

Systems for Homogeneous Electrical Fields Generation and Effects of External Bodies on Field Homogeneity

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Abstract — A procedure of modeling new systems consisting of thin toroidal electrodes placed at the surface of a sphere, for high homogeneity electric field generation, is presented in this paper. In order to investigate the influence of an external body shape and volume to the obtained field's homogeneity, the external object is modeled as a sphere, prolate and oblate ellipsoidal electrode and placed in the centre of the system. Numerical results, obtained using the software Mathematica 7.0 and Femm, are compared and shown graphically.

Index Terms — Charge simulation method, ellipsoidal conducting body, homogeneous electric field, method of images, toroidal electrodes.

I. INTRODUCTION

The problems of electric and magnetic field synthesis are of a great importance in many theoretical applications, where it is necessary to generate a field with a high accuracy and of required features. Great attention over years has been devoted to the production of a uniform magnetic field. Arrangements of coils like Helmholtz's two-coil system and Maxwell's three coils combinations are well known devices for producing a region of an approximately uniform magnetic field [1]. This case has been extended to an arbitrary number of coils placed at a virtual spherical surface [2]. A larger region of a uniform magnetic field inside a spherical coil system is achieved in [3].

Much less attention is, up to now, paid to the dual problem, i.e. to a generation of homogeneous electrical fields. A system consisting of two equal

diameter charged rings produces a uniform electrical field, [4]. A very uniform electrical field near the center of the sphere is obtained using special bowl design, [5]. This is a kind of special spherical conducting surfaces at certain potentials.

In our earlier published paper [6], we extended the problem solved in [4] and modeled complex coil systems, which consist of a larger number of primary cells. The modeling process was similar to a procedure applied in [2]. Later, we investigated an effect of an external body on achieved field homogeneity of toroidal electrodes [7]. The external body is modeled as a conducting sphere and brought into the center of the observed system.

The aim of this paper is the modeling of an external body as a prolate and oblate ellipsoidal electrode and analyzing its influence on achieved field homogeneity of a system of toroidal electrodes. Numerical results are obtained using software packages Mathematica and Femm.

II. HOMOGENEOUS ELECTRIC FIELD GENERATION

In practice, very often there is a need for getting homogeneous electric fields, with required features and of high precision. Those fields are used widely in technique: they are applied in electrical machines for high field generation, NMR spectroscopy, MHD technique, nuclear and plasma physics, so the problem of an electrostatic field is very actual from the standpoint of its realization as well as from the standpoint of its partial or complete elimination.

A. Toroidal electrodes system

By analogy to Helmholtz coils that produce an uniform magnetic field when the distance between coils is equal to the radius of the coils, it is possible to produce an uniform electrical field using two charged rings when the distance between rings is $\sqrt{6}$ times the ring radius [4].

The work described here refers to a modeling of complex systems for generating a homogeneous electric field, formed by thin coaxial toroidal electrodes. A primary cell is used as a starting element in a modeling procedure. It consists of a pair of flat toroidal electrodes which have the same radius d , separated by a distance $2h$ as in Fig. 1. The electrodes are of absolutely equal potential values, but of opposite signs. Complex systems consist of a number of primary cells placed at a virtual sphere, Fig 2.

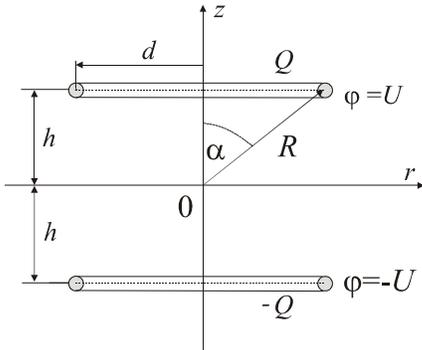


Fig. 1. A toroidal primary cell.

