Novel Characteristic Mode Basis Functions Accelerating Iteration Convergence of CMM for Analyzing Electromagnetic Scattering Problems

Fei Guo[1], Zhonggen Wang[1*], Yufa Sun[2], Wenyan Nie[3], Pan Wang[1], and Juan Wu[1]

[1]School of Electrical and Information Engineering
Anhui University of Science and Technology, Huainan, 232001, China
fguo@aust.edu.cn, zgwang@ahu.edu.cn, pwangsunny@foxmail.com, yj165@mail.ustc.edu.cn
*Corresponding author: Zhonggen Wang

[2]School of Electronics and Information Engineering
Anhui University, Hefei, 230039, China
yfsun_ahu@sina.com

[3]School of Mechanical and Electrical Engineering
Huainan Normal University, Huainan, 232001, China
wynie5240@163.com

***Abstract* –** In this study, a novel characteristic mode basis function construction method is proposed to solve the problems of low efficiency and slow convergence of the iterative solution of the characteristic mode method. First, the characteristic modes (CMs) of each extended block are calculated separately by dividing objects, the CMs construction efficiency is improved for the decreased matrix dimensionality. Next, the effective CMs are selected as basis functions according to the modal significances. Finally, to improve the poor conditional number of the reduced matrix for the block extension, the singular value decomposition is used to enhance the orthogonality among these basis functions. Several numerical calculations show that the proposed method has significant efficiency and accuracy.

***Index Terms* –** Characteristic mode, method of moments, reduced matrix, singular value decomposition.

I. INTRODUCTION

The method of moments (MoM) [1] is a popular method for solving electromagnetic scattering, which has the advantage of high computational accuracy. However, as the size of the object increases, there is a substantial escalation in computational complexity and memory usage. To alleviate this problem, various fast and effective methods have been proposed, such as the fast multipole method (FMM) [2], multilevel fast multipole method (MLFMM) [3], adaptive cross approximation (ACA) [4], and adaptive integration method (AIM) [5]. These methods effectively reduce the complexity of matrix-vector product computation, but they usually resort to iterative methods. In order to reduce the dimensionality of the matrix, macro basis function is introduced into the MoM. For example, the synthetic basis function (SBF) method [6], sub-entire-domain (SED) basis function method [7], and characteristic basis function method (CBFM) [8]. Based on the principle of domain decomposition, the CBFM divides the object into multiple blocks. Nevertheless, it demands a substantial number of incident excitations, and the construction of basis functions consumes a considerable amount of time.

The characteristic mode (CM) is an intrinsic mode adapted to an arbitrary electromagnetic structure, independent of the applied excitation [9]. It is particularly suitable for the analysis of multi-excitation problems. Thus, the CM theory is widely used in the design and simulation of antennas [10–13], while it is rarely applied in the analysis of electromagnetic scattering problems. The traditional CM method (CMM) [14] presents a high computational complexity and low efficiency in solving CMs. Subsequently, it is unsuitable for large electrical problems. To improve the efficiency of CMM analysis of electromagnetic scattering problems, the CMs are used directly as the basis functions in [15]. Nevertheless, it is difficult to apply it to electrically large problems due to the complexity of the algorithm. Consequently, a fast multipole algorithm is introduced in [16] to accelerate the solution of CMs. However, the improvement is not significant due to the large size of the impedance matrix. In addition, the CMM is combined with compressive sensing, used to analyze the bistatic scattering problems in [17], but the dimensionality of the measurement

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matrix is large. Furthermore, considering the coupling between the incident wave and the dielectric body, a new CMM is proposed to analyze the scattering problems in [18]; however, the computation complexity of CMs is relatively high. Innovatively, a method proposed in [19] utilizes principal component analysis (PCA) to accelerate the iterative solution of CMM, which improved the solution efficiency.

Unlike the previous works, based on the idea of regional blocking, this study proposes a novel characteristic mode basis function method (NCMBFM). First, the object is divided into multiple blocks and extended, selecting effective modes as basis functions. Next, the orthogonality between these basis functions is strengthened by using the singular value decomposition (SVD) [20] method. As a result, the condition number of the reduced matrix is improved, which could effectively boost the efficiency of the iterative solution. The corresponding numerical results verify the efficiency and reliability of the proposed method in the analysis of electrically large problems.

