Finite Difference Time Domain Method for the Analysis of Transient Grounding Resistance of Buried Thin Wires

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Abstract – For the analysis of grounding resistance with the finite-difference time-domain (FDTD) method for solving Maxwell's equations, an equivalent radius of a naked thin wire in a lossy medium is derived by means of the static field approximation, proposed for derivation of that of an aerial thin wire. It is 0.23 times the size of each cell employed, which is the same as that of an aerial thin wire. The validity is tested by comparing the grounding-resistance values obtained through FDTD simulation on simple buried structures with the theoretical values.

Key words – FDTD method, grounding electrode, grounding resistance, thin wire, and conductor.

I. INTRODUCTION

The role of grounding electrodes is to dissipate fault currents effectively into the soil, and thereby to prevent damage of insulations in power systems. Thus, the performance of power systems is influenced by proper functioning of grounding systems.

No formulas of impedance and admittance have been derived even for simple vertical or horizontal naked conductor buried in a homogeneous ground. Hence, transient characteristics of grounding electrodes have been investigated by experiments and recently numerical electromagnetic analyses [1 - 4] based on the method of moments (MoM), the finite element method (FEM), or the finite-difference time-domain (FDTD) method [5 - 6]. Numerical electromagnetic analyses can be performed assuming well-profiled condition that the values of conductivity and permittivity of a ground are known or set arbitrarily. Such results are useful in understanding the phenomena as well as in confirming measured results.

Numerical electromagnetic analyses based on the FDTD method are effective to analyze the transient response of a large solid conductor or electrode. The accuracy of this method, in the case of being applied to such analysis, has been fully investigated in comparison with an experiment and shown to be satisfactory [7]. As this method requires long computation time and large capacity of memory, the analysis is restricted to a rather small space. A transient analysis of a large system or a system composed of various elements still need to be performed by such tools like Electromagnetic Transients Program (EMTP) [8]. One reasonable process of study, therefore, is to investigate the physical characteristics of a grounding electrode by a numerical electromagnetic analysis, and then to represent the obtained characteristics by an equivalent circuit model or to determine the values of its parameters [3].

So far in most of the FDTD analyses of transient and steady-state grounding resistance, large solid electrodes [6], [7], which can be decomposed into small cubic cells, have been chosen and thin-wire electrodes have not been dealt with. This is because an equivalent radius of a thin wire in a lossy medium has not been made clear. In [9], a rigorous method has been shown for determining the effective radius of a single axial field component, E_x or H_x, in a two-dimensional (2-D) TM_x or TE_x FDTD grid. The method is based upon matching FDTD results for a filamentary field source with the analytical Green's function in two dimensions. It is therefore, essential to clarify the equivalent radius of a buried thin wire for more general analyses of grounding systems. In the present paper, an equivalent radius of a thin wire in lossy medium is derived with the help of the concept proposed for derivation of that of an aerial thin wire [10]. Then its validity is tested by comparing the grounding-resistance values obtained through FDTD simulations on simple buried structures with the theoretical values.

II. METHOD OF ANALYSIS

The FDTD method employs a simple way to discretize a differential form of Maxwell's equations. In the Cartesian coordinate system, it generally requires the entire space of interest to be divided into small rectangular cells and calculates the electric and magnetic fields of the cells using the discretize Maxwell's equations. As the material constant of each cell can be specified arbitrarily, a complex inhomogeneous medium can be easily analyzed. To analyze fields in an open space, an absorbing boundary has to be set on each plane which limits the space to be analyzed, so as to avoid reflection there. In the present analysis, the second-order Mur's method [11] is employed to represent absorbing planes.

