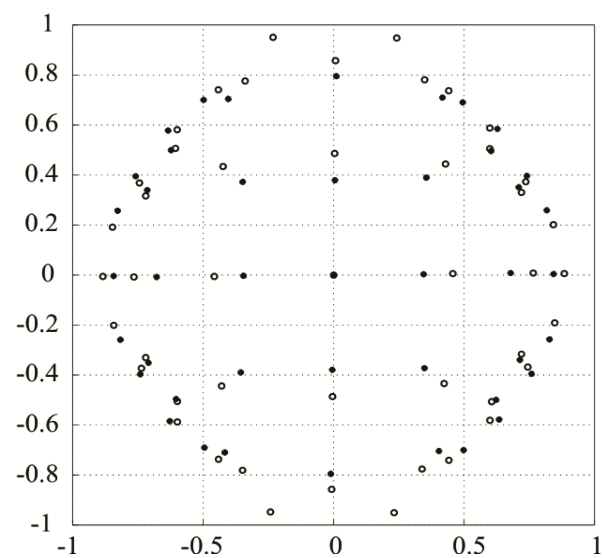

Fig. 8. Distribution of antenna elements at initial guess (white points) and final iteration (black points): AWGN model with SNR 30dB.

C. Third test case: UWN model with SNR=30dB

We consider a N = 7x7 array illuminated with L = 4 incident plane waves. The radiation pattern is known over the entire upper hemisphere. For this case, the considered noise level of an uniform noise model corresponds to SNR=30dB. The value of the initial cost function is equal to 37.32 dB. After 13 iteration steps, the final value of the cost function is 31.24 dB.

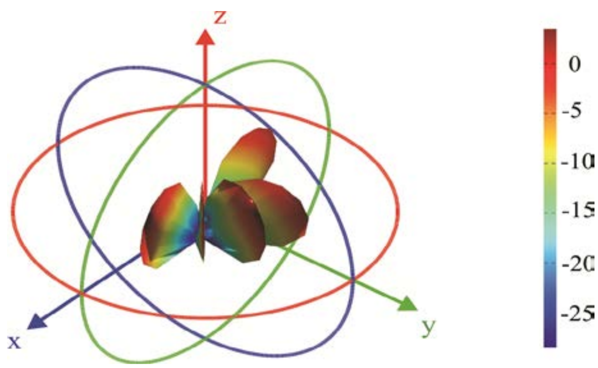

Fig. 9. Radiation pattern at final iteration (irregular distribution): AWGN model with SNR 30dB.

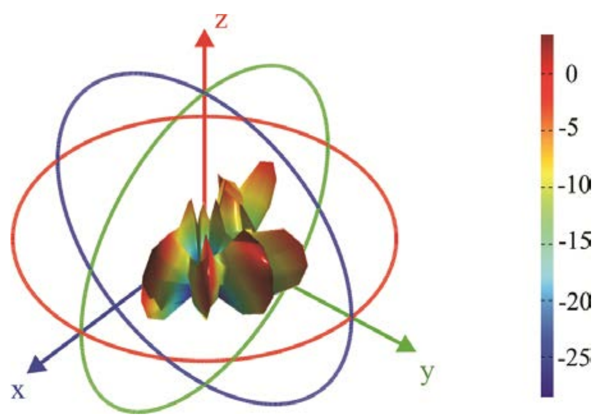

Fig. 10. Radiation pattern at initial guess (regular distribution): UWN model with SNR 30dB.

D. Fourth test case: AWGN vs. UWN

We consider a N = 7x7 array illuminated with L = 4 incident plane waves. The radiation pattern is known over the entire upper hemisphere. With this case, we compare the results obtained using an UWN model and an AWGN model, keeping in mind that the AWGN model is normally a better approximation of the real signal noise than the uniform noise.

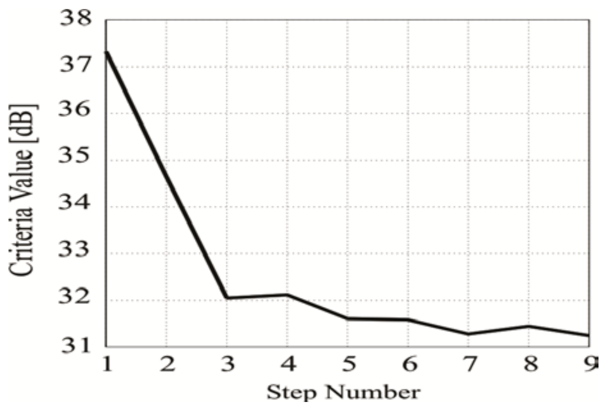

Fig. 11. Cost function with respect to the iteration step: UWN model with SNR 30dB.