II. THEORY

A. Characteristic mode method

The surface currents on the object are extended using the Rao-Wilton-Glisson (RWG) [21] basis function, and the MoM is employed to generate the matrix equation as follows:

$$\mathbf{ZJ} = \mathbf{V}, \tag{1}$$

where $\mathbf{Z}$ is an impedance matrix with the size of $N \times N$ and N is the number of unknowns; $\mathbf{J}$ and $\mathbf{V}$ represent the current coefficients vector and excitation vector, respectively. According to the CM theory, for an arbitrarily shaped PEC, the CMs are generated from its impedance matrix as follows:

$$\mathbf{Z} = \mathbf{R} + j\mathbf{X}, \tag{2}$$

$$\mathbf{X}\mathbf{J}^{\mathrm{CM}} = \lambda \mathbf{R}\mathbf{J}^{\mathrm{CM}}, \tag{3}$$

where $\mathbf{R}$ and $\mathbf{X}$ are the real and imaginary parts of $\mathbf{Z}$, respectively. $\mathbf{J}^{\mathrm{CM}}$ denotes the eigenvector, corresponding to the eigenvalue λ. Depending on the nature of the CMs, the superposition of a few low-order CMs is sufficient to approximate the surface currents. In this context, these low-order modes are chosen as basis functions. This selection approach relies on modal significances (MS), defined as

$$\mathrm{MS} = \left| \frac{1}{1 + j\lambda} \right|. \tag{4}$$

Moreover, a threshold τ_{cm} related to the MS is set, and then, a group of eigenvalues is determined based on $\mathrm{MS} > \tau_{cm}$. The corresponding significant CMs are obtained as the characteristic mode basis functions (CMBFs). Assuming that the total number of CMBFs is K, the surface currents of the object is expressed by a linear superposition of these CMBFs as

$$\mathbf{J} = \sum_{k=1}^{K} a^k \mathbf{J}^{\mathrm{CM}_k}, \tag{5}$$

where a^k is the coefficient of the CMBFs to be solved and $\mathbf{J}^{\mathrm{CM}_k}$ the kth CMBFs. Substituting equation (5) into equation (1) and multiplying both sides of the equation with the transpose of $\mathbf{J}^{\mathrm{CM}}$, a reduced matrix equation about a^k of reduced order is obtained, expressed as

$$\mathbf{Z}^{\mathrm{R}} a = \mathbf{V}^{\mathrm{R}}, \tag{6}$$

where $\mathbf{Z}^{\mathrm{R}}$ is a reduced matrix with a dimension of $K \times K$ and $\mathbf{V}^{\mathrm{R}} = (\mathbf{J}^{\mathrm{CM}})^{\mathrm{T}}\mathbf{V}$ is a $K \times 1$ vector. Furthermore, a is the matrix of extension coefficients obtained by solving equation (6).

However, when analyzing electrically large objects, as the matrix dimension increases, solving equation (3) becomes more complex, which poses challenges to the solution of the CMs.

B. Novel characteristic mode basis function method

Different from the CMM to construct CMBFs, this study builds on the idea of regional blocking to divide the object into m blocks. However, the blocking resulted in a change in the shape of the object, causing corresponding changes in the CMs of each block. As a consequence, these CMs could no longer accurately represent the original current distribution on each block. To solve this issue, each block is extended to ensure the smoothness and continuity of the characteristic current near the virtual boundary. The principle and application of blocking is shown in Fig. 1 (take a cylinder as the example). Where the blue part indicates the mutual impedance matrix, the yellow part indicates the self-impedance matrix, and the dashed line portion indicates the extended self-impedance matrix.

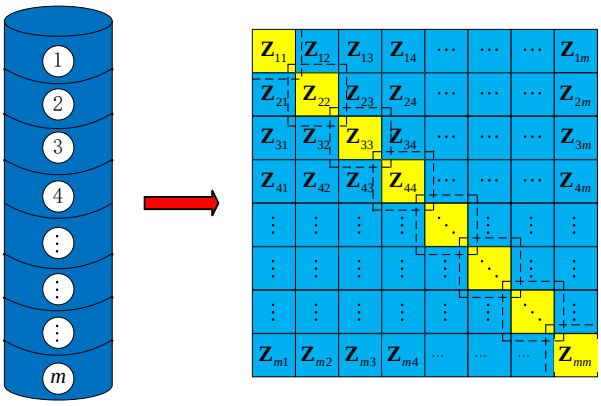

Fig. 1. The principle and application of blocking.

