Fractal Interpolation Function based Thin Wire Antennas

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Abstract – This paper presents an approach for the design of wire antennas based on fractal interpolation functions (FIFs). The interpolation points and the contraction factors of the FIFs are chosen as free parameters to modify the antenna geometry. The proposed structures' gain and radiation pattern can be optimized using FIF parameters. Producible prefractal antennas obtained in the intermediate iterations of fractal generation have compact sizes compared to classical counterparts. The error in prefractal geometry and the original fractal is bounded, and can be determined in terms of the finest producible detail's dimensions. The emerging structures have multiband behavior due to their self-similar and symmetric nature. To illustrate the approach, we have provided finite element based simulations for several prefractal antennas. $|S_{11}|$, the gain, the radiation efficiency, the radiation patterns, and feed point impedances for the demonstrated antennas are calculated numerically. The results indicate that produced antennas can be used in applications that require limited mechanical size, multiple operating bands, and controlled radiation patterns.

Index Terms – Fractal antenna, fractal interpolation functions, iterated function systems.

I. INTRODUCTION

Recent developments in wireless communications systems require more compact, wider bandwidth, multiband, and low-cost antennas. Fractal antennas can fulfill these requirements due to scale invariance, self-similarity, and space-filling properties of the fractals [1]. These properties enable the miniaturization of antenna structures [2–5]. The fractal structures can be designed to increase the effective physical length of the antennas to achieve multiband behavior in a limited space [6–8]. Basic fractal geometries such as Koch and Hilbert curves, and Sierpinski carpet have been studied for their radiation characteristics in the literature widely [9–15]. Comprehensive and up-to-date reviews can be found in [16, 17].

However, the studies rarely relate the mathematical properties of fractals to the antenna radiation characteristics. One approach is to optimize the antenna geometry over the fractal dimension using genetic algorithms [18, 19] directly. The authors present the relation between the resonant frequencies and the fractal dimension of the parameterized Koch curves in [20]. In general, the studies in the literature focus on predefined well-known fractal templates such as variants of the infamous Koch curve or the Sierpinski carpet. On the other hand, restricting the geometry a priori limits the practical applications.

As a novel approach, we present fractal wire antenna geometries based on the FIFs. In contrast to the literature, we don't assume a predefined topology in this study, and the designer is in full control of the antenna's shape by setting a few interpolation points and contraction factors. The interpolation points and the contraction factors of the FIFs can be used to optimize the antennas for a specific purpose. Then, we investigate the effects of fractal parameters on antenna radiation properties, namely the resonant frequencies, the bandwidth, the radiation patterns, gain, and input impedance.

Fractal interpolation is a technique used to construct continuous functions whose graphs are fractals based on iterated function systems (IFS) [21, 22]. Following the pioneering research, FIFs have been applied in geometric design, signal processing, and wavelet theory in the context of engineering, physics, and chemistry [23, 24]. FIFs provide non-smooth alternatives to traditional smooth interpolation techniques and are more suitable for irregular curves that display self-similarity.

Fractal interpolation is an iterative procedure, and each iteration can be considered a *prefractal*. The various antennas can be constructed associated with each of the prefractals. The skeleton of the antenna geometry can be determined by the given interpolation points. Additionally, the FIFs have free parameters that can be used to manipulate the geometry to alter the fractal dimension and the symmetry of the structure. The antenna can be

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optimized by changing the interpolation points and the free parameters. Therefore, it can be constructed without a predefined fractal template in order to optimize the antenna performance. Note that several structures such as the Koch curve can also be obtained by specific choice of FIF parameters.

To demonstrate the proposed approach, we generated a simple curved wire dipole antenna using FIFs based on affine transformations. The parameters of affine transformations consist of contraction factors on the horizontal axis and scaling factors on the vertical axis, which simply controls the antenna geometry.

The scattering parameter $|S_{11}|$, the input impedance, the gain, and the bandwidth of the constructed structure are calculated via extensive numerical simulations.