We shall take the z axis of a cylindrical coordinate system as being the axis of the coils, with the origin midway between the coils. In this case, the axial potential in area $|z| < R$ can be expanded in a power series, which contains only odd powers of the axial variable z [6]:

$$\phi(r=0, z) = \sum_{n=0}^{\infty} \phi_{2n+1} z^{2n+1}, \quad (1)$$

where

$$\phi_{2n+1} = \frac{QP_{2n+1}(\cos \alpha)}{2\pi\epsilon R^{2n+2}}, \quad (2)$$

$P_n(x)$ is the Legendre polynomial of the first kind, R denotes the electrode distance from the origin, $R = \sqrt{d^2 + h^2}$ and $\cos \alpha = h/R$.

Since the coefficients in series (1) depend on electrodes positions and dimensions, it is possible

to choose primary cell dimensions to eliminate a few subsequent terms different than linear so that a linear term becomes dominant.

For further work it is necessary to introduce the following definitions. The primary cell has N -th order, if the term ϕ_{2N+1} in expression (1) is equal to zero. This condition is satisfied if $P_{2N+1}(\cos \alpha) = 0$. This equation, for $N \geq 1$, has N positive roots, $\cos \alpha_k = x_{2N+1,k}$, for $k=1,2,\dots,N$, where $x_{2N+1,k}$ are zeros of the Legendre polynomial $P_{2N+1}(x)$. Variable k defines the degeneration degree of primary cells. So the primary cell of the N -th order has N different degeneration states, i.e. shapes. All those primary cells are placed at a surface of a virtual sphere of radius R , Fig. 2.

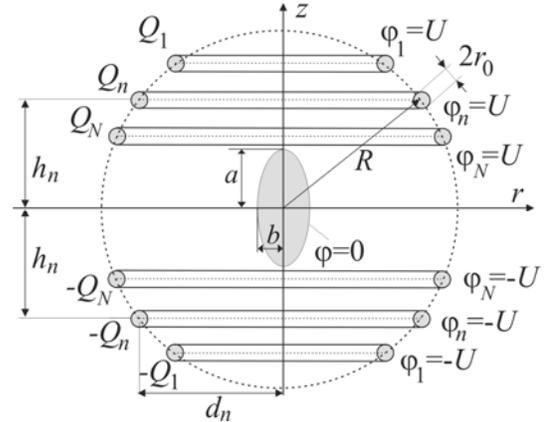


Fig. 2. An external body in the homogeneous electric field.

For the primary cell of the N -th order, the axial potential can be shown using (1), where

$$\phi_{2n+1} = \sum_{k=1}^N \frac{Q_k P_{2n+1}(x_{2k+1,k})}{2\pi\epsilon R^{2n+2}}. \quad (3)$$

The term ϕ_{2N+1} is equal to zero.

In order to eliminate the influence of other series terms from z^3 to z^{2N-1} , the following system of equations has to be satisfied,

$$\phi_{2n+1} = 0, \text{ for } n=1,2,\dots,N-1. \quad (4)$$

Charges of electrodes, Q_n , are obtained solving this system of equations using the condition that all electrodes are equipotential.

The field in a central region will be highly homogeneous if the contribution from the higher order terms is as small as possible. The rest of the

higher order terms can be neglected with respect to linear one.

B. Calculation of primary cell dimensions

Let us first consider a system consisting of one primary cell, i.e. two thin toroidal electrodes, Fig. 1. The electrodes dimensions are being obtained after assuming that the coefficient of z^3 , given by formula (2), is equal to zero, what yields: $P_3(\cos \alpha) = 0$. This equation has only one root, $\cos \alpha = 0.7745966692$, so the primary cell of the first kind has dimensions $h = 0.7745966692 R$ and $d = 0.632455532 R$. It is the same result as in [4].

