Approximate Compensation for Mutual Coupling in a Direct Data Domain Least Squares Approach using the In-situ Measured Element Patterns

(Invited Paper)

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Abstract —This paper presents a new technique for an approximate compensation of the effects of mutual coupling among the elements of an array using the measured in-situ element patterns in a direct data domain least squares algorithm. In this paper, we consider the antenna elements in the phased array to have finite dimensions, i.e., they are not omni-directional radiators. Hence, the antenna elements sample and re-radiate the incident fields resulting in mutual coupling between the antenna elements. Mutual coupling not only destroys the linear wavefront assumption for the signal of interest but also for all the interferers impinging on the array. Thus, we propose a new direct data domain approach that partly compensates for effect of mutual coupling, specifically when the jammer strengths are comparable to that of the signal. For strong interferers, a more accurate compensation for the mutual coupling is necessary using the transformation matrix through the formation of a uniform liner virtual array.

Key Words: Adaptive Processing, Direct Data Domain Approach, Element pattern, Mutual Coupling, Least Squares

I. INTRODUCTION

The principal advantage of an adaptive array is the ability to electronically steer the mainlobe of the antenna to any desired direction while also automatically placing deep pattern nulls along the specific directions of interferences. Recently, a direct data domain least squares (D3LS) algorithm has been proposed [1-3]. A D3LS approach [3] has certain advantages related to the computational issues associated with the adaptive array processing problem as it analyzes the data for each snapshot as opposed to forming a covariance matrix of the data using multiple snapshots, and then solving for the weights utilizing that information. A single snapshot in this context is defined as the array of complex voltages measured at the feed point of the antenna elements. Another advantage of the D3LS approach is that when the direction of arrival of the signal is not known precisely, additional constraints can be applied to fix the mainlobe beam width of the receiving array a priori and thereby reduce the signal cancellation problem.

Most adaptive algorithms assume that the elements of the receiving array are independent isotropic point sensors that sample, but do not reradiate, the incident fields. They further assume that the array is isolated from its surroundings. In a real system, however, each array element has physical dimensions. Therefore, the elements spatially sample and reradiate the incident fields. The reradiated fields interact with the other antenna elements causing the sensors to be mutually coupled. The effect of mutual coupling may provide erroneous results for the estimated strength of the signal of interest (SOI) and direction of arrival (DOA) of the signal. So several authors have proposed different algorithms to eliminate the effects of mutual coupling in adaptive processing.

Gupta and Ksienski [4] analyzed and compensated for the effects of mutual coupling using a statistical adaptive algorithm. Adve and Sarkar [5-7] illustrated degradation in the capabilities of the D3LS algorithms if the mutual couplings are not accounted for. Adve [7] used a transformation matrix to correct the voltages induced at the antenna elements to compensate for mutual coupling.

In this paper we propose a new technique for an approximate compensation for the effect of mutual coupling between the elements of an array using D3LS algorithms. In this algorithm we use the measured voltages across the loads connected to the antenna elements in the array without using such a transformation, as in [7]. This is equivalent to using the reciprocity theorem to link the measured in-situ far field element pattern to the voltage we are now measuring at the element terminals. We test the new algorithm using an array of dipoles. The mutual coupling among the dipole elements is computed using WIPL-D [8].

This article is organized as follows. In section II we formulate the problem. In section III we present simulation results illustrating the performance of the proposed method. Finally, in section IV we present our conclusions.

II. FORMULATION OF THE NEW DIRECT DATA DOMAIN APPROACH APPROXIMATELY COMPENSATING FOR THE EFFECTS OF MUTUAL COUPLING USING THE IN-SITU ELEMENT PATTERNS

A. Forward Method

Using the complex envelope representation for a uniform linear array where all the antenna elements are equally spaced, the $N \times 1$ complex vectors of phasor voltages $[V(t)]$ received by the antenna elements at a single time instance *t* can be expressed by

$$[V(t)] = \begin{bmatrix} V_1(t) \\ V_2(t) \\ \vdots \\ V_N(t) \end{bmatrix} = \sum_{m=1}^{M} [a(\theta_m)] s_m(t) + \left[n(t)\right] \quad (1)$$

where s_m and θ_m are the amplitude and DOA, respectively, of the m^{th} source incident on the array at the instance *t*, while $[\,a(\theta_m)\,]$ is the steering vector of the array toward direction θ_m and $[\,n(t)\,]$ is the noise vector at each of the antenna elements. We now analyze the data using a single snapshot of the voltages measured at the antenna terminals.