We have observed that even the simple structure can show multiband behavior for prefractals obtained at each iteration. Using the proposed procedure, the designer has flexibility in the determination of the skeletal structure of the antenna first. Afterwards, the vertical scaling parameters that are particularly significant on fractal properties can be used to optimize the antenna for a specific application. By means of this flexibility, the technique can be extended to design effective antennas confined in a limited space especially.

II. FRACTAL INTERPOLATION

Let the set of interpolation points, $\{[x_i, y_i]^T \in \mathbb{R}^2 : i = 0,1,2,...,N\}$ where $x_0 < x_1 < \cdots < x_N$ be given, and the continuous function $h : [x_0, x_N] \mapsto \mathbb{R}$ that satisfies $h(x_i) = y_i$, be the interpolation function.

We can construct an IFS from a set of contractive shear transformations $w_i : \mathbb{R}^2 \to \mathbb{R}^2$, $i = 1,2,\ldots,N$, of the form

$$w_i\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} \alpha_i & 0 \\ \beta_i & \gamma_i \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} u_i \\ v_i \end{bmatrix}, \quad (1)$$

such that its attractor is the graph of continuous function h. Clearly, $0 \le |\alpha_i|, |\gamma_i| < 1$, $\forall i$. The contraction factor of w_i is than $\sigma_i = \max\{|\alpha_i|, |\gamma_i|\}$, and the contraction factor of the IFS is $\sigma = \max_i \sigma_i$.

Following the steps in [23] and choosing γ_i's as free parameters, one can construct w_i's in such a way that the line segment between $[x_0, y_0]^T$ and $[x_N, y_N]^T$ is mapped to the line segment between $[x_{i-1}, y_{i-1}]^T$ and $[x_i, y_i]^T$. Therefore, the parameters must be chosen to satisfy

$$\begin{aligned} \alpha_i &= \frac{x_i - x_{i-1}}{x_N - x_0}, \quad u_i = \frac{x_N x_{i-1} - x_0 x_i}{x_N - x_0}, \\ \beta_i &= \frac{y_i - y_{i-1}}{x_N - x_0} - \gamma_i \frac{y_N - y_0}{x_N - x_0}, \\ v_i &= \frac{x_N y_{i-1} - x_0 y_i}{x_N - x_0} - \gamma_i \frac{x_N y_0 - x_0 y_N}{x_N - x_0}. \end{aligned} \quad (2)$$

Denoting $\mathscr{F}$ as the space of continuous functions $h : [x_0, x_N] \to \mathbb{R}$ such that $h(x_0) = y_0$ and $h(x_N) = y_N$ with a metric $d(h,g) = \max\{|h(x) - g(x)|,\ h,g \in \mathscr{F}\}$, lets us define a transformation $T : \mathscr{F} \to \mathscr{F}$ that satisfies

$$\begin{aligned} (Th)(x) &= \beta_i l_i^{-1}(x) + \gamma_i h(l_i^{-1}(x)) + v_i, \\ l_i(x) &= \alpha_i x + u_i \qquad i = 1,2,\ldots,N, \end{aligned} \quad (3)$$

for $x \in [x_{i-1}, x_i]$. T is a contraction in the metric space $\mathscr{F}$ with contraction factor $\sigma_T = \max\{|\gamma_i|\}$ and has a unique fixed point h^*, i.e. $(Th^*)(x) = h^*(x), \forall x \in [x_0, x_N]$ [22]. For any $h^{[0]} \in \mathscr{F}$, the sequence of functions for $k = 1,2,\ldots$

$$h^{[k]}(x) = (Th^{[k-1]})(x) \quad \forall x \in [x_0, x_N], \quad (4)$$

converges to h^*, i.e.,

$$\lim_{k\to\infty} h^{[k]}(x) = h^*(x), \quad \forall x \in [x_0, x_N]\,. \quad (5)$$