III. DERIVATION OF EQUIVALENT RADIUS OF BURIED THIN WIRE

In [8], it has been shown that an aerial thin wire has some equivalent radius in the case that the electric-field elements along the thin wire are set to zero in an orthogonal and uniform-spacing Cartesian grid. When the size of cubic cells employed is Δs, the equivalent radius is 0.23Δs. In the present paper, an equivalent radius of a naked thin wire in a lossy medium is derived. Note that in [10] an equivalent radius of an aerial thin wire has been shown to be 0.135Δs. In a quasi-steady state, however, 0.23Δs is more appropriate than 0.135Δs as an equivalent radius [8] which is very close to the effective radius 0.2Δs [7].

Figure 1 illustrates the cross section of a long thin wire surrounded by a cylindrical sheath conductor. The radii of the thin wire and the sheath are a and b, respectively. The conductivity and the relative permittivity of a medium between the thin wire and sheath conductor are assumed to be σ and ε_s, respectively. In this condition, the conductance G and the susceptance B between the thin wire and the sheath are given as follows,

$$G = \frac{2\pi\sigma}{\ln(b/a)}, \quad B = \frac{2\pi\varepsilon_0\varepsilon_s\omega}{\ln(b/a)}. \tag{1}$$

Note that ε_0 is the permittivity of vacuum and ω is the angular frequency. Therefore, the conductance becomes equal to the susceptance when the frequency f is

$$f_0 = \sigma/(2\pi\varepsilon_0\varepsilon_s). \tag{2}$$

For instance, f_0 is 1.5 or 7.5 MHz for a medium of ε_s = 12 and σ = 1 mS/m or 5 mS/m, respectively.

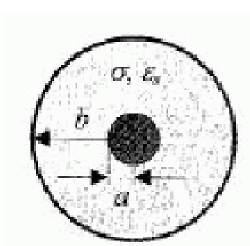


Fig. 1. Cross section of a thin wire surrounded by a cylindrical sheath.

Figure 2 shows the cross section of a thin wire surrounded by a rectangular sheath conductor for an FDTD simulation. Both the thin wire core and the sheath are perfectly conducting. The cross-sectional area of the sheath is 2.5 X 2.5 m^2 and the length is 25 m. The conductor system is represented with cubic cells whose side Δs is 0.25 m. A voltage, which has a rise-time of 20 ns and a magnitude of 100 V, is applied between the thin wire and the sheath at its one end. The other end is open. The response is calculated up to 10 μs with a time increment of 0.4 ns.

Figure 3 shows the time-variations of the ratios of E_1, E_2 and E_3 to E_2 which are radial electric fields calculated for 0.5Δs, 1.5Δs, and 2.5Δs, at 12.5 m from the ends of the conductor. It is found that the ratios settle down after 100 ns or so, and they are almost equal to those calculated for a thin wire in air [10]: 2.21, 1.00 and 0.59. This is natural because both the conductance and the susceptance of a thin wire follow similar expressions as shown in equation (1). Furthermore, the ratios change a little even if a different conductivity such as 0.2 or 10 mS/m is employed and a different time increment 0.25 or 0.48 ns is used. Thus electric field around the thin wire can also be approximated by the following function [10],

$$E = 3\Delta s/(2x). \tag{3}$$

Note that x is the distance from the centre of the thin wire. In this function, the electric field $\boldsymbol{E}$ is normalized so that $\boldsymbol{E}$ should be unity at x =1.5 Δs. Figure 4 shows the radial electric fields calculated by this function and those obtained by the FDTD simulation.

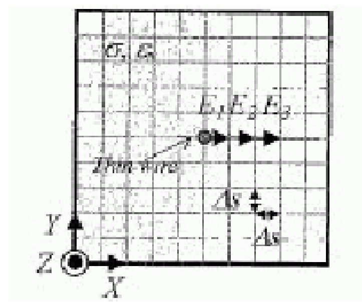


Fig. 2. Electric field around a thin wire in a rectangular sheath to be used for an FDTD simulation.

