

A Single-Field FDTD Formulation for Electromagnetic Simulations

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Abstract — A set of general purpose single-field finite-difference time-domain (FDTD) updating equations for solving electromagnetic problems is derived. The formulation uses a single-field expression for full-wave solution. This formulation can provide numerical results similar to those obtained using the traditional Yee algorithm with less computer resources. The traditional FDTD updating equations are based on Maxwell's curl equations whereas the single-field FDTD updating equations, used here, are based on the vector wave equation. Performance analyses of the single-field formulation in terms of CPU time, memory requirement, stability, dispersion, and accuracy are presented. It was observed that the single-field method is significantly efficient relative to the traditional one in terms of speed and memory requirements.

Index Terms — FDTD, single-field approach.

I. INTRODUCTION

The first paper on finite-difference time-domain (FDTD) was published in 1966 by Yee [1]. Since then, the FDTD has become widely used in computational electromagnetics [2]. Extensive research has been reported to improve the accuracy and speed of the method and different absorbing boundary conditions (ABCs) are developed to provide more accurate results [3, 4, 5]. An improvement in speed of the method,

however, has relied almost solely on progresses in computer hardware and software architecture.

This paper investigates the single-field approach based on the vector wave equation (VWE) to derive the FDTD updating equations in a way that only one field component will be calculated and updated inside the iteration loop to eliminate iteration steps required to update the other field component. Since one field (E or H) can be calculated from the other field, whenever needed, the proposed method, hence, is able to provide simulation results similar to that obtained from traditional FDTD updating equations.

There is not much published work investigating VWE-based updating equations as a complete alternative to the traditional Yee algorithm; Aoyagi et al. investigated a possible combination of scalar and vector wave equations as well as scalar wave equation and Maxwell's equations [6], however both approaches lose generality since they require partitioning of the problem domain; Okoniewski discussed the application of the vector wave equation approach to inhomogeneous wave-guide structure in terms of stability by using transverse field components [7]. Chu et al. studied the FDTD modeling of optical guided-wave devices based on the Yee algorithm and investigated scalar wave equation and its semivectorial version for the simulation of optical guided-wave devices, but the vector nature of the electromagnetic waves is either completely or partially ignored [8,9].

The single-field FDTD is an effort at reduction of FDTD variables in a Yee grid to only the three components of a single field variable, either \mathbf{E} or \mathbf{H} , while maintaining the ability to analyze full vector source injection. To compare the proposed updating equations with the traditional Yee algorithm, 2D TM, TE, and 1D electromagnetic problems are solved. Results of performance analyses: CPU time, memory requirement, stability, dispersion, and accuracy, are presented. It was observed that for 1D and 2D problems, the single-field method has advantages over the traditional one in terms of speed and memory requirements.

II. FORMULATIONS

The single-field formulation is derived by starting with Maxwell's curl equations:

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} - (\mathbf{M}_i + \sigma^m \mathbf{H}), \quad (1)$$

$$\nabla \times \mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t} + (\mathbf{J}_i + \sigma^e \mathbf{E}). \quad (2)$$

where \mathbf{E} is the electric field strength, \mathbf{H} is the magnetic field strength, \mathbf{J}_i is the impressed electric current density, \mathbf{M}_i is the impressed magnetic current density, ε is the permittivity, and μ is permeability, σ^e and σ^m are the electric and magnetic conductivity, respectively. Taking the curl of (1) we have:

$$\nabla \times \nabla \times \mathbf{E} = -\mu \frac{\partial}{\partial t} (\nabla \times \mathbf{H}) - \nabla \times \mathbf{M}_i - \sigma^m (\nabla \times \mathbf{H}). \quad (3)$$

Replacing the curl of \mathbf{H} in (3) with the right hand side of (2), (3) can be rewritten as:

$$\nabla \times \nabla \times \mathbf{E} = -\mu \frac{\partial}{\partial t} \left(\varepsilon \frac{\partial \mathbf{E}}{\partial t} + (\mathbf{J}_i + \sigma^e \mathbf{E}) \right) - \nabla \times \mathbf{M}_i - \sigma^m \left(\varepsilon \frac{\partial \mathbf{E}}{\partial t} + (\mathbf{J}_i + \sigma^e \mathbf{E}) \right). \quad (4)$$