Furthermore, the points on the attractor of the IFS is determined by the function h^* since

$$(Th)(\alpha_i x + u_i) = \beta_i x + \gamma_i h(x) + v_i,\ \forall x \in [x_{i-1}, x_i]. \quad (6)$$

We consider each set $\{[x, h^{[k]}(x)]^T \in \mathbb{R}^2, \forall x \in [x_0, x_N]\}$ associated with $h^{[k]}$ as a *prefractal* and a candidate antenna. Given the transformations w_i, and $h^{[0]}(x) \equiv 0$, we can construct the geometry of the antenna using (3) and the random iteration algorithm for IFS [22]. The convergence rate to final attractor depends on the contraction factor σ_T. Given $0 < \varepsilon \ll 1$, the convergence can be assumed if

$$\begin{aligned} d(h^{[k]}, h^{[k-1]}) &\le \sigma_T^{k-1} d(Th^{[0]}, h^{[0]}) \\ &= \sigma_T^{k-1} \max_{x\in[x_0,x_N]} \left\{ \left| \beta_i \frac{x - u_i}{\alpha_i} + v_i \right| \right\}_{i=1}^{N} \le \varepsilon, \end{aligned} \quad (7)$$

is satisfied. Clearly, σ_T depends on the number of interpolation points and chosen γ_i; hence the designer has two means of controlling how fast the convergence to h^* is. ε can be chosen according to the finest detail that can be manufactured in practice, and the necessary number of iterations, k, in (3) determined accordingly. We also have

$$d(h^*, h^{[k]}) \le \frac{\sigma_T}{1 - \sigma_T} d(h^{[k-1]}, h^{[k]}), \quad (8)$$

in order to measure how close the prefractal associated with $h^{[k]}$ is to the fractal associated with h^*.

Note that the selection γ_i's has a significant impact on the overall topology of the FIF, as depicted in Fig. 1.

The fractal dimension, D, of the final attractor of the associated IFS satisfies

$$D = 1 + \begin{cases} \frac{\log\left(\sum_{i=1}^{N} |\gamma_i|\right)}{\log(N)}, & \sum_{i=1}^{N} |\gamma_i| > 1 \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

Hence $1 \le D < 2$ if the interpolation points are spaced equally. Clearly, we have absolute control of the fractal's dimension and the complexity [22].

If $\gamma_i = \gamma = 0, \forall i$, h^* corresponds to the linear interpolator. Besides, the small contraction factor ($\sigma_T \to 0$) yields 1D fractals without much detail, and associated prefractals are not of much interest. The choice of