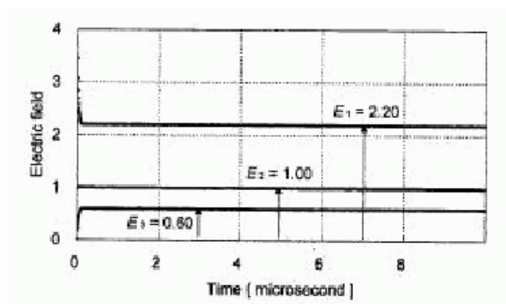


Fig. 3. Time-variation of the ratios of E_1, E_2 and E_3 to E_2 calculated by the FDTD method in the case of σ = 5 mS/m and ε_s = 12.

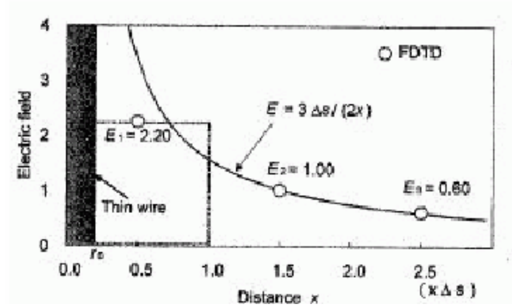


Fig. 4. Radial electric fields around the thin wire.

If the equivalent radius of the thin wire now in question is assumed to be $\boldsymbol{r_0}$ and the electric field is assumed to follow the above function, the potential difference between $x = r_0$ and $x = \Delta s$ is given as follows,

$$\int_{r_0}^{\Delta s} \mathrm{E}\ \mathrm{d}x = \frac{3\Delta s}{2} \ln \frac{\Delta s}{r_0}\,. \tag{4}$$

If the above expression is equated to 2.2 Δs, which is the potential difference obtained by the FDTD simulation, the equivalent radius r_0 is given as,

$$r_0 = 0.23\,\Delta s\,. \tag{5}$$

This is an equivalent radius of a naked thin wire in a lossy medium.

IV. COMPARISON WITH SUNDE'S FORMULA ON GROUNDING RESISTANCE

A. Models for Analysis

Figure 5 shows a side view of an analysis model, which is composed of two naked vertical thin wires and an overhead horizontal thin wire. The buried portion of vertical thin wires is 3 or 5 m. The horizontal thin wire is 30 m long and 1 m high over the surface of a homogeneous ground. The conductor system is excited by a voltage source at a connection point between the horizontal wire and one of the buried vertical wires. The voltage source produces a steep-front wave having a rise-time of 10 ns, after which it maintains a magnitude of 100 V, [12 - 16].

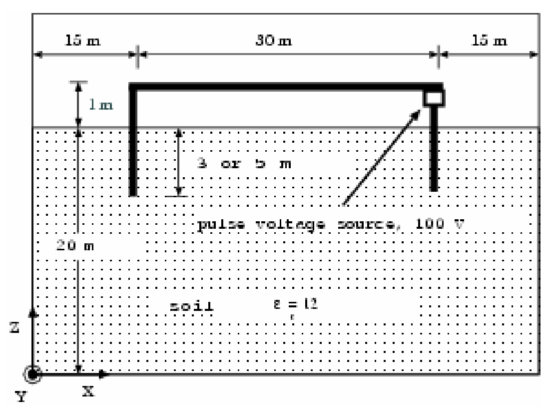


Fig. 5. Two buried vertical thin wires connected by an overhead horizontal wire to be analyzed by the FDTD method.

The conductivity of the homogeneous ground σ is set to 0.2 mS/m, 1.0 mS/m, and 5 mS/m in order to visualize the moisture contained in the soil, where the conductors are buried. The thickness and relative permittivity (ε_s) of the ground are set to 20 m and 12, respectively. For the FDTD simulation, the conductor system shown in Fig. 5 is accommodated by a large rectangular analysis space of 80 ×120 × 60 m^3 with space length Δs = 0.5 m. The voltage in the gap which exists between the horizontal wire and one of the buried vertical wires represents conductor-top voltage. The gap length is maintained as the space length of the conductor system. The time-step was determined by equation (14) found in [10] with α = 0.01, and all the six boundaries of the cell were treated as the second-order Liao's absorbing boundary.