Alternatively, taking the curl of (2) we have:

$$\nabla \times \nabla \times \mathbf{H} = \varepsilon \frac{\partial}{\partial t} (\nabla \times \mathbf{E}) - \nabla \times \mathbf{J}_i - \sigma^m (\nabla \times \mathbf{E}). \quad (5)$$

Replacing the curl of \mathbf{E} in (5) with the right hand side of (1), (5) can be written as:

$$\nabla \times \nabla \times \mathbf{H} = \varepsilon \frac{\partial}{\partial t} \left(-\mu \frac{\partial \mathbf{H}}{\partial t} - (\mathbf{M}_i + \sigma^m \mathbf{H}) \right) - \nabla \times \mathbf{J}_i - \sigma^m \left(-\mu \frac{\partial \mathbf{H}}{\partial t} - (\mathbf{M}_i + \sigma^m \mathbf{H}) \right). \quad (6)$$

To implement (4) or (6) as FDTD updating equations, we have to write them in scalar form for each Cartesian component.

A. 2D single-field E-based updating equations

If we assume no variation with respect to the z direction, i.e. $\frac{\partial}{\partial z} = 0$, the x -component of the electric field from (4) is given as:

$$\frac{\partial^2 E_x}{\partial y^2} - \frac{\partial^2 E_y}{\partial x \partial y} = \sigma^m \sigma^e E_x + (\mu \sigma^e + \varepsilon \sigma^m) \frac{\partial E_x}{\partial t} + \mu \varepsilon \frac{\partial^2 E_x}{\partial t^2} + \frac{\partial M_{zi}}{\partial y} + \sigma^m J_{ix} + \mu \frac{\partial J_{ix}}{\partial t}. \quad (7)$$

To derive the FDTD updating equations for the x -component of the electric field, we have to evaluate all the spatial derivatives in (7) at their corresponding electric field node, i.e. E_x . Central difference formula is used to discretize the derivatives. Care must be taken for $\frac{\partial^2 E_y}{\partial x \partial y}$ term. Four field components need to be used to evaluate the second derivative of E_y at the corresponding electric field node as shown in Fig. 1.

$$\frac{\partial^2 E_y(i,j)}{\partial x \partial y} = \frac{E_y^n(i+1,j) - E_y^n(i+1,j-1) - E_y^n(i,j) + E_y^n(i,j-1)}{\Delta x \Delta y}. \quad (8)$$

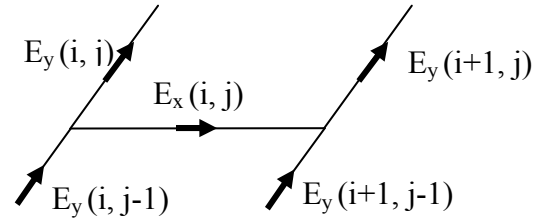


Fig. 1. Electric field components in 2D.

$$\begin{aligned} E_x^{n+1}(i,j) = & C_{ex}^{ex,n}(i,j)[E_x^n(i,j)] \\ & + C_{ex}^{ex,n-1}(i,j)[E_x^{n-1}(i,j)] \\ & + C_{ex}^{ex,n,y}(i,j)[E_x^n(i,j+1) + E_x^n(i,j-1)] \\ & + C_{ex}^{ey,n,xy}(i,j)[E_y^n(i+1,j) - E_y^n(i,j) \\ & \quad - E_y^n(i+1,j-1) + E_y^n(i,j-1)] \\ & + C_{ex}^{Mz,n,y}(i,j)[M_{iz}^n(i,j) - M_{iz}^n(i,j-1)] \\ & + C_{ex}^{Jx,n}(i,j)[J_{ix}^n(i,j)] \\ & + C_{ex}^{Jx,t}(i,j)[J_{ix}^{n+1}(i,j) - J_{ix}^{n-1}(i,j)]. \quad (9) \end{aligned}$$

The final expression of the updating equation for the x -component of the electric field is given in (9), where the C terms are constant coefficients as given in Appendix A. The source terms are only included at the source locations whereas the field terms are included in the updating equation throughout the entire problem domain.