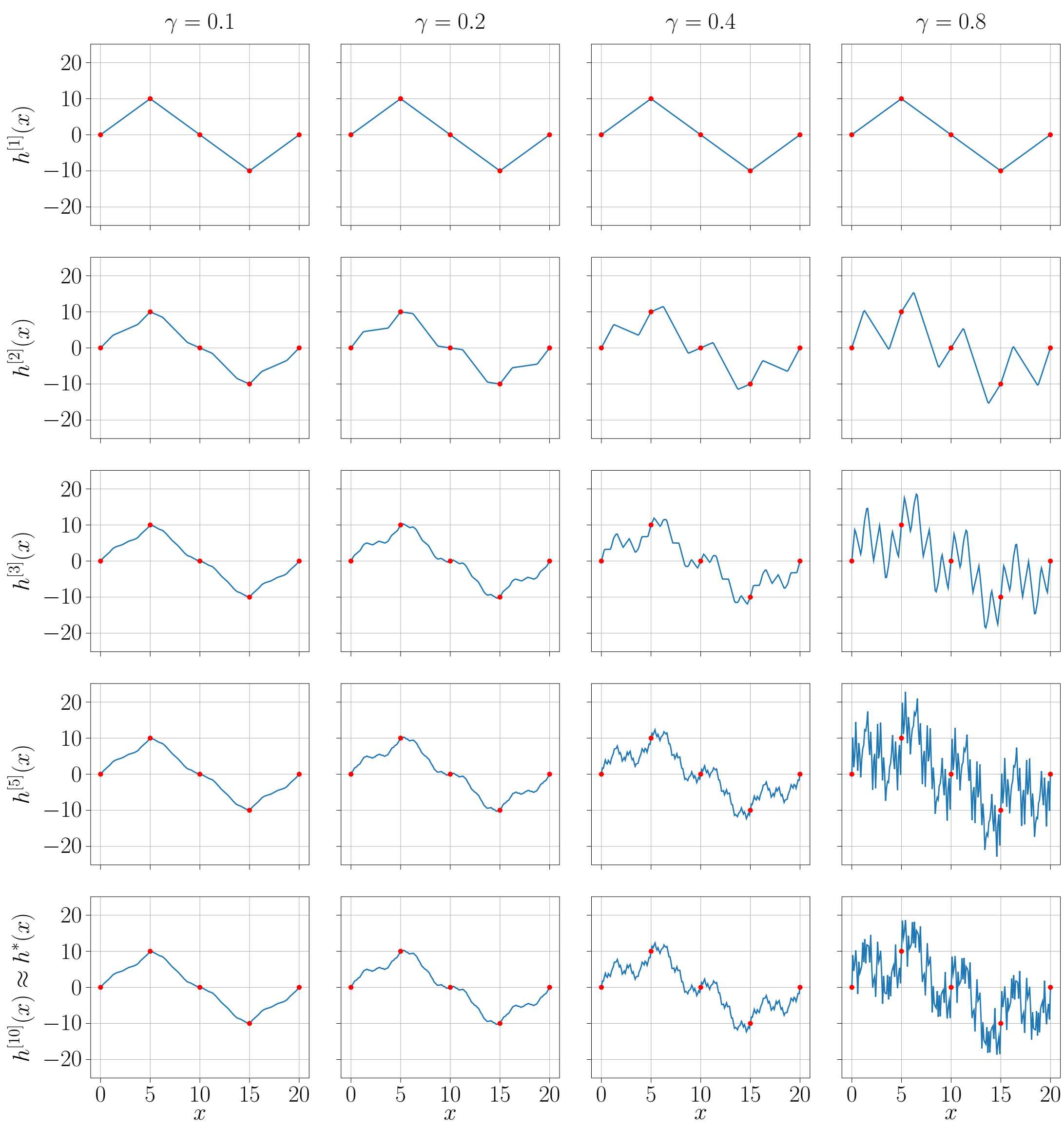


Fig. 1. FIF's corresponding to prefractals for the parameters in Table 1. The red dots indicate interpolation points. The fractal dimensions for $\gamma_i = \gamma \leq 0.25$, $\gamma = 0.4$, and $\gamma = 0.8$ are $D = 1$, $D \approx 1.34$, and $D \approx 1.84$ for the graph of $h^*(x)$ respectively.

Table 1: The parameters of FIF wire prefractal antennas for $\gamma = 0.2$ and 0.4 in Fig. 1

Parameter	**Value**	**Description**
N	5	Number of interpolation points
$\begin{bmatrix} z_i \\ y_i \end{bmatrix}$, $i = 1, 2, ..., 5$	$\begin{bmatrix} 0.5 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 5 & 10 & 15 & 20 \\ 0 & 10 & 0 & -10 & 0 \end{bmatrix}$	Interpolation points (mm), $y_i = h^{[k]}(z_i)$
α_i, $i = 1, 2, 3, 4$	0.25	z-scaling factor
β_i, $i = 1, 2, 3, 4$	$\dfrac{y_i - y_{i-1}}{20}$	y-scaling factor
u_i, $i = 1, 2, 3, 4$	z_{i-1}	z-translation
v_i, $i = 1, 2, 3, 4$	y_{i-1}	y-translation
r_0	0.005	Radius of the antenna wire (mm)

$\sum_{i=1}^{N} |\gamma_i| > 1$ results in fractals with self similar and symmetric structures with fine details. On the other hand, $|\gamma_i| \to 1$ implies $\sum_{i=1}^{N} |\gamma_i| \to N$ and as a result $D \to 2$. The attractor associated with the underlying IFS has finer details in this case. However, the details of the associated prefractals may not be suitable for manufacturing for large k.