By setting the coefficient of z^5 to zero, we get the equation that has two solutions, which define dimensions of the second order primary cell:

$$d_1 = 0.422892524 R, h_1 = 0.906179846 R,$$

$$d_2 = 0.8426544122 R, h_2 = 0.538469310 R.$$

In a general case, by setting the coefficient of z^{2N+1} to zero in (1), the primary cell of the N -th order is obtained.

Table 1 presents the dimensions of primary cells of the first to fifth order.

Table 1: Optimal system dimensions for N^{th} order primary cell

	d_5/R	d_4/R	d_3/R	d_2/R	d_1/R	
N	0.963	0.855	0.683	0.462	0.208	5
1	0.775	0.946	0.790	0.549	0.250	4
2	0.906	0.538	0.914	0.671	0.315	3
3	0.949	0.742	0.406	0.843	0.423	2
4	0.968	0.836	0.613	0.324	0.632	1
5	0.978	0.887	0.730	0.519	0.270	N
	h_1/R	h_2/R	h_3/R	h_4/R	h_5/R	

For the system dimensions presented in Table 1, the primary cells of the first and the second order are formed.

In Fig. 3, the normalized electric field along the z axis of the system with primary cells of the first and second order is shown. It is evident that the high field homogeneity is obtained in close vicinity of the central system area.

Below will be the examined effect of an external body on achieved field homogeneity of toroidal electrodes.

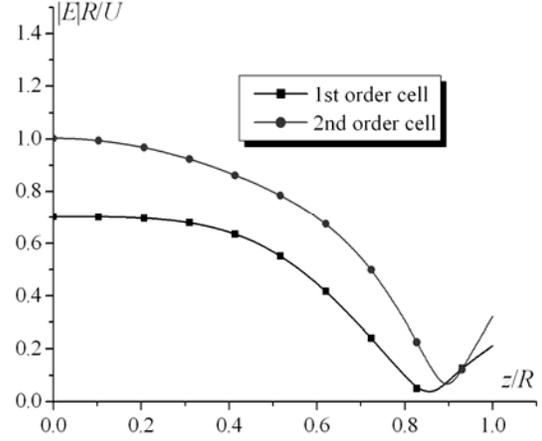


Fig. 3. Normalized electric field along z axis using primary cell of the first and second order.

III. MATHEMATICAL BACKGROUND

A. Rings charges calculation of ellipsoidal electrode

Consider a system presented in Fig. 2. The external body is firstly modeled as a prolate ellipsoidal electrode of semi-axes a and b , $a > b$, and placed into the central system area. Its centre coincides with the coordinate origin. In order to find the affection of the external prolate ellipsoid brought into the homogeneous electric field of the toroidal primary cell of N -th order, the charge simulation method [8] is used. Thin toroidal electrodes of radii

$$r_n = fb\sqrt{1 - (n/(N+1))^2}$$

are used as fictitious sources and placed inside the conducting body at positions $z_n = n/(N+1)fa$, where f is the coefficient, $0 < f < 1$.

The potential at any point is given by the following expression:

$$\phi = \frac{1}{2\pi^2 \epsilon} \sum_{n=1}^N \sum_{m=1}^4 Q_{nm} \left(\frac{K(\pi/2, k_{nm})}{R_{nm}} \right), \quad (5)$$

where $K(\pi/2, k)$ is the complete elliptic integral of the first kind of moduli:

$$k_{n1}^2 = \frac{4r r_n}{R_{n1}^2}, \quad k_{n2}^2 = \frac{4r r_n}{R_{n2}^2},$$

$$k_{n3}^2 = \frac{4r d_n}{R_{n3}^2}, \quad k_{n4}^2 = \frac{4r d_n}{R_{n4}^2},$$

where

$$\begin{aligned} R_{n1} &= \sqrt{(r+r_n)^2 + (z-z_n)^2} \\ R_{n2} &= \sqrt{(r+r_n)^2 + (z+z_n)^2}, \\ R_{n3} &= \sqrt{(r+d_n)^2 + (z-h_n)^2}, \\ R_{n4} &= \sqrt{(r+d_n)^2 + (z+h_n)^2}. \end{aligned}$$