Similarly, updating equations for E_y and E_z can be derived in the same manner and are given below for completeness.

$$\begin{aligned}
 E_y^{n+1}(i, j) = & C_{ey}^{ey,n}(i, j)[E_y^n(i, j)] \\
 & + C_{ey}^{ey,n-1}(i, j)[E_y^{n-1}(i, j)] \\
 & + C_{ey}^{ey,n,x}(i, j)[E_y^n(i+1, j) + E_y^n(i-1, j)] \\
 & + C_{ey}^{ex,n,xy}(i, j)[E_x^n(i, j+1) - E_x^n(i, j) \\
 & \quad - E_x^n(i-1, j+1) + E_x^n(i-1, j)] \\
 & + C_{ey}^{Mz,n,x}(i, j)[M_{i,z}^n(i, j) - M_{i,z}^n(i-1, j)] \\
 & + C_{ey}^{Jy,n}(i, j)[J_{i,y}^n(i, j)] \\
 & + C_{ey}^{Jy,t}(i, j)[J_{i,y}^{n+1}(i, j) - J_{i,y}^{n-1}(i, j)]. \quad (10)
 \end{aligned}$$

and

$$\begin{aligned}
 E_z^{n+1}(i, j) = & C_{ez}^{ez,n}(i, j)[E_z^n(i, j)] \\
 & + C_{ez}^{ez,n-1}(i, j)[E_z^{n-1}(i, j)] \\
 & + C_{ez}^{ez,n,x}(i, j)[E_z^n(i+1, j) + E_z^n(i-1, j)] \\
 & + C_{ez}^{ez,n,y}(i, j)[E_z^n(i, j+1) + E_z^n(i, j-1)] \\
 & + C_{ez}^{Miy,n,x}(i, j)[M_{i,y}^n(i, j) - M_{i,y}^n(i-1, j)] \\
 & + C_{ez}^{Mix,n,y}(i, j)[M_{i,x}^n(i, j) - M_{i,x}^n(i, j-1)] \\
 & + C_{ez}^{Jz,n}(i, j)[J_{i,z}^n(i, j)] \\
 & + C_{ez}^{Jz,t}(i, j)[J_{i,z}^{n+1}(i, j) - J_{i,z}^{n-1}(i, j)]. \quad (11)
 \end{aligned}$$

B. 1D single-field E-based updating equations

FDTD updating equations for the one-dimensional field components can be easily obtained from the two-dimensional updating equations by further assuming $\frac{\partial}{\partial y} = 0$. The y -component of the electric field is then given as:

$$\begin{aligned}
 \frac{\partial^2 E_y}{\partial x^2} = & \sigma^e \sigma^m E_y + (\mu \sigma^e + \varepsilon \sigma^m) \frac{\partial E_y}{\partial t} + \mu \varepsilon \frac{\partial^2 E_y}{\partial t^2} - \\
 & \frac{\partial M_{i,z}}{\partial x} + \sigma^m J_{i,y} + \mu \frac{\partial J_{i,y}}{\partial t}, \quad (12)
 \end{aligned}$$

which yields the following updating equation

$$\begin{aligned}
 E_y^{n+1}(i) = & C_{ey}^{ey,n}(i)[E_y^n(i)] \\
 & + C_{ey}^{ey,n-1}(i)[E_y^{n-1}(i)] \\
 & + C_{ey}^{en,n,x}(i)[E_y^n(i+1) + E_y^n(i-1)]
 \end{aligned}$$

$$\begin{aligned}
 & + C_{ey}^{Mz,n,x}(i)[M_{i,z}^n(i) - M_{i,z}^n(i-1)] \\
 & + C_{ey}^{Jy,n}(i)[J_{i,y}^n(i)] \\
 & + C_{ey}^{Jy,t}(i)[J_{i,y}^{n+1}(i) - J_{i,y}^{n-1}(i)]. \quad (13)
 \end{aligned}$$

Similarly, updating equations for E_z , H_y , and H_z can be derived in the same manner.

In the solution of 2D and 1D electromagnetic problems, one can utilize the proper updating equation to find the field in the problem domain at each time step. Moreover, frequency domain solution and scattering parameters can also be obtained by using the time domain field solution.

III. PERFORMANCE ANALYSIS

A. Memory/speed analysis of a 1D problem

Next, we examine a one-dimensional electromagnetic problem given in [10]. The electric field components, due to a z -directed electric current sheet placed at the center of a problem space filled with air between two parallel perfect electric conducting plates extending to infinity in y and z directions, are computed. Figure 2 shows the comparison of the CPU time required by the single-field and the traditional formulations for the same cell size and number of cells. The required number of floating-point addition operation per node (FLAOPn), floating-point multiplication operation per node (FLMOPn), and memory allocation needed for the field terms per node (MAFTn) are tabulated in Table 1.