The parameter ε in (7) can be chosen with respect to the finest producible detail. Hence, k can be determined automatically. Then the error estimate between the prefractal at k^{th} iteration and the attractor can be estimated by (8). Although the fractal structures are more interesting in terms of radiation properties as $D \to 2$, the practical realization can be cumbersome for a given accuracy due to large number of required iterations, yielding extremely fine details.

The main advantage of using FIF as the basis for the structure of the antenna is its flexibility. The designer can optimize the structure by a few number of points on the structure ($\{x_i, h(x_i)\}_{i=1}^{N}$) and altering the free parameters ($\{\gamma_i\}_{i=1}^{N}$) for desired radiation properties.

III. ILLUSTRATIVE EXAMPLE

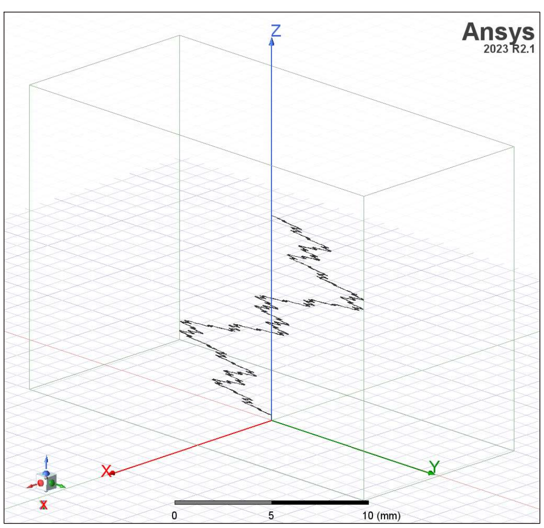
To illustrate the approach, we present FIF perfectly conducting thin wire prefractal antennas oriented along z-axis embedded in yz-plane as a proof of concept.

The simulations were carried out using Ansoft High Frequency Structural Simulator (HFSS)™, on an Intel Xeon based workstation with 32 physical cores and 256 Gb of memory.

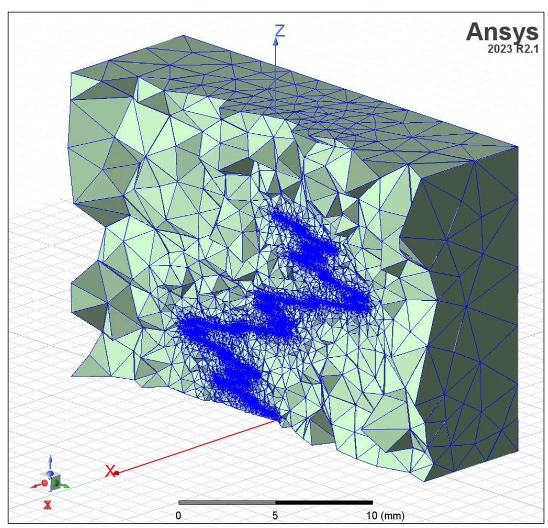
The antennas have been fed through a gap of 1 mm located at the origin with a 50 Ω lumped port. The common parameters for the design are listed in Fig. 1. The prefractal curves that form the antennas are obtained by running iterations in (4) with Julia programming language [25]. The generated curves are imported to HFSS for further processing. A circle of radius $r_0 = 5\ \mu m$ has been extruded along the imported path to create the 3D model. The 3D model is simplified to exclude irrelevant details with respect to operating wave length. The 3D models are simulated using finite element method (FEM). The mesh used in FEM has been fine-tuned with adaptive meshing. Only half $z \geq 0$ plane is considered with an electric symmetry boundary at xy-plane (Fig. 2 (a)). The largest FEM model had 852,603 mixed order tetrahedral elements for the case with $\gamma = 0.4$ and $k = 10$ (Fig. 2 (b)).