Let us assume that the signal is coming from the angular direction θ_s and our objective is to estimate its amplitude while simultaneously rejecting all other interferences. The signal arrives at each sensor at different times dependent on the DOA of the target and the geometry of the array. At each of the *N* antennas, the received signal is the sum of the SOI, interference, clutter and thermal noise. The interference may consist of coherent multipaths of the SOI along with clutter and thermal noise. Therefore, by suppressing the time dependence in phasor notation, we can reformulate (1) as

$$[V] = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix} = \alpha_s \begin{bmatrix} a_1(\theta_s) \\ a_2(\theta_s) \\ \vdots \\ a_N(\theta_s) \end{bmatrix} + \sum_{m=1}^{M-1} s_m \begin{bmatrix} a_1(\theta_m) \\ a_2(\theta_m) \\ \vdots \\ a_N(\theta_m) \end{bmatrix} + [n] \quad (2)$$

where α_s is the complex amplitude of the SOI, to be determined. The column vectors in this equation explicitly show the various components of the signal induced in each of the *N* antenna elements. In (2), a_n represents the voltage induced at the n^{th} antenna element due to a 1-Volt signal arriving from the particular direction θ. There are $M - 1$ undesired signal components in addition to the SOI. For the conventional adaptive array system, using each of *K* weights W_k, we can estimate the SOI through the following weighted sum

$$y = \sum_{k=1}^{K} W_k \; V_k \quad (3)$$

or in compact matrix form as

$$y = [W]^T [V] \quad (4)$$

where the superscript *T* denotes the transpose of a matrix and *K* is equal to number of weights. In the present method, $K = (N + 1)/2$ [3]. Also *K* has to be greater than the number of interferers $M - 1$, i.e. $K \geq M$.

Let us define another matrix [*S*]

$$[S] = \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_N \end{bmatrix} = \begin{bmatrix} a_1(\theta_s) \\ a_2(\theta_s) \\ \vdots \\ a_N(\theta_s) \end{bmatrix} \quad (5)$$

where a_n represents the voltage induced at the n^{th} antenna element due to the SOI only, with an assumed amplitude of 1V. However, the actual complex amplitude of the SOI is not 1V but α_s which is to be determined. This SOI is arriving from the particular direction θ_s. So the value of S_n in the absence of mutual coupling is

$$S_n = \exp\left[j2\pi \frac{nd}{\lambda} \cos\theta_s \right], \quad n = 1, 2, ..., N. \quad (6)$$

Then, $V_2/S_2 - V_1/S_1$ will have no components of the SOI, moreover, there are only undesired signal components left in it [6]. In a real environment, however, there is mutual coupling among the antenna elements. In this case the elements of the vector [*S*] should be the measured voltages due to the SOI in the antenna array with an assumed amplitude of 1V. So, if we use the actual voltages from the real antenna array for the vector [*S*] and [*V*], then $V_2/S_2 - V_1/S_1$ contains undesired signal components and mutual coupling due to both the SOI and the undesired signals.

Therefore one can form a reduced rank matrix $[T]_{(K-1)\times K}$, generated from the vector [*V*] and [*S*], such that

$$\begin{bmatrix} \frac{V_2}{S_2}-\frac{V_1}{S_1} & \frac{V_3}{S_3}-\frac{V_2}{S_2} & \cdots & \frac{V_{K+1}}{S_{K+1}}-\frac{V_K}{S_K} \\ \frac{V_3}{S_3}-\frac{V_2}{S_2} & \frac{V_4}{S_4}-\frac{V_3}{S_3} & \cdots & \frac{V_{K+2}}{S_{K+2}}-\frac{V_{K+1}}{S_{K+1}} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{V_K}{S_K}-\frac{V_{K-1}}{S_{K-1}} & \frac{V_{K+1}}{S_{K+1}}-\frac{V_K}{S_K} & \cdots & \frac{V_N}{S_N}-\frac{V_{N-1}}{S_{N-1}} \end{bmatrix}_{(K-1)\times K} \times \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_K \end{bmatrix}_{K\times 1} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{(K-1)\times 1} \tag{7}$$

