Scattering and Diffraction Evaluated by Physical Optics Surface Current on a Truncated Cylindrical Conductive Cap

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Abstract – In this study physical optics (PO) surface current is obtained by using the Malyughinetz solution to get the scattered field expression for a truncated cylindrical conductive cap satisfying the related boundary conditions given throughout this paper. This is done by using the inverse edge point method for the transformation from the Malyughinetz solution for the wave diffraction by a half plane to the wave diffraction by a truncated conductive cylinder. This transformation method can be used to examine the diffraction and scattering phenomena for curved surfaces having discontinuities as dealt with in this work. Total scattered field comprising the incident and scattered fields is plotted with respect to the observation angle for some parameters of the problem. The obtained results are examined numerically for the same parameters.

Index Terms – conductive surface, diffraction, physical optics, scattering, truncated cylindrical cap.

I. INTRODUCTION

Scattering and diffraction by the objects of curved shapes have been under investigation for decades. Franz and Klante examined the diffraction phenomenon by the variable-curvature surfaces by applying the integral equation method to a convex cylinder [1]. Hong presented a method to obtain successive terms in short wavelength asymptotic expansions of the diffracted field by a smooth convex surface on which a plane acoustic or electromagnetic wave is incident [2]. Bahar derived a solution for the diffracted fields around a convex cylindrical boundary having a varying radius of curvature and surface impedance [3]. In the study of Idemen and Felsen, whispering gallery mode diffraction of a thin, concave, cylindrically curved surface was analyzed by using the Fourier transform method [4]. Idemen and Erdoğan examined the diffraction of creeping waves by a spherical reflector, and derived diffraction coefficient expressions [5]. In the study of Serbest, diffraction of whispering gallery modes by a conducting spherical reflector was examined, and diffraction or transformation coefficients were obtained [6]. Büyükaksoy studied the diffraction at high frequencies by the edge of a cylindrically curved surface whose convex and concave sides show soft and hard boundary conditions [7]. New diffraction coefficients for the mixed boundary conditions were also defined in the study. Hansen and Shore obtained the inremental length diffraction coefficients at the shadow boundaries for a perfectly electric conducting convex cylinder [8]. Yalçın investigated the scattering by a perfectly conducting cylindrical reflector by using the Modified Theory of Physical Optics (MTPO) [9]. By using the surface integrals of the MTPO, Umul investigated the scattering phenomenon by a cylindrical parabolic impedance reflector [10]. Umul transformed Malyughinetz solution for the scattering by a half screen of equal face impedances to a Physical Optics (PO) integral to employ it in the problem of diffraction by a truncated impedance cylinder [11]. Andronov examined the diffraction of a high-frequency plane wave by an infinite cylinder having a strongly prolate ellipse cross-section [12]. Andronov and Lavrov investigated the scattering by an elliptic cylinder having strongly elongated cross-section by obtaining uniform asymptotic expressions [13]. Başdemir investigated the diffraction of inhomogeneous plane waves for a truncated cylindrical cap [14]. Shanin and Korolkov analyzed the diffraction by a parabola by introducing a Volterra-type boundary integral equation [15]. We investigated the scattering of plane waves by a cylindrical, parabolic, perfectly electric conductor reflector [16]. In this study which will be a new work in the literature, our aim is to use the field expression of a plane wavediffracted by a half plane for

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obtaining the PO surface current on the truncated cylindrical conductive cap. The novelty of this study is that, as a new approach, instead of solving the scattering problem in a conventional way, we will transform the solution of wave scattering problem by a half plane into PO integral via inverse edge point method for a truncated cylindrical conductive cap. PO current that can be used in wide variety of problems will be employed to express the scattered field. For scattering, PO method could be widely used in various situations including mesh-type antennas and cross-section applications.

II. THEORY

The geometry of the problem is given in Fig. 1 where the radius of the cylinder is a, and the incident plane wave is taken as

$$E_i = E_0 e^{jkx}, \tag{1}$$

where E_0 is the amplitude, and k is the wavenumber.

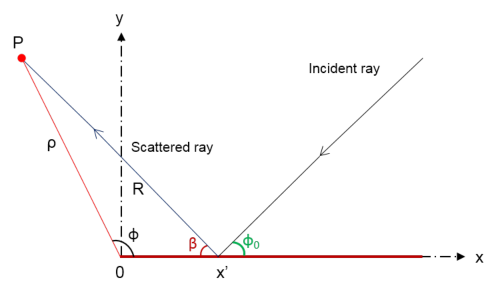

Fig. 1. Scattered beam geometry for the conductive half-plane.