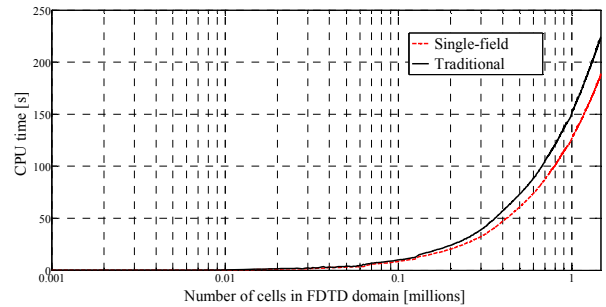


Fig. 2. Comparison of performance in 1D.

B. Memory/speed analysis of a TM problem

A two-dimensional problem is constructed as free space with a z -directed impressed electric current located at the origin. The current density has a Gaussian waveform with magnitude of 1 [Amp/m]. Electric field generated by the

traditional and the single-field formulations are compared in time and frequency domains, the stability and dispersion analyses are also performed for both. Since the real benefit of the single-field formulation is the time required to run the simulation and required memory size, the two formulations are compared for different domain sizes. Figure 3 shows the CPU time verses domain size for both formulations for the same cell size and number of cells. To get a better insight for the simulation time and memory usage, the required number of FLAOPn, FLMOPn, and (MAFTn) are tabulated in Table 1. As for the memory allocation, only the field terms and their coefficients are taken into account since the source terms are updated at only source points, therefore the required memory for the source terms and their coefficients are negligible compared to the field terms.

C. Memory/speed analysis of a TE problem

A two dimensional problem is constructed as free space with a z -directed impressed magnetic current located at the origin. The current density has a Gaussian waveform with magnitude of 1 [V/m]. Magnetic field generated by the traditional and the single-field formulations are compared in time and frequency domains, stability and dispersion analyses are, also, performed for both. Due to the symmetry in the formulation and duality in the problem, merits for CPU time, memory requirements, stability, and dispersion are the same as the previous TM problem. Therefore, Fig. 3 and Table 1 show the performance of the single-field formulation for 2D TE problems as well.

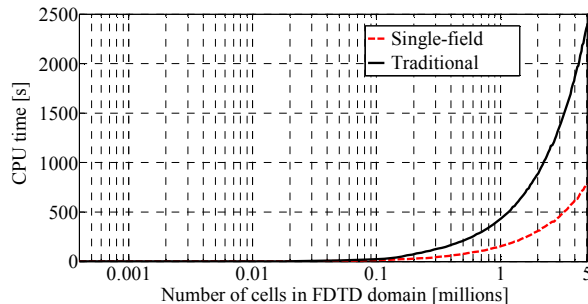


Fig. 3. Comparison of performance in 2D.

A speed up factor is calculated according to the formula given in (14) for different problem sizes and plotted in Fig. 4.

$$\text{Speed up Factor} = \frac{\text{CPU Time (Traditional)}}{\text{CPU Time (Single-field)}} \quad (14)$$

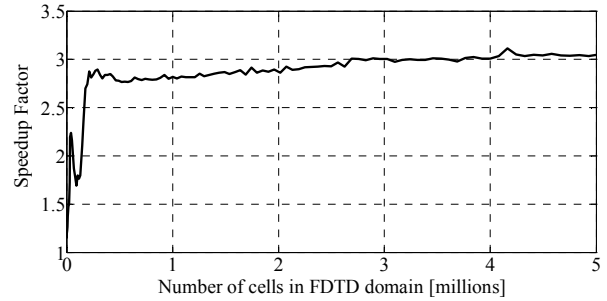


Fig. 4. Speed up factor in 2D.

The single-field formulation appears to be faster than the traditional one, especially for greater domain sizes.