As a result, equation (1) changed as follows:

$$\begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} & \cdots & \mathbf{Z}_{1m} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \cdots & \mathbf{Z}_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{Z}_{m1} & \mathbf{Z}_{m2} & \cdots & \mathbf{Z}_{mm} \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_m \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \vdots \\ \mathbf{V}_m \end{bmatrix}, \quad (7)$$

where the self-impedance matrix $\mathbf{Z}_{ii}$ and mutual impedance matrix $\mathbf{Z}_{ij}$ are used to represent the interactions between different blocks. The current coefficient vector $\mathbf{J}_i$ and excitation vector $\mathbf{V}_i$ are associated with block i. The CMs of each extension block are solved by equations (8) and (9), selected effective modes as basis functions by MS, as follows:

$$\mathbf{Z}_{ii} = \mathbf{R}_{ii} + j\mathbf{X}_{ii}, \quad (8)$$

$$\mathbf{X}_{ii}\mathbf{J}_i^{\mathrm{CM}} = \lambda \mathbf{R}_{ii}\mathbf{J}_i^{\mathrm{CM}}. \quad (9)$$

It is worth noting that as the number of blocks increases, the dimension of the self-impedance matrix for each block becomes smaller, leading to improved efficiency in solving CMs. However, the increase in the number of CMs results in a higher dimensionality of the reduced matrix, necessitating iterative solutions. Furthermore, even though blocking and extending could improve the efficiency of solving CMs, they weaken the orthogonality between CMs, which eventually leads to a worse condition number of the reduced matrix, and the iterative solution converges slowly.

For this reason, after removing the extension and selecting effective CMs using the MS, the SVD is applied to these CMs. The process of SVD is as follows:

$$\mathbf{J}_i^{\mathrm{CM}} = \mathbf{U}\mathbf{W}\mathbf{V}^{\mathrm{T}}, \quad (10)$$

where $\mathbf{J}_i^{\mathrm{CM}}$ represent the set of all basis functions on the block i with a dimension of $N_i^e \times P_i$; N_i^e and P_i denote the number of unknowns and initial basis functions after extending on block i, respectively. $\mathbf{U}$ and $\mathbf{V}^{\mathrm{T}}$are both orthogonal matrixes with the dimension of $N_i^e \times N_i^e$ and $P_i \times P_i$, respectively. $\mathbf{W}$ is a diagonal matrix with the dimension of $N_i^e \times P_i$. The elements of the diagonal are arranged from largest to smallest with a rapid decay trend, all of which are singular values of $\mathbf{J}_i^{\mathrm{CM}}$. Afterwards, the left singular value vectors in $\mathbf{U}$ are retained as the novel characteristic mode basis functions (NCMBFs). Supposing that the number of NCMBFs retained on each block is L_i, the current of block i is expressed as

$$\mathbf{J}_i = \sum_{l=1}^{L_i} a_i^l \mathbf{J}_i^{\mathrm{NCM}_l}, \quad (11)$$

where a_i^l is the coefficient matrix to be solved and $\mathbf{J}_i^{\mathrm{NCM}_l}$ is the lth NCMBFs of block i. On this basis, the original current of the object is expressed as

$$\mathbf{J} = \sum_{i=1}^{M} \sum_{l=1}^{L_i} a_i^l \mathbf{J}_i^{\mathrm{NCM}_l}, \quad (12)$$

where a_i^l is obtained by solving a reduced matrix with the size of $\sum_{i=1}^{M} L_i \times \sum_{i=1}^{M} L_i$.

In general, solving equation (6) is categorized into direct and iterative methods. However, the construction of the reduced matrix $\mathbf{Z}^{\mathrm{R}} = \left(\mathbf{J}^{\mathrm{NCM}}\right)^{\mathrm{T}} \mathbf{Z}\mathbf{J}^{\mathrm{NCM}}$ is associated with the CMs. For electrically large problems, as the number of dividing blocks increases, the count of CMs increases, resulting in a larger dimension of the reduced matrix, thereby making direct method solution difficult. Based on this situation, the SVD technique and an iterative method are employed to solve the problem in this paper. Compared to the CMM, the condition number of the reduced matrix constructed using NCMBFs is diminished, leading to a notably improved convergence rate in the iterative solution of the reduced matrix.