The truncated cylindrical cap is located symmetrically with respect to the x-axis, $\phi \in [-\phi_0, \phi_0]$ and $z \in (-\infty, \infty)$. Boundary conditions for the conductive surface are given as

$$\vec{n} \times (\vec{H}_1 - \vec{H}_2) \mid_S = 0, \tag{2}$$

and

$$\vec{n} \times (\vec{n} \times \vec{H}_1) \mid_S = R_m \vec{n} \times (\vec{E}_2 - \vec{E}_1) \mid_S, \tag{3}$$

where $\vec{n}$ is the unit vector of the surface, and $\vec{E}$ and $\vec{H}$ are the electric and magnetic fields respectively. $\vec{E}_1$ and $\vec{H}_1$ are the fields on the convex side, and $\vec{E}_2$ and $\vec{H}_2$ are the ones on the concave side of the cylinder. The diffracted field in [18, 19], can be rewritten for a conductive half plane as

$$E_d = -\frac{E_0 2cos(\frac{\phi}{2})cos(\frac{\phi_0}{2})}{sin\theta(cos\phi + cos\phi_0)} K_+(\phi,\theta)K_+(\phi_0,\theta) \times \frac{exp(-j\frac{\pi}{4})}{\sqrt{2\pi}} \frac{exp(-jk\rho)}{\sqrt{k\rho}}, \tag{4}$$

where

$$sin\theta = 2R_m Z_0. \tag{5}$$

R_m is the conductivity of the surface, and Z_0 is the characteristic impedance of free space. Scattered field is expressed as

$$E_s = E_0 \frac{kexp(-\frac{j\pi}{4})}{\sqrt{2\pi}} \times \int_0^\infty J_{PO} \frac{exp(-jkR)}{\sqrt{kR}} exp(jkx' cos\phi_0) dx', \tag{6}$$

where J_{PO} is the physical optics surface current, and

$$R = \sqrt{\rho^2 + x'^2 - 2\rho x' cos\phi}. \tag{7}$$

Diffracted field at the edge point can be written as

$$E_d = -\frac{1}{jk} \frac{kE_0 exp(\frac{j\pi}{4})}{\sqrt{2\pi}} \times J_{PO}(x'=0) \frac{exp(-jk\rho)}{\sqrt{k\rho}} \frac{1}{cos\phi + cos\phi_0}. \tag{8}$$

At the edge point J_{PO} is concluded as

$$J_{PO}(x'=0) = 2cos(\frac{\phi}{2})cos(\frac{\phi_0}{2})K_+(\phi,\theta)K_+(\phi_0,\theta), \tag{9}$$

where K_+ is the split function first given by Senior [17]. Split function is expressed by means of Malyughinetz function by Senior [18], and Senior and Volakis [19]. Split functions are used for the transition from geometrical optics fields to diffracted fields. K_+ has the form of

$$K_+(\alpha,\beta) = \frac{4\sqrt{sin\beta}sin(\frac{\alpha}{2})}{(1+\sqrt{2}cos(\frac{\frac{\pi}{2}-\alpha+\beta}{2}))(1+\sqrt{2}cos(\frac{\frac{3\pi}{2}-\alpha-\beta}{2}))} \times (\frac{\psi_\pi(\frac{3\pi}{2}-\alpha-\beta)\psi_\pi(\frac{\pi}{2}-\alpha+\beta)}{(\psi_\pi(\frac{\pi}{2}))^2})^2, \tag{10}$$

where $\psi_\pi(\delta)$ is the Malyughinetz function that is written as

$$\psi_\pi(a) = exp(-\frac{1}{8\pi}\int_0^a \frac{\pi sinx - 2\sqrt{2}\pi sin(\frac{x}{2}) + 2x}{cosx} dx). \tag{11}$$

In Fig. 2, β_e is the diffraction angle of the ray at the edge point, and incident ray is parallel to the x-axis. Scattered field is written as

$$E_s = E_0 \frac{kexp(-\frac{j\pi}{4})}{\sqrt{2\pi}} \times \int_{-\phi_0}^{\phi_0} J_{PO} \frac{exp(-jkR)}{\sqrt{kR}} exp(jkacos\phi_0) d\phi', \tag{12}$$

where

$$R = \sqrt{\rho^2 + a^2 - 2\rho a' cos(\phi - \phi')}. \tag{13}$$

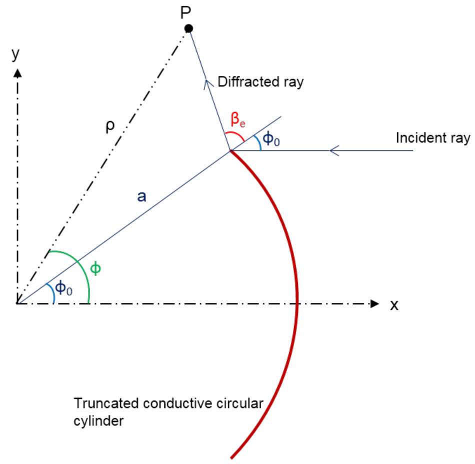

Fig. 2. Diffracted beam geometry for a truncated conductive circular cylinder.