Table 1: Summary of the required number of FLAOPn, FLMOPn, and MAFTn

Formulation	# FLAOPn	# FLMOPn	# MAFTn
2D Single-field	5	4	6
2D Traditional	8	7	10
1D Single-field	3	3	5
1D Traditional	2	4	6

D. Numerical validation

An infinite line of a constant electric current is placed parallel and in the vicinity of a circular conducting cylinder of infinite length. We will examine the scattering of the cylindrical waves by the cylinder for $\rho \geq \rho'$. The analytical solution for the total electric field is given in [11] as

$$E_{\rho}^t = E_{\varphi}^t = 0, \quad (15)$$

$$E_z^t = -\frac{\beta^2 I_e}{4\omega\epsilon} \sum_{n=-\infty}^{+\infty} H_n^{(2)}(\beta\rho) \left[J_n(\beta\rho) - \frac{J_n(\beta a)}{H_n^{(2)}(\beta\rho')} H_n^{(2)}(\beta\rho') \right] e^{jn(\varphi-\varphi')}, \quad (16)$$

where ρ is the distance from the center of the cylinder to the field point, its range is 0.1-1.1 m, ρ' is the distance from the center of the cylinder to

the source point, its value is 0.1 m, φ is the azimuth angle of the field point, and φ' is the azimuth angle of the source point; its value is 0, a is the radius of the conducting cylinder and its value is 0.01 m. For the numerical simulation, the spatial and temporal steps used are $\Delta x = \Delta y = 1$ mm and $\Delta t = 2.2407$ ps. The cylinder is modeled in FDTD domain by stair-casing. Electric field is computed with the single-field and the traditional formulation at 1000 different spatial points in time domain and converted to frequency domain to compare with the analytical solution results. The single-field and the traditional formulation show similar performance in terms of accuracy as shown in Figs. 5 and 6.

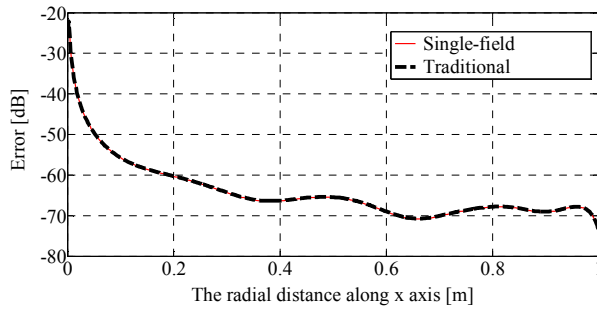


Fig. 5. Comparison of the numerical solutions with the analytical solution; magnitude.

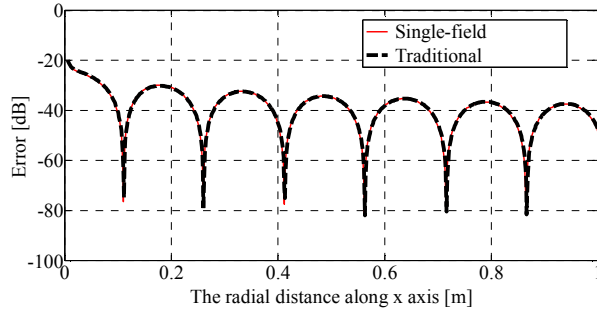


Fig. 6. Comparison of the numerical solutions with the analytical solution; phase.

E. Stability comparison

Stability comparison was conducted by changing the value of the time-increment (Δt) and observing the change in the field values generated by the single-field and the traditional formulations.

The Courant-Friedrichs-Lewy (CFL) condition [12] requires that the time increment Δt be 2.35 ps for a stable result if the space increments in both directions, Δx and Δy , are 1 mm. Figure 7 shows the field comparison of such stable simulation

results calculated at point (8, 8) mm in a 20 mm x 20 mm free-space problem domain as described in Section B. If we set Δt to 2.37 ps, the single-field and the traditional formulation shows divergence from optimum field values. Figure 8 shows the divergence in terms of absolute value of the field versus time step. The single-field formulation provides comparatively less divergent results than the traditional formulation does, since it requires less numerical computation.

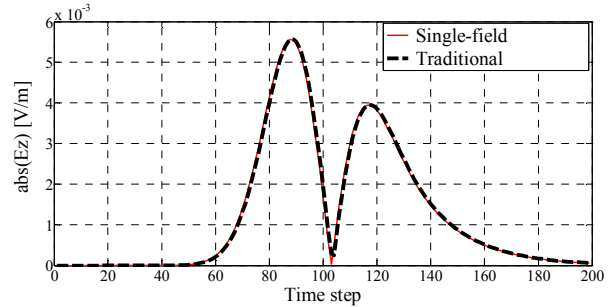


Fig. 7. Field comparison for $\Delta t = 2.35$ ps.