III. COMPLEXITY ANALYSIS

The calculation of CMM and NCMBFM mainly primarily involve three processes, generating basis functions, constructing the reduced matrix, and solving the reduced matrix. For simplicity, the effect of the extension in NCMBFM is ignored, and the number of NCMBFs and unknowns in each block are assumed to be P and N/M, respectively. The computational complexity is analyzed for the CMM and NCMBFM as follows:

1. Generating basis functions: In the CMM, equation (3) is commonly solved using the implicitly restarted Arnoldi method (IRAM), the most time-consuming of which is the LU decomposition [22], with the complexity of $O\left(N^3\right)$. However, constructing NCMBFs in NCMBFM mainly consists of LU decomposition and SVD; both of them have the complexity of $O\left(M\left(N/M\right)^3\right)$, so the complexity of the NCMBFM is $O\left(2M\left(N/M\right)^3\right) \approx O\left(\left(N/M\right)^3\right)$. Since N is always greater than N/M, NCMBFM can expedite the generation of basis functions, and the acceleration factor is M^3.

2. Constructing the reduced matrix: Because of the different number of basis functions, the complexity of constructing the reduced matrix in the CMM and NCMBFM is $O\left(KN^2\right)$ and $O\left(PMN^2\right)$, respectively. Since K and PM are constants, the computational complexities of both are close.

3. Solving the reduced matrix equation: In CMM, the small number of solved CMs leads to a smaller dimension of the reduced matrix, which is suitable for using LU decomposition with complexity $O\left(N_c^3\right)$ to solve it directly, where N_c is the dimension of the reduced matrix. However, in CMB_iLU (CMB_iLU means the incomplete LU decomposition preconditioning method is used to accelerate the iterative convergence of the blocked CMM) and NCMBFM, as the number of unknowns increases,

more CMs are obtained, resulting in the increased dimensionality of the reduced matrix that requires iterative method with a complexity of $O\left(N_{iter}N_r^2\right)$ to solve. N_{iter} and N_r represent the number of iterations and the size of the reduced matrix, respectively. For NCMBFM, the condition number of the reduced matrix is better and the required N_{iter} is smaller, resulting in an acceleration factor of $N_{iter}^{'}$ ($N_{iter}^{'}$ is the difference in the number of iterations).

In summary, compared to CMM, the blocking operation in NCMBFM accelerates the generation of basis functions, albeit resulting in larger dimension of the constructed reduced matrix. In comparison to CMB_iLU, the introduction of the SVD process enhances the orthogonality between basis functions, optimizes the condition number of reduced matrices, and accelerates the iterative solution of the reduced matrix equation.

IV. NUMERICAL RESULTS

To verify the efficiency of the proposed method, the scattering properties of three objects are analyzed using the MoM, CMM, CMB, and NCMBFM. For convenience of expression, the CMM of blocking is defined as CMB. The generalized minimal residual (GMRES) method is used for the iterative algorithm. The threshold of the iLU decomposition is empirically set to 0.001 and the tolerance of the GMRES to 1E-05. In addition, to estimate the accuracy of the calculation results, the relative error is introduced as follows:

$$\mathrm{Err} = \left(\frac{\|\sigma_{\mathrm{cal}} - \sigma_{\mathrm{MoM}}\|_2}{\|\sigma_{\mathrm{MoM}}\|_2} \right) \times 100\%, \qquad (13)$$

where σ_{cal} and σ_{MoM} represent the radar cross section (RCS), calculated via the used method and MoM, respectively. Since CMM generates fewer basis functions and has a lower reduced matrix dimension, the reduced matrix equation is solved using a direct method. Meanwhile, the calculated results of CBFM are added as a comparison. Besides, we have included a comparison of the effects between NCMBFM and CM-PCA [19] in this section.

Initially, the bistatic RCS of a cylinder with a radius of 0.2 m and a height of 1 m is analyzed at an incident frequency of 1.8 GHz. The cylinder's surface is discretized using RWG functions, resulting in 18,478 triangles and 27,711 unknowns. With the threshold τ_{cm} of the MS set to 0.001, 1404 effective modes are generated in CMM. Subsequently, the object is divided into 16 blocks, and the extension size is set to 0.15λ in NCMBFM, creating 3520 effective modes and 41244 unknowns.