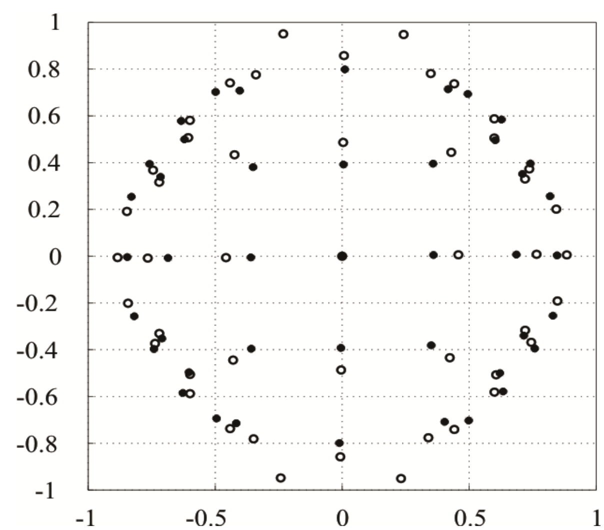

Fig. 12. Distribution of antenna elements at initial guess (white points) and final iteration (black points): UWN model with SNR 30dB.

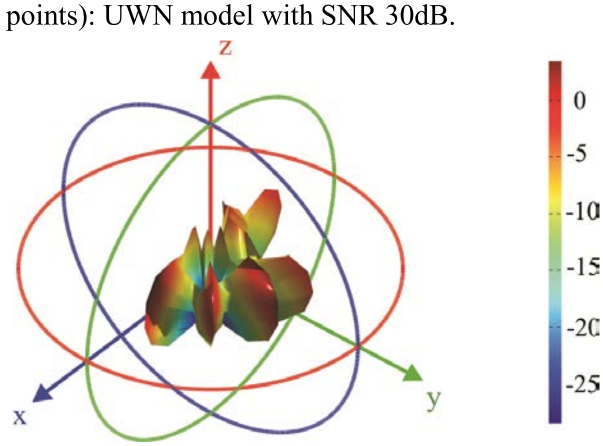

Fig. 13. Radiation pattern at final iteration (irregular distrib.): UWN model with SNR 30dB.

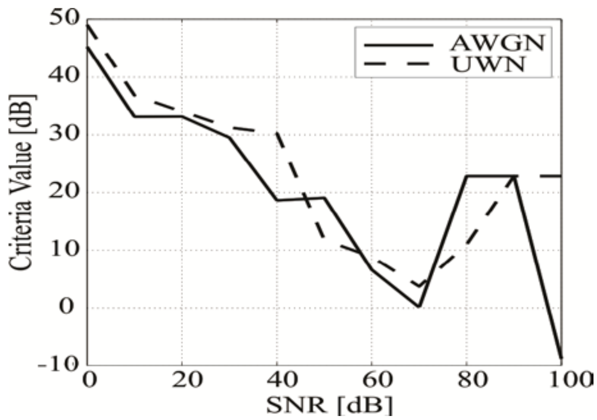

Fig. 14. Cost function (criteria values) with respect to the SNR value: AWGN vs UWT model.

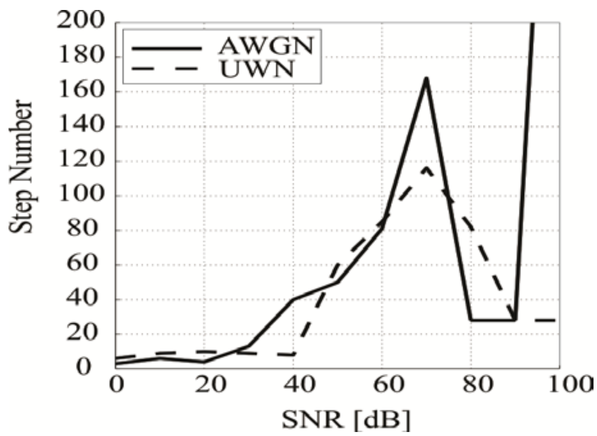

Fig. 15. Number of iterations with respect to the SNR value: AWGN vs UWT model.