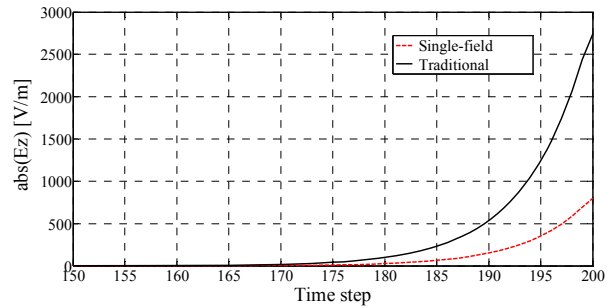


Fig. 8. Field comparison for $\Delta t = 2.37$ ps.

F. Dispersion analysis

Dispersion is defined as the variation of a propagating wave's velocity with frequency. The analysis is done for E_z component of the electric field for 2D case under the assumption of lossless medium and monochromatic traveling wave solution

$$E_z^n(i, j) = E_{z0} e^{j(\omega n \Delta t - k_x i_x \Delta x - k_y i_y \Delta y)}, \quad (17)$$

where k_x and k_y are the x and y components of the numerical wavevector, i_x and i_y are space indices. By substituting this field expression into the electric field updating equation for E_z , and using the points per wavelength discretization (PPW) $= \frac{\lambda_n}{h}$, $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ and $\frac{c \Delta t}{h} = 0.5$, one may obtain

$$\frac{c_n}{c_0} = \frac{\lambda_n}{\lambda} = \frac{2PPW}{\pi} \sin^{-1} \left[\frac{1}{4} - \left[\frac{1}{4} \cos \left(\frac{\pi}{PPW} (\cos \alpha + \sin \alpha) \right) \cos \left(\frac{\pi}{PPW} (\cos \alpha - \sin \alpha) \right) \right] \right]^{1/2}. \quad (18)$$

where c_n is the numerical velocity, λ_n is the numerical wavelength, and α is the angle between the direction of the propagating wave and the positive x -axis. Equation (18) gives the ratio of the velocities or wavelengths as a function of PPW and α for the 2D case. Figure 9 shows the variation of the normalized numerical phase velocity (c_p/c_0) versus PPW in two-dimensional FDTD grid for a plane wave travelling at 0 degree angle i.e., $\alpha = 0$.

Dispersion performance of the single-field formulation shows a characteristic identical to the traditional formulation as given in [13].

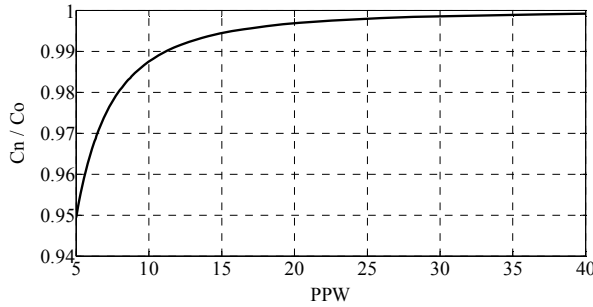


Fig. 9. Dispersion performance of the single-field formulation.

IV. CONCLUSION

The single-field finite-difference time-domain updating equations have been derived for two and one dimensional electromagnetic problems. In the traditional approach, electric field components are updated at integer time increments whereas magnetic field components need to be updated after a half-time-increment. Since the proposed updating equations are based on a single field only, updating field components takes place only at integer time increments [14]. Liao's ABCs are used for both formulations in the verification of the examples presented [4]. One-dimensional case of the single-field formulation is evaluated with example geometry, and it is observed that the single-field formulation is about 20% faster than the traditional one, and provides around 20% memory reduction for solving the same size problem. The single-field formulation has great

advantage in the two-dimensional case. A two-dimensional TM_z problem is constructed with an electric current source, and the field away from the source is calculated by the single-field and the traditional formulations. First, the stability and dispersion analyses are performed. Then, the speed and memory analyses follow; the single-field formulation happens to be around three times faster for reasonably big problem sizes and requires around 43% less memory than its traditional counterpart. A two-dimensional TE_z problem evaluation is also discussed to show that the single-field formulation is advantageous for two-dimensional TM_z as well as TE_z problems. The 3D case is being worked on for non-dispersive and dispersive media. The results will be reported in a future article.