To investigate the effect of different extended sizes on the accuracy, the change of RCS Err with τ_{cm} under different extended sizes is given in Fig. 2. It can be observed that as the extended size increases, the accuracy increases, but so does the number of unknowns and the number of NCMBFs. To balance time and accuracy, the extended size and τ_{cm} are set to 0.15λ and 0.001, respectively. To test the effect of with and without SVD on the orthogonality between NCMBFs, the changes in the condition number of the reduced matrix with different extended sizes are shown in Table 1. It can be found that the condition number of the reduced matrix, constructed without SVD, increases significantly as the extended size grows, eventually reaching the point of becoming an ill-conditioned matrix. However, when NCMBFs are handled with SVD, the condition number of their constructed reduced matrix is significantly decreased. Therefore, the SVD helps to strengthen the orthogonality between basis functions to achieve the effect of optimizing the condition number of the reduced matrix.

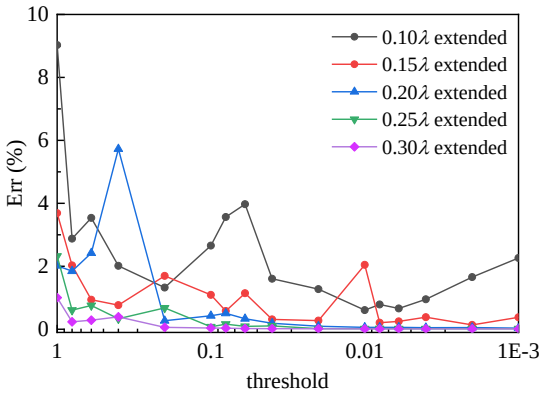

Fig. 2. Err of NCMBFM with different extended sizes.

Table 1: Condition number of reduced matrix constructed with and without SVD for different extended sizes

Extended Size	Condition Number of Reduced Matrix	
	Without SVD	With SVD
0.10λ	6.0388E+05	1.4050E+04
0.15λ	3.8820E+06	1.2081E+04
0.20λ	9.6412E+06	1.5381E+04
0.25λ	9.8231E+06	6.5468E+03
0.30λ	2.4077E+07	4.7326E+03

The iteration number of several different methods for the reduced matrix solution is compared in Fig. 3. Compared to these methods, the proposed approach exhibits the fewest iterations, resulting in a more efficient solution to the reduced matrix equation. The bistatic RCS of cylinder horizontal polarization is plotted in Fig. 4. It is shown that the proposed method is in excellent agreement with the calculated results of the MoM, CMM, and CBFM.

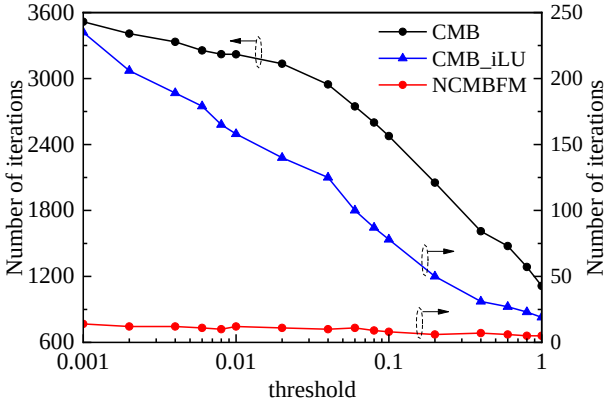

Fig. 3. The number of iterations for solving the reduced matrix using different methods.

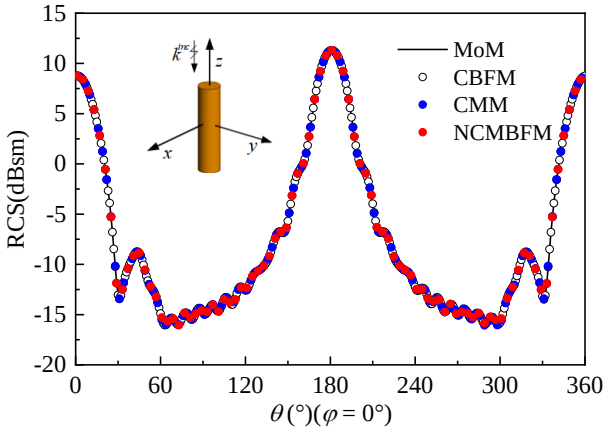

Fig. 4. Bistatic RCS of cylinder in horizontal polarization.