Q_{n1} and Q_{n2} are the charges of fictitious sources placed inside the prolate ellipsoidal electrode volume, for $z > 0$ and $z < 0$, respectively. Q_{n3} are the charges of ring electrodes of potential U and Q_{n4} are the charges of ring electrodes of potential $-U$. After matching the total potential to the value of zero at the ellipsoidal electrode surface and to the potential U at upper electrodes, i.e. to the value $-U$ at the lower electrodes, all the unknown charges can be determined.

B. Rings charges calculation of spherical electrode

We should now consider the system presented in Fig. 2 if $a=b$. In this case, the external body is modeled as a conducting sphere and placed into the central area. Using the method of images into the sphere mirror, this problem is observed in [7]. Dimensions of n -th degenerated primary cell of N -th order are h_n and d_n , and their images into the sphere mirror are $(a/R)^2 h_n$ and $(a/R)^2 d_n$, where $R = \sqrt{(h_n)^2 + (d_n)^2}$. If $Q_{n1} = Q_n$ are the charges of the toroidal electrodes of potential U , $Q_{n2} = -Q_n$ are the charges of the toroidal electrodes of potential $-U$, then $Q_{n3} = -Q_{n4} = -aQ_n/R$ are the charges of their images into the sphere mirror, respectively, for $n=1,2,\dots,N$.

The potential, which produce charges on the toroidal electrodes and their images into the sphere mirror, can be calculated using (5), where

$$\begin{aligned} k_{n1}^2 &= \frac{4d_n r}{R_{n1}^2}, \quad k_{n2}^2 = \frac{4d_n r}{R_{n2}^2}, \\ k_{n3}^2 &= \frac{4(a/R)^2 d_n r}{R_{n3}^2}, \quad k_{n4}^2 = \frac{4(a/R)^2 d_n r}{R_{n4}^2}, \end{aligned}$$

are moduli of complete elliptic integral of the first kind and:

$$\begin{aligned} R_{n1} &= \sqrt{(r+d_n)^2 + (z-h_n)^2}, \\ R_{n2} &= \sqrt{(r+d_n)^2 + (z+h_n)^2}, \\ R_{n3} &= \sqrt{(r+(a/R)^2 d_n)^2 + (z-(a/R)^2 h_n)^2}, \\ R_{n4} &= \sqrt{(r+(a/R)^2 d_n)^2 + (z+(a/R)^2 h_n)^2}. \end{aligned}$$

Using the condition that the toroidal electrodes are of equal potential U for $z > 0$, their unknown charges can be determined after matching the total potential to the potential U on them.

IV. NUMERICAL RESULTS

In accordance with the analysis presented above, a general numerical program has been developed using the software Mathematica 7.0 for different shapes of conducting bodies brought into the homogeneous electric field: prolate ellipsoid, oblate ellipsoid, and sphere conducting electrode. All these problem geometries are modeled using the software Femm [9] and the results are compared with analytical results.

A. Prolate ellipsoidal electrode

Let us observe a system consisting of a number of thin coaxial toroidal electrodes, which generate a homogeneous electric field, and prolate ellipsoidal electrode brought into the field. On the basis of a procedure presented in Section III A, numerical results for the prolate ellipsoidal electrode brought into the homogeneous electric field are obtained. A normalized value of the electric field strength along the z axis, for the case of an ellipsoidal body of normalized semi-axes $a/R=0.4$ and $b/R=0.2$, for primary cells of the first to fifth order is presented in Fig. 4.

It can be noticed that an excellent agreement of the results obtained using the analytical approach and the program FEMM is achieved. Equipotential lines for the same prolate ellipsoidal body and the number of primary cells $N=1$ and $N=3$ are presented in Figs. 5a and 5b, respectively. Only the region of $z > 0$ is shown as the electrical field symmetric.