It may be believed that the FDTD method is a time-consuming method. However, the progress of computers in terms of speed and memory is considerable, and even a personal computer can be used for the FDTD calculation. In fact, the simulation presented in this paper were performed by a personal computer with Intel Pentium 4, 2.80 GHz CPU and 512 MB RAM. Responses are calculated up to 1.5 μs with a time increment of 0.9 ns. Therefore, the computation time for one case is about 3 hours.

B. Analyzed Results

Figures 6 and 7 show both voltage and current waveforms at the vertical conductor-top, respectively, i.e., at the injection point calculated for the model of Fig. 5 in case of the vertical thin wires are buried up to 3 m and 5 m with different conductivity of the earth soil. Tables I and II summarizes the values of transient grounding resistance R_{GV} of the 3-m and 5-m vertical thin wires evaluated at 1.5 μs for Figs. 6 and 7. They are simply calculated from the following relation: $I_s = V_s / R_{GV}$. Note that V_s is the magnitude of the voltage and I_s is the current of the circuit.

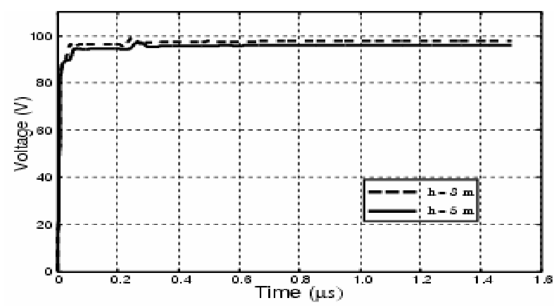
Figure 8 shows the propagation of the current at different heights of the 6 m-vertical electrodes, which are buried up to 5 m and with different conductivity. These currents are simulated at 5.5 m, 2.5 m and at the bottom of the electrode in which the source is applied and thus treated as upper, middle and lower currents. It is noted that the middle and lower currents are characterized by the ground parameters. The magnitudes of current waveforms are increasing with the increase of the conductivity and thus the time required to settle down the currents is increasing. It is also noted that as the conductivity gets higher, the wavefronts of voltage and current become less steep. The waveform of a voltage of the buried naked conductor is not similar to that of a current, particularly around the injection point. If the buried conductor is insulated, the waveform of a voltage is almost identical to that of a current just, as if it is a coaxial cable [17].

C. Discussion

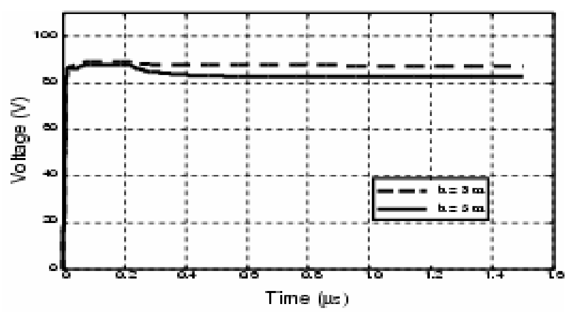
The wavelength of an electromagnetic field, which corresponds to the evaluation time (1.5 μs), is several hundred meters. It is ten times longer than the length of the conductor system shown in Fig. 5. Hence, it is considered that the transient-resistance value at 1.5 μs is close to the resistance in the steady state. Sunde [18] has derived a theoretical formula for the DC resistance of a vertical conductor buried in a homogeneous ground. It is expressed as