The resonant frequencies, 10 dB bandwidths, peak gains, and feed point impedances are listed in Table 2. The radiation efficiencies have been confirmed to be unity in all cases listed, as the antennas have been assumed to be perfect electric conductors. The frequency sweep analysis in the range of 0.8GHz $\leq f \leq$ 8GHz is shown in Fig. 3. Note that the case with $k = 1$ corresponds to linear interpolation over the set $\{x_i, h(x_i)\}_{i=1}^{N}$, and its shape is independent of the contraction factors $\{\gamma_i\}_{i=1^N}$. It is a simple bend wire dipole antenna. The decrease in the first mode's frequency and the emergence of several other modes is apparent with respect to the reference bend wire dipole.

When the contraction factor is close to 0, the prefractals in each iteration converge to a simple wire antenna with slight decrease in the resonant frequencies for increasing k (Table 2). This is expected since the



(a)



(b)

Fig. 2. HFSS 3D model for $\gamma = 0.4$, $k = 10$. (b) Corresponding mesh. yz-plane is set to be a perfect electric symmetry boundary to for a smaller FEM model. The other outer surfaces of the mesh region is set to be radiation boundaries.

Table 2: The properties of the simulated FIF wire prefractal antennas

γ	k	**Frequency** [GHz]	**Bandwidth** [MHz]	**Gain**	**Impedance** [Ω]
-	1	1.807	126	0.47	$67.2 - j2.0$
		5.277	146	1.36	$53.3 - j0.0$
0.2	3	1.653	108	0.40	$64.4 - j1.4$
		4.838	115	1.27	$45.5 - j0.1$
	10	1.602	90	0.38	$64.2 - j0.5$
		4.636	94	1.01	$46.6 - j0.1$
0.4	3	1.088	56	0.2	$57.4 - j0.8$
		3.043	37	0.42	$34.4 - j0.1$
		4.832	49	2.09	$78.9 - j0.4$
		6.484	47	2.09	$83.5 + j0.1$
	10	0.866	42	0.13	$55.2 - j0.9$
		2.468	28	0.29	$35.4 + j0.3$
		3.824	39	1.40	$62.3 - j2.8$
		5.058	45	1.90	$46.8 - j1.2$