E. Fifth test case: measurement points with AWGN model

We consider a N = 7x7 array illuminated with L = 4 incident plane waves. The radiation pattern is known over the entire upper hemisphere. For this case, we compare five different sets of measurement points using the AWGN model. The Figure 16 shows the convergence of the cost function (criteria values) with respect to the SNR value for different sets of measurement points. The Figure 17 shows the number of iterations with respect to the SNR value for different sets of measurement points. When examining the figures 16 and 17, the optimum set of measuring points is found to be 4x9=36. Although the continuous black curve may seem to the best one, we have to be careful before concluding. As the final radiation pattern has many local minima, we have a certain risk that the measuring points be close or correspond to these minima.

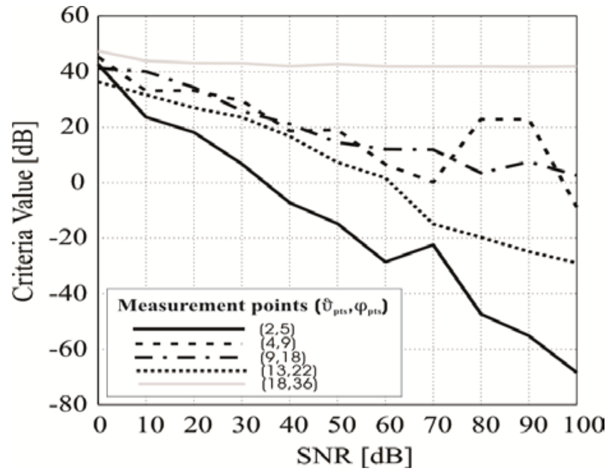

Fig. 16. Cost function (criteria values) with respect to the SNR value for different sets of measurement points: AWGN model.

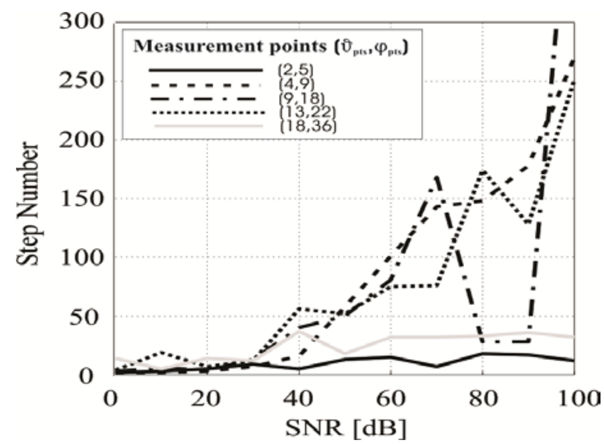

Fig. 17. Number of iterations with respect to the SNR value: AWGN model.

That may lead to a rapid convergence but to a wrong solution. In conclusion, some precaution has to be done in decreasing the number of measurement points. For the continuous gray curve, corresponding to the set of 18x36=648 measurement points, we may expect to obtain the best convergence due to the high number of measurement points but we still observe a strong non-convergence. This is simply due to an excess of information. In conclusion, we chose the dashed black curve corresponding to 4x9=36 as the optimum set of measuring points.

F. Sixth test case: number of plane waves with AWGN model

We consider a N = 7x7 element array illuminated with L = 4 incident plane waves. The radiation pattern is known over the entire upper hemisphere. For this case, we compare the results for three different numbers of incident plane waves using the AWGN mode. The Figure 18 shows the convergence of cost function (criteria values) with respect to the SNR value for different sets of measurement points and number of incident plane waves.

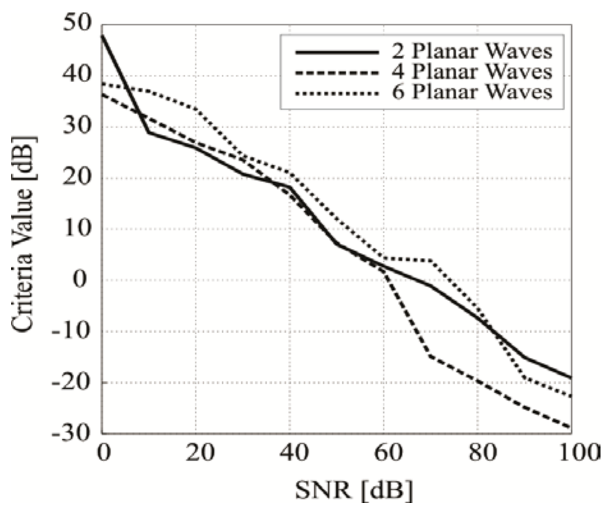

Fig. 18. Cost function (criteria values) with respect to the SNR value: AWGN model.