Considering Fig. 1 at the edge point, it is written that $J_{PO}(\beta)$= $J_{PO}(\pi-\phi)$. At any point x', exact J_{PO} is expressed with respect to β as

$$J_{PO_{exact}}(\beta)=2\sin(\frac{\beta}{2})cos(\frac{\phi_0}{2})K_+(\phi,\theta)K_+(\phi_0,\theta), \tag{14}$$

which can be used in the scattered field expression in equation (12). By considering the equations (8) and (14), the incident diffracted field can be concluded as

$$E_{id}=\frac{J_{PO_{exact}}(\beta)}{\sin\phi_0}f_1, \tag{15}$$

where

$$f_1=E_0sin(\frac{\beta_e+\phi_0}{2})exp(jka\cos(\phi_0)) \\ \times exp(-jkR_e\cos(\beta_e+\phi_0))sign(\xi_{id})F[|\xi_{id}|], \tag{16}$$

and

$$\xi_{id}=-\sqrt{2kR_e}\sin(\frac{\phi+\phi_0}{2}), \tag{17}$$

where R_e is the distance between the edge and observation points. The reflected diffracted field is written as

$$E_{rd}=\frac{J_{PO_{exact}}(\beta)}{\sin\phi_0}f_2, \tag{18}$$

where

$$f_2=E_0cos(\frac{\beta_e-\phi_0}{2})exp(jka\cos(\phi_0)) \\ \times exp(-jkR_e\cos(\beta_e-\phi_0))sign(\xi_{rd})F[|\xi_{rd}|], \tag{19}$$

and

$$\xi_{rd}=-\sqrt{2kR_e}\sin(\frac{\beta_e-\phi_0}{2}). \tag{20}$$

III. NUMERICAL RESULTS

In this section we will plot the total scattered field, which is the combination of incident and scattered fields, for some parameters of the problem. ρ is the observation distance, λ is the wavelength, and a is the radius of the cylinder cap. E_0 is taken as unity for simplicity. For the Figs. 3-6, total scattered fields are plotted with respect to the observation angle ϕ.

In Fig. 3, total scattered field variation according to theta is shown. $a=2\lambda$, $\rho=10\lambda$, and $\phi_0=\pi/3$ are taken. For $\phi\in[-\pi/6,\pi/6]$ and $\phi\in[5\pi/6,7\pi/6]$ total scattered field intensity almost does not change according to θ variation. But for For $\phi\in[\pi/6,5\pi/6]$ and $\phi\in[7\pi/6,11\pi/6]$, a change in the intensity occurs in a way that when the θ decreases, the intensity increases. As expected, intensity decays as the value of θ increases when considering equation (12) in which J_{PO} causes this variation. Also, an increase in θ implies an increase in the conductivity which results in a decrease in the total scattered field.

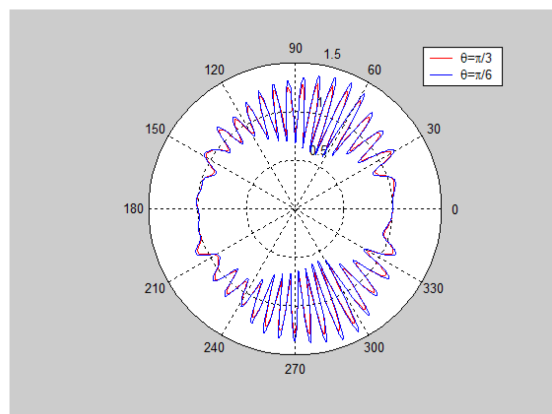

Fig. 3. Total scattered field variation for some θ values.

In Fig. 4, total scattered field is plotted according to cylinder radius a for the parameter values of $\rho=6\lambda$, and $\phi_0=\theta=\pi/3$. It is observed that between $\phi=-\pi/3$ and $\phi=\pi/3$ the field intensity decreases as the radius of the cylinder increases. The intensities between $\phi=-\pi/2$ and $\phi=\pi/2$ scattered field becomes negligible, and incident wave determines the resultant field.