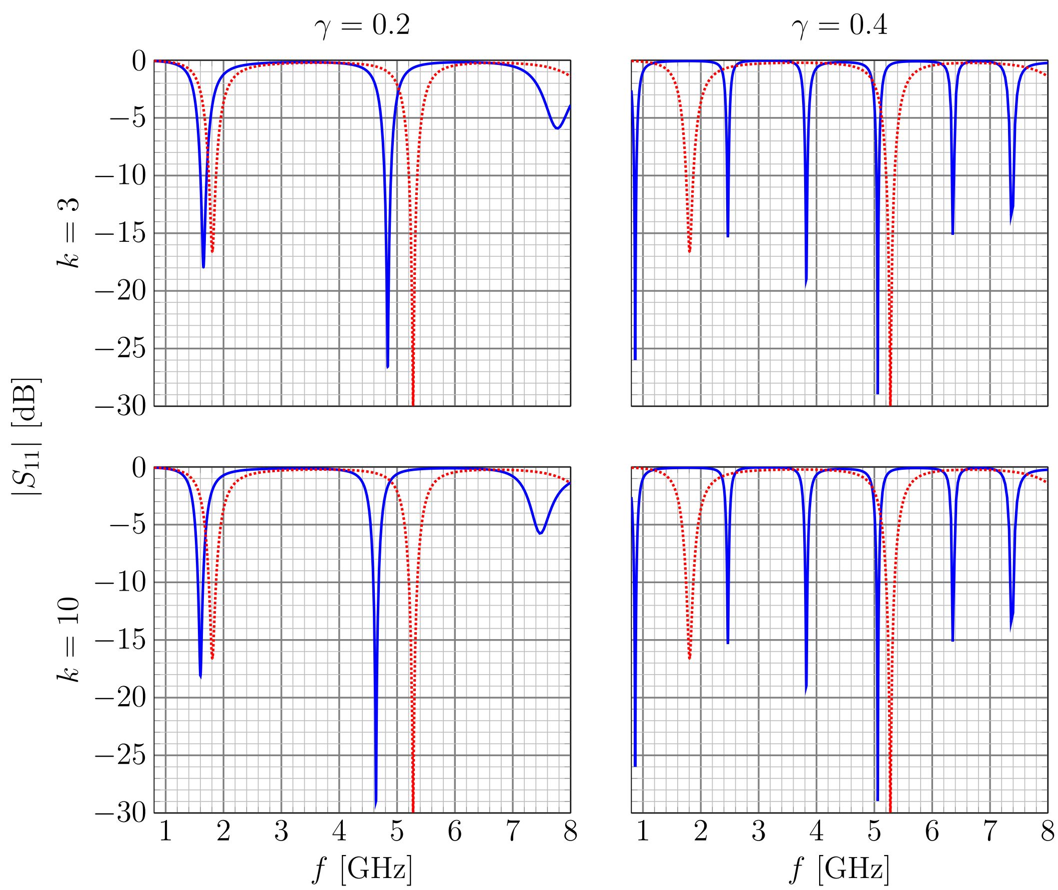


Fig. 3. $|S_{11}|$ versus the frequency f for varying k and γ. (—) and (······) indicates the prefractal and the reference bend wire antenna corresponding to $k = 1$ case, respectively.

physical length of the antenna increases with successive iterations of FIF as well. More interesting results are observed when $\gamma = 0.4$. Several new bands of operation with excellent matching emerge as k increases. Besides, the deviation in resonant frequencies is more pronounced compared to $\gamma = 0.2$. For larger contraction factors, the antenna is still confined to the same space compactly, although it is electrically longer.

The normalized radiation patterns for $\gamma = 0.4$ are presented in Fig. 4. The multi-directional radiations