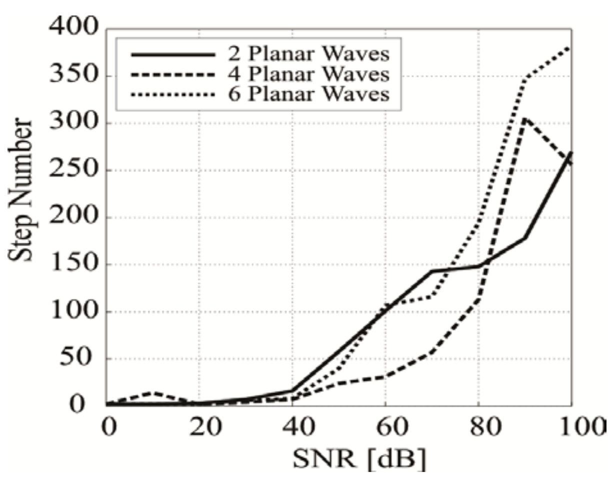

Fig. 19. Number of iteration steps with respect to the SNR value: AWGN model.

Figure 18 shows the number of iteration steps with respect to the SNR value for different number of plane waves and sets of measurement points. We can observe a similar behavior between the cases using two, four, and six incident plane waves.

Nevertheless, we prefer to choose results having a stronger convergence results despite of results obtained with less number of iteration steps. Therefore, we choose for the optimal configuration, the results obtained with four incident waves (corresponding to the dashed black line) which represents the best convergence with respect to SNR value with only few additional iterations steps.

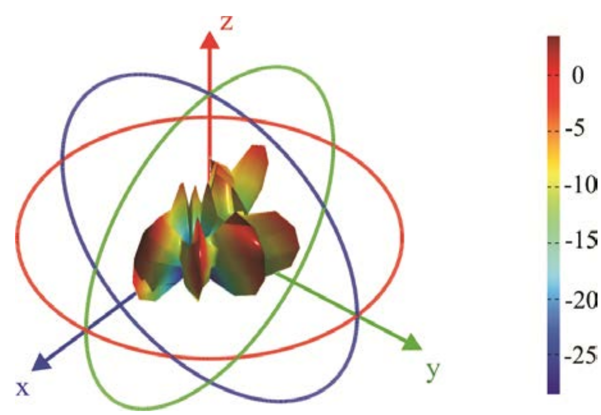

Fig. 20. Radiation pattern at initial guess (regular distribution): AWGN model with SNR 70dB.

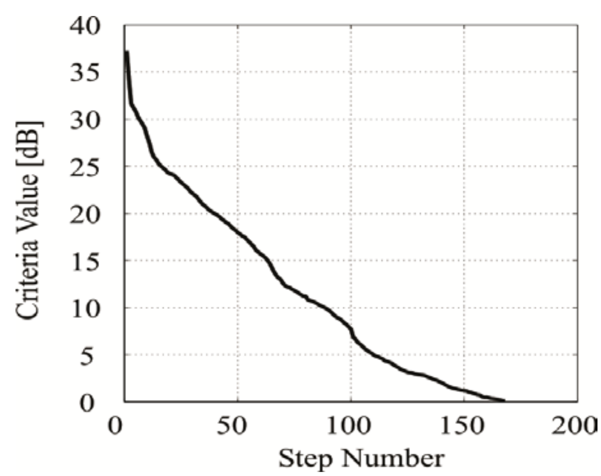

Fig. 21. Cost function (criteria values) with respect to the iteration step: AWGN model with SNR 70dB.

G. Seventh test case: optimal configuration

As we can see from Figs. 14 and 15, the best results, in terms of convergence of the cost function, are given from the AWGN model with a SNR level of 70dB. Therefore, the optimal configuration in terms of number of measurement points and incident plane waves, when observing the results achieved at points D. F. G., corresponds to the following one: Gaussian signal noise model (AWGN); four measurement points in θ, nine measurement points in ϕ over the upper hemisphere; and four incident plane waves.