$$R_{GV_{SUNDE}} = \frac{1}{2\pi\sigma d}(\ln\frac{4d}{r} - 1), \qquad (6)$$

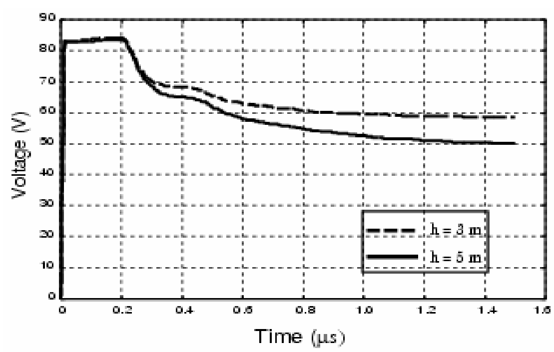
where, d is the length and r is the radius of the electrode. The values of grounding resistance calculated by this theoretical formula are also included in Tables I and II. The values of the transient grounding resistance obtained by the FDTD simulation are only 8 % lower than those calculated by Sunde's formula regardless of the ground conductivity.



(a) σ = 0.2 mS/m.

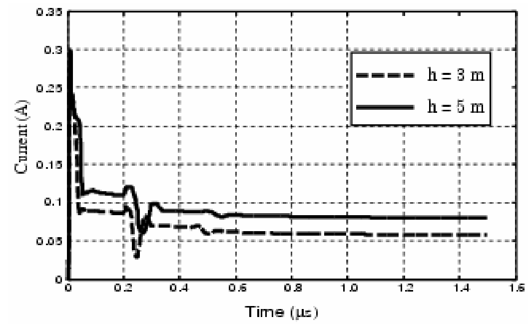


(b) σ = 1 mS/m.

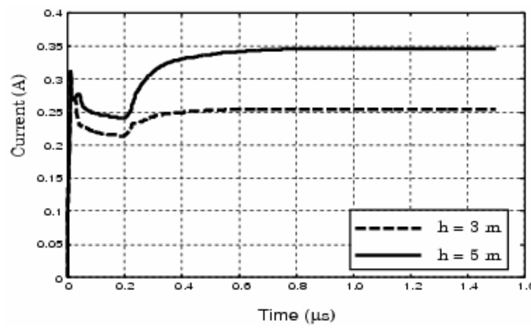


(c) σ = 5 mS/m.

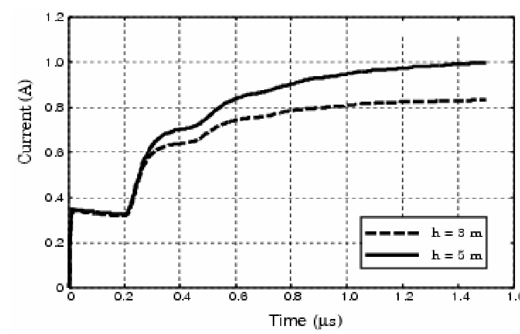
Fig. 6. Voltages evaluated at the injection point of vertical thin electrodes of Fig. 5 buried up to 3 m and 5 m with different ground conductivity.



(a) σ = 0.2 mS/m.



(b) σ = 1 mS/m.



(c) σ = 5 mS/m.

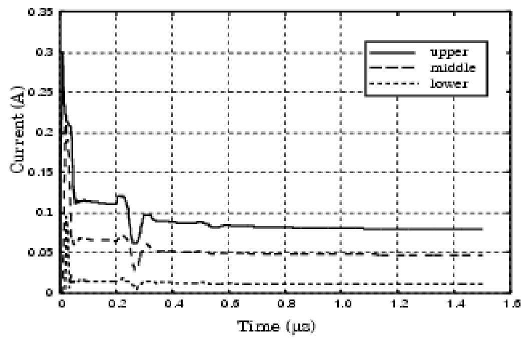
Fig. 7. Calculated current waveforms at the injection point of the model of Fig. 5 with different conductivity in the case that the vertical thin wires are buried up to 3 and 5 m.

Table I. Transient grounding resistance of a 3-m vertical electrode obtained by the FDTD analysis and the DC resistance calculated by Sunde's formula.