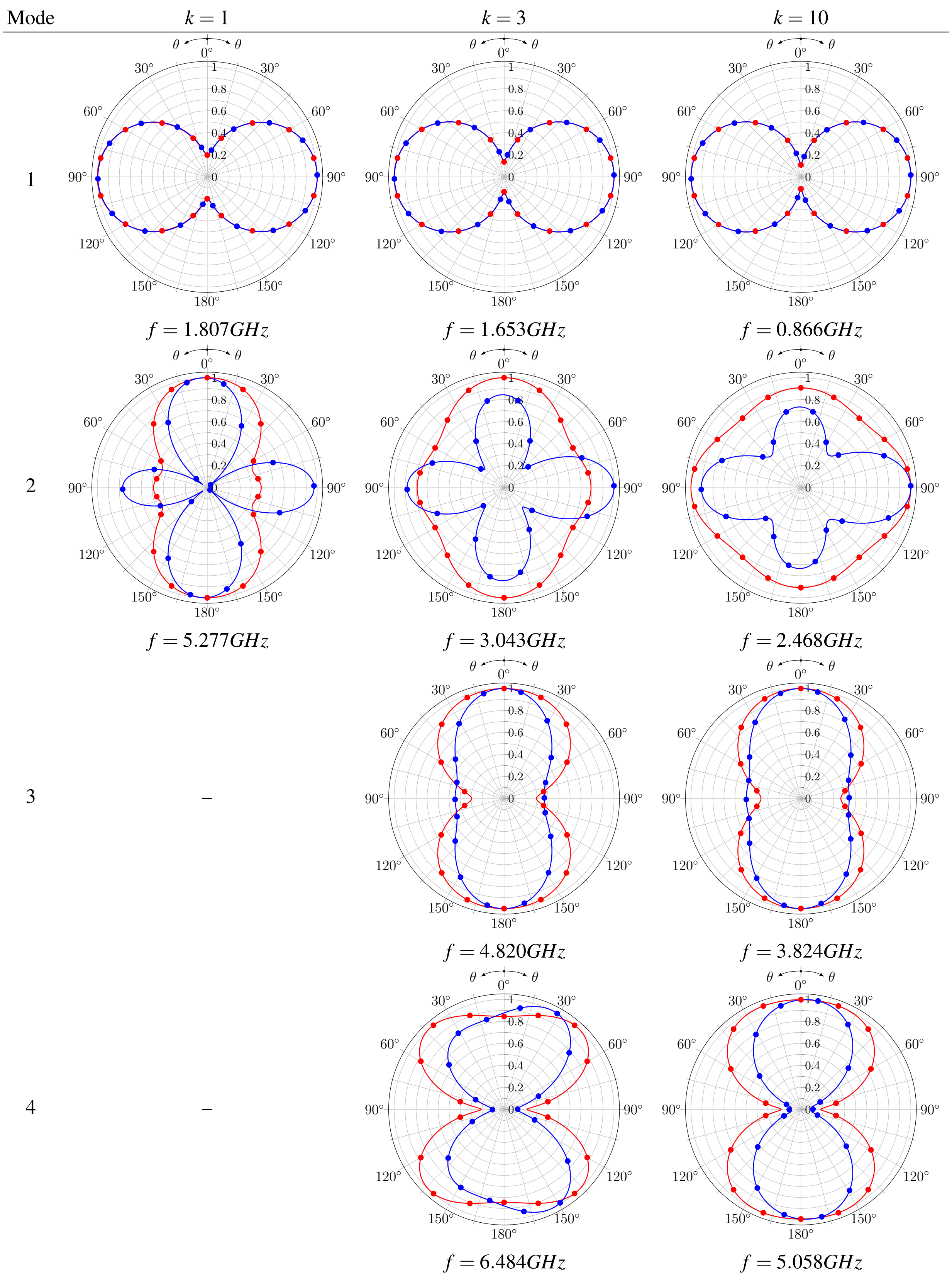


Fig. 4. The H-plane (: $\phi = 0^\circ$), and E-plane (: $\phi = 90^\circ$) normalized radiation patterns for the first four modes in frequency range $0.8GHz \leq f \leq 8GHz$ with $\gamma = 0.4$.

patterns emerge for various operating frequencies. Note that although the antenna's orientation was kept fixed along z-axis, the radiation patterns are almost perpendicular in mode 1 ($f = 0.866$ GHz) and mode 4 ($f = 5.058$ GHz) for $k = 10$ and $\gamma = 0.4$. The fractal structure allows such possibilities, which would not be available in classical wire antennas.

IV. CONCLUSION

In this article, we propose an approach based on FIFs to design fractal wire antennas. In this approach the geometry need not be predetermined, but can be altered flexibly, in contrast to the fractal antenna studies in the literature. The geometry of the antenna can be controlled by the free parameters, i.e., the interpolation points and the contraction factors of the FIF. Therefore, the antenna's radiation properties can be controlled directly. The FIF parameters can be adjusted to optimize the performance for the desired antenna properties in terms of gain, radiation pattern, and matching. Furthermore, the optimization can be carried out for multiple bands of operation under spatial constraints.

One of the advantages of the proposed approach is the possibility to bound errors between the prefactals obtained in the intermediate iterations of fractal generation and ideal fractals. This is particularly important because it is impossible to manufacture the infinite details of the ideal fractal. We can determine the required number of iterations a priori for a given manufacturing tolerance based on the constructiveness of the underlying transformations leading to FIF for given antenna performance measures.

The fractal nature of the designed geometries allows the apparent electrical length of the antenna to be larger than the equivalent dipole fitted to the same limited space. In other words, the antenna size can be miniaturized relative to classical structures while operating at low frequencies. Additionally, the self-similarity of the generated fractals results in multiband behavior. These properties render proposed antennas suitable for mobile and wearable wireless applications that require long-range communication especially.

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