So if we find the weighting vectors which satisfy above matrix equation, we can then eliminate all the undesired signals. Mutual coupling due to the undesired signals is also partially compensated for when we use the actual measured voltage from the real antenna array for the elements of vector [*V*] and [*S*]. To achieve a perfect compensation we have to use a transformation matrix [3] that transforms the measured voltages [*V*] to an equivalent set of voltages that is induced in a uniform linear virtual array consisting of isotropic point radiators radiating in free space. This transformation takes care of the effect of the dissimilarity in the values in the self terms of the port admittance matrix of the array which is reduced to an identity matrix when dealing with isotropic point radiators operating in free space.

In order to make the matrix full rank, we fix the gain of the array to be *C* along the direction of θ_s. This provides an additional equation resulting in

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ \frac{V_2}{S_2}-\frac{V_1}{S_1} & \frac{V_3}{S_3}-\frac{V_2}{S_2} & \cdots & \frac{V_{K+1}}{S_{K+1}}-\frac{V_K}{S_K} \\ \frac{V_3}{S_3}-\frac{V_2}{S_2} & \frac{V_4}{S_4}-\frac{V_3}{S_3} & \cdots & \frac{V_{K+2}}{S_{K+2}}-\frac{V_{K+1}}{S_{K+1}} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{V_K}{S_K}-\frac{V_{K-1}}{S_{K-1}} & \frac{V_{K+1}}{S_{K+1}}-\frac{V_K}{S_K} & \cdots & \frac{V_N}{S_N}-\frac{V_{N-1}}{S_{N-1}} \end{bmatrix}_{K\times K} \times \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_K \end{bmatrix}_{K\times 1} = \begin{bmatrix} C \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{K\times 1} \tag{8}$$

or, in short,

$$[F][W]=[Y]. \tag{9}$$

Once the weights are solved by using (9), the complex amplitude of SOI α_s may be estimated from

$$\alpha_s = \frac{1}{C}\sum_{k=1}^{K}\frac{V_k}{S_k}W_k\ . \tag{10}$$

For the solution of [*F*][*W*]=[*Y*] in (9), the conjugate gradient method starts with an initial guess $[W]_0$ for the solution and continues with the calculation of the following [1-3]

$$[P]_0 = -b_{-1}[F]^H[R]_0 = -b_{-1}[F]^H\{[F][W]_0-[Y]\} \tag{11}$$

where superscript *H* denotes the conjugate transpose of a matrix. At the k^{th} iteration the conjugate gradient method develops the following:

$$c_k = \frac{1}{\|[F][P]_k\|^2}\ , \tag{12}$$

$$[W]_{k+1} = [W]_k + c_k[P]_k\ , \tag{13}$$

$$[R]_{k+1} = [R]_k + c_k[F][P]_k\ , \tag{14}$$

$$b_k = \frac{1}{\|[F]^H[R]_{k+1}\|^2}\ , \tag{15}$$

$$[P]_{k+1} = [P]_k - b_k[F]^H[R]_{k+1}\ . \tag{16}$$

The norm is defined by

$$\|[F][P]_k\|^2 = [P]_k^H[F]^H[F][P]_k\ . \tag{17}$$

The above equations are applied in an iterative fashion till the desired error criterion for the residuals $\left\|[R]_k\right\|$, is satisfied, where $[R]_k = [F][W]_k - [Y]$. In our case, the error criterion is defined by

$$\frac{\left\|[R]_k\right\|}{\left\|[Y]\right\|} = \frac{\left\|[F][W]_k - [Y]\right\|}{\left\|[Y]\right\|} \leq 10^{-6}. \quad (18)$$