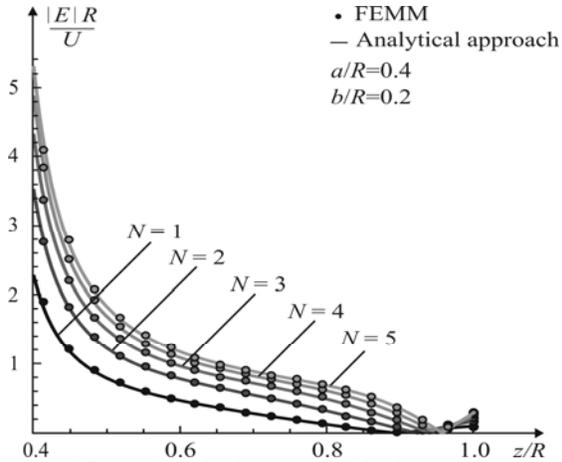


Fig. 4. Normalized electric field along z axis for different number of primary cells and ellipsoidal electrode of semi-axes $a/R=0.4$ and $b/R=0.2$.

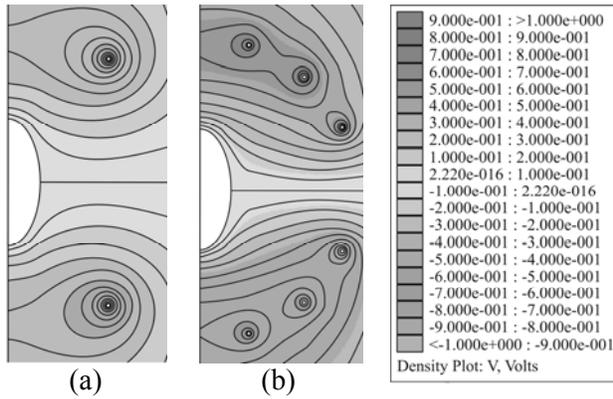


Fig. 5. Equipotential lines for different number of primary cells, N , and ellipsoidal electrode of semi-axes $a/R=0.4$, $b/R=0.2$, $N=1$ (a) and $a/R=0.4$, $b/R=0.2$, $N=3$ (b).

B. Oblate ellipsoidal electrode

For the case of an oblate ellipsoidal electrode brought into the homogeneous electric field, of normalized semi-axes $a/R=0.2$ and $b/R=0.4$, normalized value of the electric field strength along z axis, $|E|R/U$, is presented in Fig. 6, for a different number of primary cells, from first to fifth.

Equipotential lines for the same ellipsoidal body and the number of primary cells $N=1$ and $N=3$ are presented in Figs. 7a and 7b, respectively.

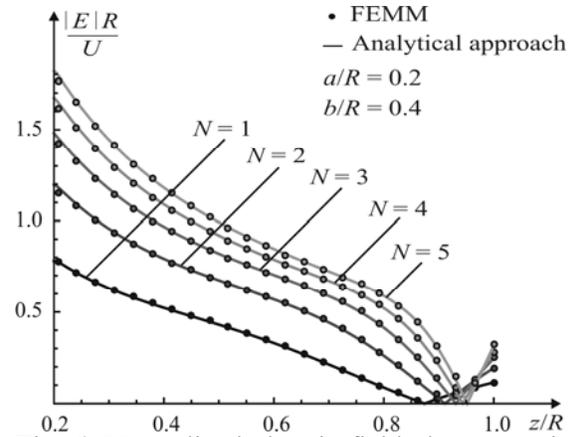


Fig. 6. Normalized electric field along z axis for different number of primary cells and ellipsoidal electrode of semi-axes $a/R=0.2$ and $b/R=0.4$.