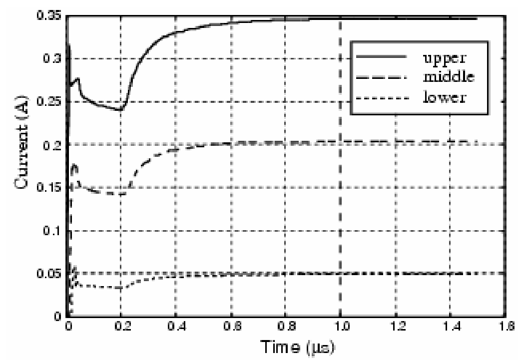
	σ=0.2mS/m	*σ=1mS/m*	*σ=5 mS/m*
FDTD	900	178	36
Theory	967	193	38.7
Difference	6.9%	7.7%	7%

Table II. Transient grounding resistance of a 5-m vertical electrode obtained by the FDTD analysis and the DC resistance calculated by Sunde's formula.

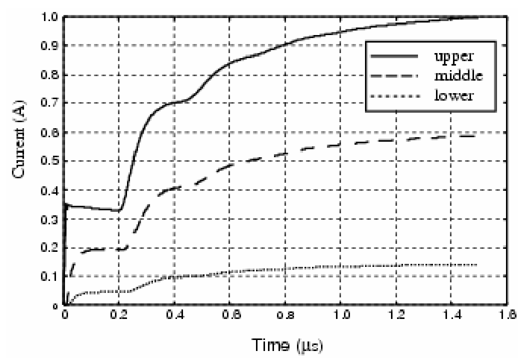
	σ=0.2mS/m	*σ=1mS/m*	*σ=5 mS/m*
FDTD	615	121	21.5
Theory	661	131	26.5
Difference	7%	8%	7.5%



(a) σ = 0.2 mS/m.



(b) σ = 1 mS/m.



(c) σ = 5 mS/m.

Fig. 8. Propagating current observed at a different height of the vertical thin wire with different conductivity (5 m buried vertical thin wires).

When the length of the overhead horizontal thin wire is shortened or enlarged from 30 m to 20 m or 40 m, the transient resistance decreases only by 0.5 Ω (1.7\ %) or increases by 0.4 Ω (1.3\ %) for a 5-m buried vertical thin wire in a ground having the conductivity of 5 mS/m, as shown in Table III. Therefore, it is clear that the influence of the 30-m distance between the two electrodes is insignificant than the properties and the depth of the lossy ground.

As a consequence, it has become clear that the 0.23 Δs is valid as the equivalent radius of a thin wire buried in a lossy ground. Note that Sunde has proposed a theoretical formula of resistance also for a horizontal cylindrical electrode [18]. As it is a function of the natural logarithm of the square root of ***r***, the resistance value of a horizontal thin electrode is not so sensitive to the radius of the electrode. This is the reason why a horizontal electrode is not employed for comparison.

Table III. Dependency of the transient grounding resistance of a 5-m vertical electrode, calculated by the FDTD analysis on the distant two electrodes.

Distance	20 m	30 m	40 m
Resistance	25.8	26.5	27.7

V. CONCLUSIONS

In the present paper, for the analysis of grounding resistance with the FDTD method, an equivalent radius of a naked vertical thin wire in a lossy medium has been investigated with the help of the static-field concept proposed for an aerial thin wire. It is 0.23 times the side of cells employed, which is the same as that of the aerial thin wire. The validity has also been examined by comparing the grounding-resistance value obtained through FDTD simulations on simple buried structures with the theoretical values, and are shown to be satisfactory.

VI. ACKNOWLEDGMENT

The authors are indebted to T. Noda and Y. Baba for their technical support and providing useful information.

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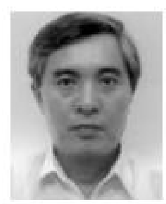

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