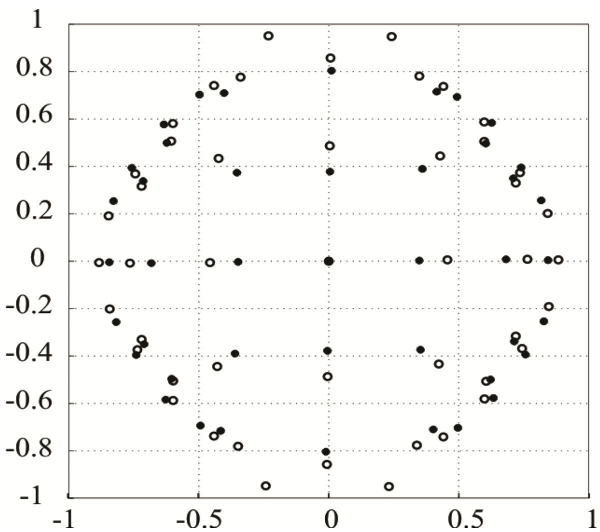

Fig. 22. Distribution of antenna elements at initial guess (white points) and final iteration (black points): AWGN model with SNR 70dB.

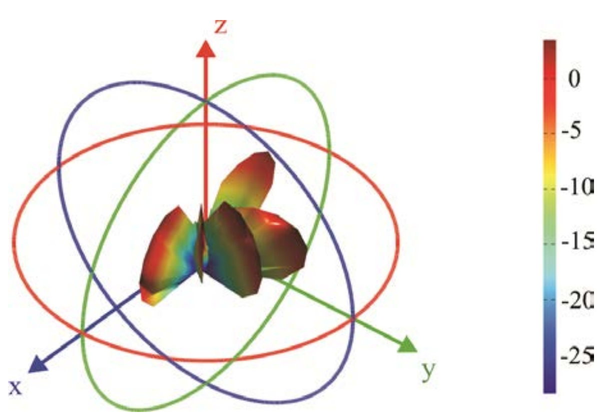

Fig. 23. Radiation pattern at final iteration (irregular distribution): AWGN model with SNR 70dB.

We consider a N = 7x7 array and the radiation pattern is known over the upper hemisphere. The value of the initial cost function is 37.30 dB. After 168 iteration steps, the final value of the cost function is 0.13 dB.

V. CONCLUSION

An optimization technique has been developed for solving an inverse scattering problem in order to retrieve the location of N antenna elements modeled by sources located on a planar surface, illuminated by plane waves. We have investigated the robustness of the algorithm with noise-corrupted data, using average Gaussian noise (AWGN) and uniform white noise (UWN) models. Different numerical results for testing the performance of the optimization technique have been presented in terms of the noise model, the number of measurement points number and number of incident planar waves. The radiation pattern is assumed to be known over the upper hemisphere.

REFERENCES

[1] M. Zoppi, C. Dedeban, Ch. Pichot, S. Selleri, and G. Pelosi, "Optimal Location of Multi-Antenna System using a Conjugate Gradient Method," *2011 Computational Electromagnetics International Workshop (CEM'11)*, Izmir, Turkey, August 10-13, 2011.

[2] M. Zoppi, C. Dedeban, Ch. Pichot, S. Selleri, and G. Pelosi, "Antenna Shape Synthesis of Various Planar Configurations using a Hybrid Conjugate Gradient Method," *2010 IEEE International Conference on Wireless Information Technology and Systems (IEEE ICWITS 2010)*, Honolulu, USA. Paper WITS1258.pdf, 28 August-3 September 2010.

[3] M. Farmahini-Farahani, R. Faraji-Dana, and M. Shahabadi, "Fast and Accurate Cascaded Particle Swarm Gradient Optimization Method for Solving 2-D Inverse Scattering Problems," *Applied Computational Electromagnetics Society (ACES) Journal*, vol. 24, no. 4, pp. 511 – 517, October 2009.

[4] R. E. Ziemer and W. H. Tranter, "Principles of Communication Systems, Modulation and Noise," *Haughton Mifflin Company*, Boston 1976.