In Fig. 5, $\phi_0=\pi/3$, $a=3\lambda$, and $\theta=\pi/6$ are taken. Total scattered field is observed with respect to ϕ as the observation distance changes. Scattered field intensity tends to decrease with the increase in the observation distance.

In Fig. 6, total scattered field is plotted with respect to ϕ for two ϕ_0 values when a and θ are kept the same as for Fig. 5, $\rho=10\lambda$ is taken. For $\phi\in[-\pi/4,\pi/4]$ and $\phi\in[3\pi/4,5\pi/4]$ intervals, scattered field does not change when $\phi_0=\pi/3$ is replaced by $\pi/4$. However, it

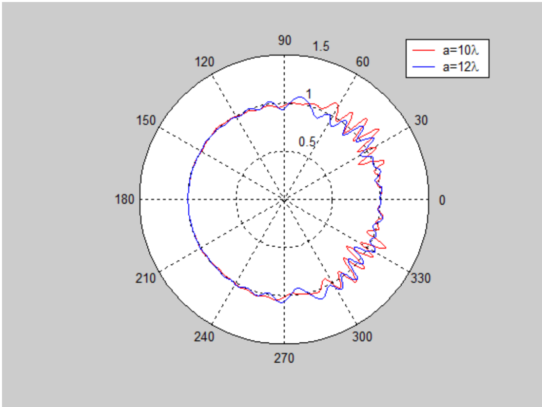

Fig. 4. Total scattered field variation for cylinder radius (a) values.

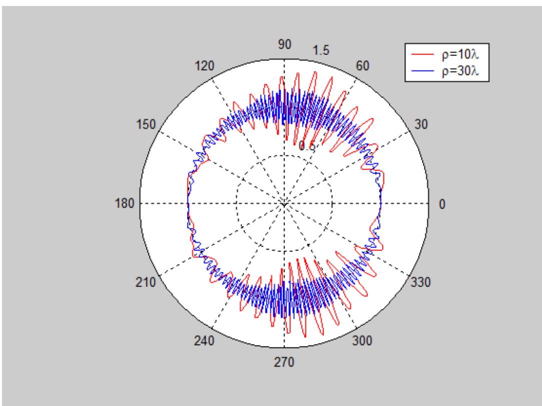

Fig. 5. Total scattered field variation according to observation distance ρ.

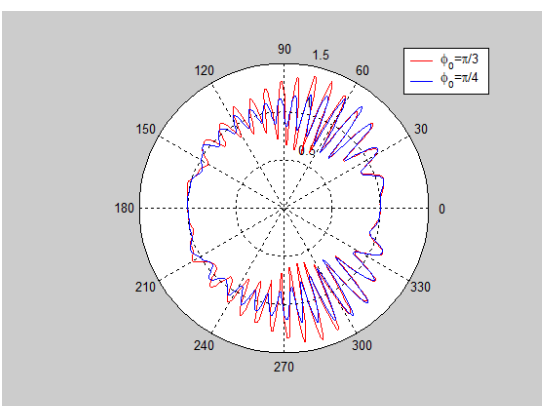

Fig. 6. Total scattered field variation according to angle of incidence ϕ_0.

is observed that for the intervals of $\phi \in [\pi/4, 3\pi/4]$ and $\phi \in [5\pi/4, 7\pi/4]$ field intensity at $\phi_0 = \pi/3$ decreases when ϕ_0 is replaced by $\pi/4$.

IV. CONCLUSION

In this work we obtained the exact physical optics (PO) surface current J_{PO} for a truncated conductive cylindrical cap by using the diffracted field expression for a conductive half plane. Diffracted field expression for the conductive half plane is obtained by employing the Malyughinetz solution which is used by Umul [11] to transform it to a PO integral for the diffraction problem of a truncated impedance cylinder. Ultimate scattered field is obtained by substituting the exact PO current J_{PO} into the scattering integral. Total scattered fields according to observation distance and radius of the cylinder cap are examined. It is concluded that for both parameters, total scattered field intensity approaches the intensity of incident plane wave as the observation distance or the cylinder radius increases. The intensity variation according to theta is observed as inversely proportional in some intervals of ϕ, and almost constant in other intervals. Finally, it is observed that as ϕ_0 increases, fringing in the intensity increases.

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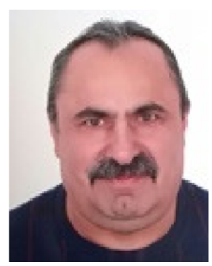

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