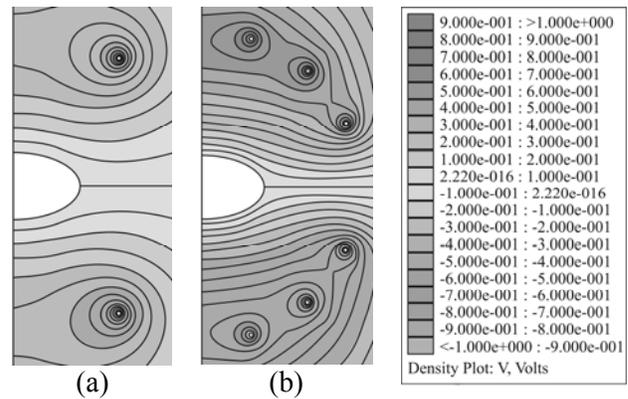


Fig. 7. Equipotential lines for different number of primary cells, N , and ellipsoidal electrode of semi-axes $a/R=0.2$, $b/R=0.4$, $N=1$ (a) and $a/R=0.2$, $b/R=0.4$, $N=3$ (b).

C. Spherical electrode

On the basis of the procedure presented in Section III B, the problem of the spherical electrode brought into the homogeneous electric field of toroidal electrodes is investigated [7].

Let us observe firstly a system consisting of one primary cell. The normalized value of the electric field strength along z axis, with and without the external sphere conducting body of normalized radius $a/R=0.1$, is presented in Fig. 8.

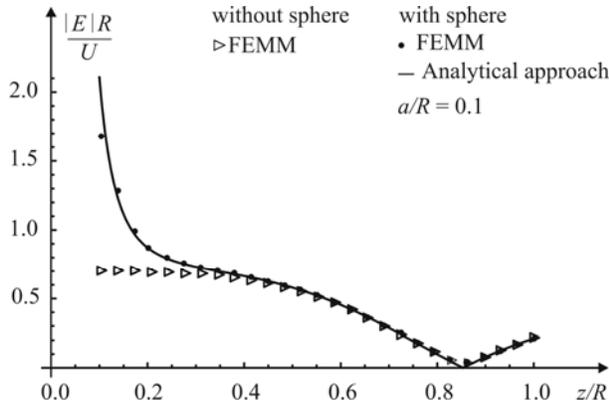


Fig. 8. Normalized electric field along z axis using one primary cell with and without external sphere of normalized radius $a/R = 0.1$.

Presented distributions show that an external conducting body influences on field distribution only in the close vicinity of the body brought into the homogeneous electric field.

The electric field distribution along the system axis for a different number of primary cells, by application of both Mathematica and Femm, is presented in Fig. 9. A high homogeneous electric field is attained using primary cells of high order.

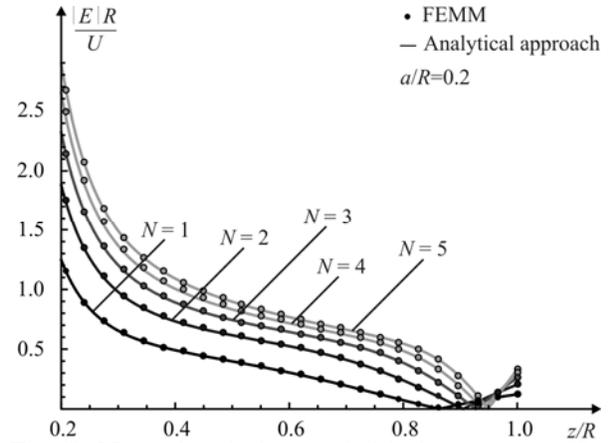


Fig. 9. Normalized electric field along z axis for different number of primary cells and $a/R = 0.2$.

The influence of the external body dimension on the achieved field homogeneity is shown in Fig. 10. An increase of sphere size disturbs realized field homogeneity.