APPENDIX A

The Complete Expressions of the Coefficients

$$C_x(i, j) = -\frac{2(\Delta t)^2}{\Delta t(\mu\sigma^e + \varepsilon\sigma^m) + 2\mu\varepsilon} \quad (19)$$

$$C_{ex}^{ex, n}(i, j) = C_x(i, j) \left(\frac{2}{(\Delta y)^2} - \frac{2\mu\varepsilon}{(\Delta t)^2} + \sigma^m\sigma^e \right) \quad (20)$$

$$C_{ex}^{ex, n-1}(i, j) = C_x(i, j) \left(\frac{\mu\varepsilon}{(\Delta t)^2} - \frac{(\mu\sigma^e + \varepsilon\sigma^m)}{2\Delta t} \right) \quad (21)$$

$$C_{ex}^{ex, n, y}(i, j) = -C_x(i, j) \left(\frac{1}{(\Delta y)^2} \right) \quad (22)$$

$$C_{ex}^{ey, n, xy}(i, j) = C_x(i, j) \left(\frac{1}{\Delta x \Delta y} \right) \quad (23)$$

$$C_{ex}^{mz, n, y}(i, j) = C_x(i, j) \left(\frac{1}{\Delta y} \right) \quad (24)$$

$$C_{ex}^{jx, n}(i, j) = C_x(i, j) (\sigma^m) \quad (25)$$

$$C_{ex}^{jx, t}(i, j) = C_x(i, j) \left(\frac{\mu}{2\Delta t} \right) \quad (26)$$

$$C_y(i, j) = -\frac{2(\Delta t)^2}{\Delta t(\mu\sigma^e + \varepsilon\sigma^m) + 2\mu\varepsilon} \quad (27)$$

$$C_{ey}^{ey, n}(i, j) = C_y(i, j) \left(\frac{2}{(\Delta x)^2} - \frac{2\mu\varepsilon}{(\Delta t)^2} + \sigma^m\sigma^e \right) \quad (28)$$

$$C_{ey}^{ey, n-1}(i, j) = C_y(i, j) \left(\frac{\mu\varepsilon}{(\Delta t)^2} - \frac{(\mu\sigma^e + \varepsilon\sigma^m)}{2\Delta t} \right) \quad (29)$$

$$C_{ey}^{ey, n, x}(i, j) = -C_y(i, j) \left(\frac{1}{(\Delta x)^2} \right) \quad (30)$$

$$C_{ey}^{ex, n, xy}(i, j) = C_y(i, j) \left(\frac{1}{\Delta x \Delta y} \right) \quad (31)$$

$$C_{ey}^{mz, n, y}(i, j) = -C_y(i, j) \left(\frac{1}{\Delta x} \right) \quad (32)$$

$$C_{ey}^{jy, n}(i, j) = C_y(i, j) (\sigma^m) \quad (33)$$

$$C_{ey}^{jy,t}(i,j) = C_y(i,j) \left(\frac{\mu}{2\Delta t} \right) \quad (34)$$

$$C_z(i,j) = -\frac{2(\Delta t)^2}{\Delta t(\mu\sigma^e + \varepsilon\sigma^m) + 2\mu\varepsilon} \quad (35)$$

$$C_{ez}^{ez,n}(i,j) = C_z(i,j) \left(\frac{2}{(\Delta x)^2} + \frac{2}{(\Delta y)^2} - \frac{2\mu\varepsilon}{(\Delta t)^2} + \sigma^m\sigma^e \right) \quad (36)$$

$$C_{ez}^{ez,n-1}(i,j) = C_z(i,j) \left(\frac{\mu\varepsilon}{(\Delta t)^2} - \frac{(\mu\sigma^e + \varepsilon\sigma^m)}{2\Delta t} \right) \quad (37)$$

$$C_{ez}^{ez,n,x}(i,j) = -C_z(i,j) \left(\frac{1}{(\Delta x)^2} \right) \quad (38)$$

$$C_{ez}^{ez,n,y}(i,j) = -C_z(i,j) \left(\frac{1}{(\Delta y)^2} \right) \quad (39)$$

APPENDIX B

2D Single-Field H-Based Updating Equations

If we assume no variation with respect to the z direction, i.e. $\frac{\partial}{\partial z} = 0$, the x -component of the magnetic field from (6) is given as:

$$\frac{\partial^2 H_x}{\partial y^2} - \frac{\partial^2 H_y}{\partial x \partial y} = \sigma^m \sigma^e H_x + (\mu\sigma^e + \varepsilon\sigma^m) \frac{\partial H_x}{\partial t} + \mu\varepsilon \frac{\partial^2 H_x}{\partial t^2} + \frac{\partial J_{iz}}{\partial y} + \sigma^E M_{i,x} + \mu \frac{\partial M_{i,x}}{\partial t}. \quad (12)$$

To derive the FDTD updating equations for the x -component of the magnetic field, we have to evaluate all the spatial derivatives in (12) at their corresponding magnetic field node, i.e. H_x . Central difference formula is used to discretize the derivatives.