Next, a missile with a length of 1 m is calculated for bistatic RCS at 3 GHz. The surface of the missile is discretized by the RWG functions to generate 21,880 triangles and 32,820 unknowns. Meanwhile, the threshold τ_{cm} of MS is set to 0.001. In NCMBFM, the missile is divided into 26 blocks, and an extension of 0.15λ is chosen to create 52,662 unknowns. 1609 effective modes are obtained in CMM, while 4663 effective modes are acquired in NCMBFM. Compared with the CMB_iLU, the condition number of the reduced matrix constructed by NCMBFs decreased from 1.1309E+08 to 1.6832E+04 in NCMBFM. Moreover, the proposed method diminished the number of iterations in solving the reduced matrix equation from 277 to 17. Hence, the solution efficiency is significantly improved. The bistatic RCS of vertical polarization for several methods is shown in Fig. 5. The results show that the proposed method is consistent with the calculation results of the MoM and CMM with high accuracy.

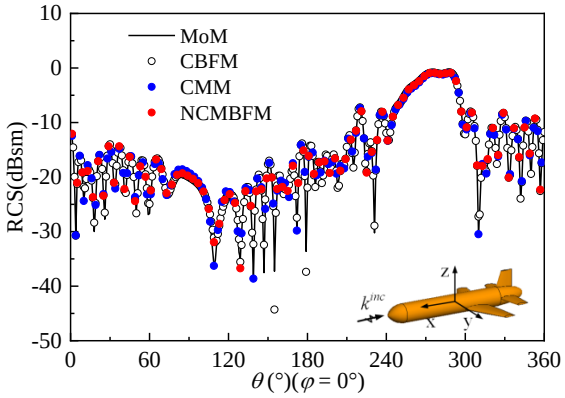

Fig. 5. Bistatic RCS of missile in vertical polarization.

Finally, the bistatic RCS with a cone-sphere with a gap at an incident frequency of 6.2 GHz is calculated. The RWG basis functions discretize the target surface to obtain 27,898 triangles and 41,847 unknowns. At the same time, the threshold τ_{cm} of MS is set to 0.001. In the CMM, 1959 effective modes are obtained. In NCMBFM, 6357 effective modes and 73,638 unknowns are acquired by dividing the target into 45 blocks and extending each block by 0.15λ. Compared to CMB_iLU, the proposed approach reduces the matrix condition number from 6.2385E+08 to 1.1048E+04, while it decreases to 4.8327E+04 in CM-PCA. In addition, the iteration number for solving the reduced matrix equation dropped from 530 to 30, while it is 42 in CM-PCA. The efficiency of iterative solution is greatly improved. The horizontal polarized bistatic RCS of the cone-sphere with a gap is shown in Fig. 6. It can be concluded that the proposed method is in good agreement with the results of the MoM.

Although both SVD and PCA are featured to enhance orthogonality within the data, but there are

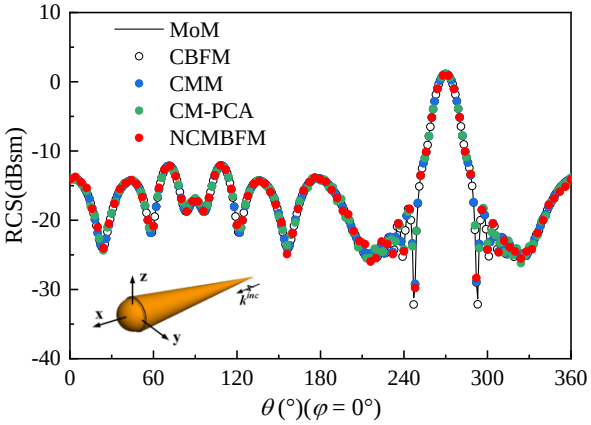

Fig. 6. Bistatic RCS of cone-sphere with a gap in vertical polarization.