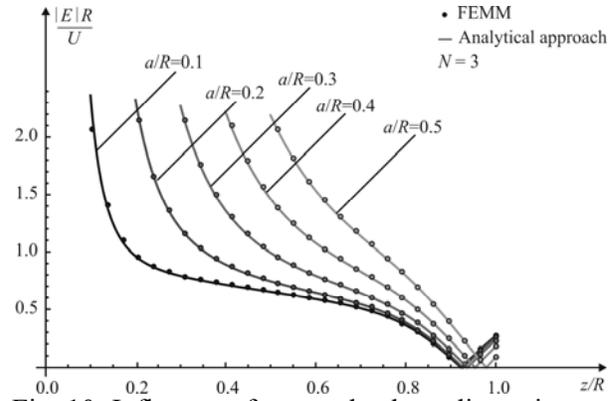


Fig. 10. Influence of external sphere dimension on realized field homogeneity using $N = 3$ primary cells.

Equipotential lines for $a/R = 0.2$, $N = 1$ and $a/R = 0.5$, $N = 3$, are shown in Figs. 11(a) and 11(b), respectively.

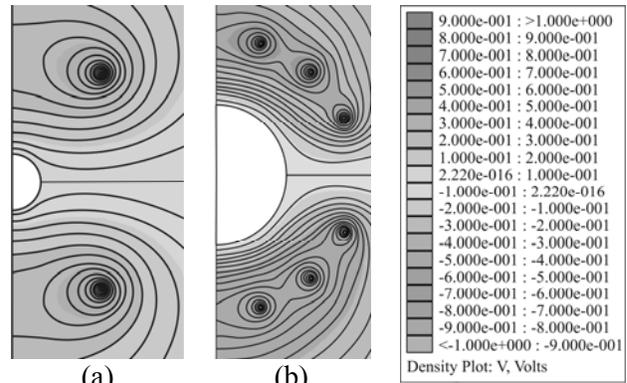


Fig. 11. Equipotential lines for different number of primary cells, N , and spherical electrode of radius $a/R = 0.2$, $N = 1$ (a) and $a/R = 0.5$, $N = 3$ (b).

V. CONCLUSION

Complex toroidal electrostatic systems for the generation of homogeneous electrical fields and effects of different external conducting bodies on the achieved field homogeneity, are observed in this paper.

A potential along the system axis between two toroidal electrodes may be presented by a series in which all the coefficients are functions of an electrodes dimensions. With an appropriate choice of coil positions and dimensions, it is possible to eliminate as many as possible subsequent terms that differ from a linear one, which represent a

desired field. The homogeneity criterion is a number of series terms that vanish.

As the number of used electrodes increase, more members in the series vanish increasing the area of the homogeneous electric field. Keeping this in mind, a primary cell of the third order, $N = 3$, is recommended for a practical usage.

Based on the theoretical investigation, an affection of an external body, modeled as a spherical, prolate and oblate ellipsoidal electrode, is observed. External bodies are brought into the central region of the system of toroidal electrodes. The homogeneity is disturbed mostly in the close vicinity of the body.

More primary cells will provide a larger region of the homogeneous electrical field. Decrease of the external body influence on realized field homogeneity is accomplished by an increase of the primary cells number.

A dimension of an external conducting body has a strong influence on the achieved field homogeneity. The bigger the body, the smaller the area of the homogeneous electrical field.

The further investigation will apply to the system of biconical electrodes, which also can successfully be used for a generation of electrical fields of high homogeneity. Different shaped external bodies brought into the homogeneous electrical field will be investigated, too.

In addition to the need of making a space of the homogeneous electric field, very often there is a need for space in which there is no field. When such toroidal electrodes are grounded and placed in the external electric field, then they can be used for the space protection against the overflow electrical fields strengths.

ACKNOWLEDGEMENT

The authors want to thank Dr. Slavoljub Aleksic, full professor at the Faculty of Electronic Engineering of Nis, University of Nis, Serbia, for helpful discussions and comments. This research was partially supported by funding from the Serbian Ministry of Science (project No. 18019).

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