The final expression of the updating equation for the x -component of the magnetic field becomes

$$\begin{aligned} H_x^{n+1}(i,j) = & C_{hx}^{hx,n}(i,j)[H_x^n(i,j)] \\ & + C_{hx}^{hx,n-1}(i,j)[H_x^{n-1}(i,j)] \\ & + C_{hx}^{hx,n,y}(i,j)[H_x^n(i,j+1) + H_x^n(i,j-1)] \\ & + C_{hx}^{hy,n,xy}(i,j)[H_y^n(i+1,j) - H_y^n(i,j) \\ & \quad - H_y^n(i+1,j-1) + H_y^n(i,j-1)] \\ & + C_{hx}^{jiz,n,y}(i,j)[J_{i,z}^n(i,j) - J_{i,z}^n(i,j-1)] \\ & + C_{hx}^{Miz,n}(i,j)[M_{i,z}^n(i,j)] \\ & + C_{hx}^{Miz,t}(i,j)[M_{i,z}^{n+1}(i,j) - M_{i,z}^{n-1}(i,j)], \quad (14) \end{aligned}$$

where the C terms are constant coefficients. Similarly, updating equations for H_y and H_z can be obtained in the same manner and their final expressions are

$$\begin{aligned} H_y^{n+1}(i,j) = & C_{hy}^{hy,n}(i,j)H_y^n(i,j) \\ & + C_{hy}^{hy,n-1}(i,j)[H_y^{n-1}(i,j)] \\ & + C_{hy}^{hy,n,y}(i,j)[H_y^n(i,j+1) + H_y^n(i,j-1)] \\ & + C_{hy}^{hy,n,xy}(i,j)[H_y^n(i,j+1) - H_y^n(i,j) \\ & \quad - H_y^n(i-1,j+1) + H_y^n(i-1,j)] \\ & + C_{hy}^{jiz,n,x}(i,j)[J_{i,z}^n(i+1,j) - J_{i,z}^n(i,j)] \\ & + C_{hy}^{Miy,n}(i,j)[M_{i,y}(i,j)] \\ & + C_{hy}^{Miy,t}(i,j)[M_{i,y}^{n+1}(i,j) - M_{i,y}^{n-1}(i,j)]. \quad (15) \end{aligned}$$

$$\begin{aligned} H_z^{n+1}(i,j) = & C_{hz}^{hz,n}(i,j)[H_z^n(i,j)] \\ & + C_{hz}^{hz,n-1}(i,j)[H_z^{n-1}(i,j)] \\ & + C_{hz}^{hz,n,x}(i,j)[H_z^n(i+1,j) + H_z^n(i-1,j)] \\ & + C_{hz}^{hz,n,y}(i,j)[H_z^n(i,j+1) + H_z^n(i,j-1)] \\ & + C_{hz}^{jy,n,x}(i,j)[J_{i,y}^n(i+1,j) - J_{i,y}^n(i,j)] \\ & + C_{hz}^{jx,n,y}(i,j)[J_{i,x}^n(i,j+1) - J_{i,x}^n(i,j-1)] \\ & + C_{hz}^{Miz,n}(i,j)[M_{i,z}(i,j)] \\ & + C_{hz}^{Miz,t}(i,j)[M_{i,z}^{n+1}(i,j) - M_{i,z}^{n-1}(i,j)]. \quad (16) \end{aligned}$$

APPENDIX C

The Computing System Information

All of the simulations presented in this paper are performed using a system whose specifications are given in the table below.

Table 2: The computing system specifications

Processor	Intel(R) Core(TM) i7 CPU 920 @ 2.67 GHz
Memory	6.00 GB
System Type	64-bit OS
Operation System	Windows 7 Pro
Programming Language	Matlab R2009a (32-bit)

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