Table 2: The simulation time of different processes and RCS Err

Model	Method	Basis Functions Construction Time (s)	Number of Basis Functions	Reduced Matrix Filling Time (s)	Solving Time (s)	Total Time (s)	RCS Err (%)
Cylinder	CBFM	716.4	4303	495.0	6.3	2163.6	0.34
	CMM	3325.3	1404	47.5	2.1	4834.7	0.21
	CMB_iLU	108.1	3520	131.1	247.4	1931.4	0.25
	NCMBFM	117.7	3520	132.1	9.1	1702.6	0.37
Missile	CBFM	1148.8	5798	933.7	9.2	4118.5	2.78
	CMM	5336.1	1609	76.5	2.5	7516.1	1.52
	CMB_iLU	185.9	4663	441.8	282.8	3650.6	1.87
	NCMBFM	223.8	4663	440.9	9.5	3375.9	2.11
Cone-sphere with a gap	CBFM	1889.4	7875	1508.2	29.7	5638.3	2.63
	CMM	8136.3	1959	323.8	3.2	11618	1.33
	CM-PCA	541.3	6357	627.7	41.8	4382.4	1.97
	CMB_iLU	336.1	6357	630.7	914.1	5071.2	1.60
	NCMBFM	374.9	6357	629.4	33.3	4229.1	1.81

differences in the implementation and advantages. First, SVD is applicable to various types of data and provides an optimal low-rank approximation of the data, which has global optimality and stability. In contrast, PCA is a statistical method based on the covariance matrix, which may be influenced by the distribution of the data. Second, SVD is a rigorous mathematical method that precisely decomposes a matrix into a product between two orthogonal matrixes and a diagonal matrix, which maximizes the enhancement of orthogonality. Third, SVD has a wider application, such as data noise reduction, data reconstruction, and matrix approximation. Finally, SVD can be viewed as a special case of PCA, which also has data interpretability.

In addition, the simulation times of different processes and RCS Err corresponding to Figs. 4, 5, and 6 are given in Table 2. Compared to the CMM, the proposed method greatly reduces the time of generating basis functions and solving the reduced matrix equation. The total time for the three simulations is decreased by 65%, 56% and 63%, respectively. Thus, the solution efficiency is substantially improved. However, in CM-PCA, the covariance matrix is constructed by computing the covariance, and then the principal components are obtained through SVD. Subsequently, data reconstruction is performed. In contrast, the proposed method handles CMs faster and with greater efficiency.

Furthermore, both the proposed method and CBFM are based on the idea of regional decomposition; the CBFM compared in the paper constructs the characteristic basis functions (CBFs) by multilevel plane wave and SVD. But the proposed method has the following advantages: In CBFM, the construction of the basis functions is very time-consuming since it requires massive incident excitations and uses SVD to remove redundancy. In contrast, CM is independent of excitation, applicable to objects of any shape, and the basis functions are faster to construct. Simultaneously, multiple experimental results have demonstrated that, under comparable accuracy conditions, the proposed method generates fewer basis functions. Therefore, the proposed method constructs the reduced matrix faster.

V. CONCLUSION

In this study, a novel method is proposed to accelerate the analysis of electromagnetic scattering properties of large electrical objects. By introducing the idea of blocking to decrease the dimensionality of the impedance matrix, the efficiency of solving CMs is significantly improved. Innovatively, SVD is applied to process the CMs, thereby enhancing their orthogonality, improving the condition number of the reduced matrix, and achieving rapid convergence in the iterative solution of the reduced matrix equation. Compared to CMM, the proposed method significantly reduces computational time while maintaining accuracy, as evidenced by the complexity analysis and numerical simulation results. Simultaneously, it provides a new way to analyze the electromagnetic scattering properties of electrically large objects.

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Fei Guo received the B.E degree from Anqing Normal University in 2021. He is currently pursuing the M.S degree in Anhui University of Science and Technology. His current research interest includes computational electromagnetics.

Zhonggen Wang received the Ph.D. degree in electromagnetic field and microwave technique from the Anhui University of China (AHU), Hefei, P. R. China, in 2014. Since 2014, he has been with the School of Electrical and Information Engineering, Anhui University of Science and Technology. His research interests include computational electromagnetics, array antennas, and reflect arrays.

Wenyan Nie is a professor at Huainan Normal University. She received the B.S. and M.S degrees from Anhui University of Science and Technology in 2007 and 2012, respectively. Her research interests include computational electromagnetic methods, antenna theory and design.