B. Backward Method

Next we reformulate the problem using the same data to obtain a second independent estimate for the solution. This is achieved by reversing the data sequence and then complex conjugating each term of that sequence. It is well-known in the parametric spectral estimation literature that a sampled sequence which can be represented by a sum of exponentials with purely imaginary argument can be used either in the forward or in the reverse direction resulting in the same value for the exponent. From physical considerations we know that if we solve a polynomial equation with the weights W_i as the coefficients then its roots provide the DOA for all the unwanted signals including the interferers. Therefore whether we look at the snapshot as a forward sequence as presented in the last section or by a reverse conjugate of the same sequence the final results for W_i must be the same. Hence for these classes of problems we can observe the data either in the forward direction or in the reverse direction. This is equivalent to creating a virtual array of the same size but located along a mirror symmetry line. Therefore, if we now conjugate the data and form the reverse sequence, then one gets an independent set of equations similar to (8) for the solution of the weights $[W]$. This is represented by

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ \frac{V_N^*}{S_N^*} - \frac{V_{N-1}^*}{S_{N-1}^*} & \frac{V_{N-1}^*}{S_{N-1}^*} - \frac{V_{N-2}^*}{S_{N-2}^*} & \cdots & \frac{V_K^*}{S_K^*} - \frac{V_{K-1}^*}{S_{K-1}^*} \\ \frac{V_{N-1}^*}{S_{N-1}^*} - \frac{V_{N-2}^*}{S_{N-3}^*} & \frac{V_{N-2}^*}{S_{N-2}^*} - \frac{V_{N-3}^*}{S_{N-3}^*} & \cdots & \frac{V_{K-1}^*}{S_{K-1}^*} - \frac{V_{K-2}^*}{S_{K-2}^*} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{V_{K+1}^*}{S_{K+1}^*} - \frac{V_K^*}{S_K^*} & \frac{V_K^*}{S_K^*} - \frac{V_{K-1}^*}{S_{K-1}^*} & \cdots & \frac{V_2^*}{S_2^*} - \frac{V_1^*}{S_1^*} \end{bmatrix}_{K\times K} \times \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_K \end{bmatrix}_{K\times 1} = \begin{bmatrix} C' \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{K\times 1} \quad (19)$$

or in a matrix form as

$$[B][W] = [Y] \quad (20)$$

and the complex amplitude of signal α_s can again be determined by

$$\alpha_s = \left[\frac{1}{C'}\sum_{k=1}^{K} W_k \frac{V_{K-k+1}^*}{S_{K-k+1}^*}\right]^*. \quad (21)$$

Note that for both the forward and the backward methods described in Sections II.A and II.B, we have $K = (N+1)/2$. Hence the degrees of freedom are the same for both the Forward and the Backward method. However, we have two independent solutions for the same adaptive problem. In a real situation when the solution is unknown two different estimates for the same solution may provide a level of confidence on the quality of the solution.

C. Forward-Backward Method

Finally, in this section we combine both the forward and the backward method to double the given data and thereby increase the number of weights or the degrees of freedom significantly over that of either the forward or the backward method. This provides a third independent solution. In the forward-backward model we double the amount of data by not only considering the data in the forward direction but also conjugating it and reversing the direction of increment of the independent variable. This type of processing can be done as long as the series to be approximated can be fit by exponential functions of purely imaginary arguments. This is always true for the adaptive array case. So by considering the data set V_n and V^*_{-n} we have essentially doubled the amount of data without any penalty, as these two data sets for our problem are linearly independent.

An additional benefit accrues in this case. For both the forward and the backward method, the maximum number of weights we can consider is given by $(N + 1)/2$, where N is the number of the antenna elements. Hence, even though all the antenna elements are being utilized in the processing, the number of degrees of freedom available for this approach is essentially $(N + 1)/2$. For the forward-backward method, the number of degrees of freedom can be significantly increased without increasing the number of antenna elements. This is accomplished by considering the forward and backward versions of the array data. For this case, the number of degrees of freedom Q, can reach $(N + 0.5)/1.5$. This, Q, is approximately equal to 50% more weights or number of degrees of freedom than the two previous cases of K. The equation that needs to be solved for the weights is given by combining (8) and (19), with C′ = C, into

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ \frac{V_2}{S_2}-\frac{V_1}{S_1} & \frac{V_3}{S_3}-\frac{V_2}{S_2} & \cdots & \frac{V_{Q+1}}{S_{Q+1}}-\frac{V_Q}{S_Q} \\ \frac{V_3}{S_3}-\frac{V_2}{S_2} & \frac{V_4}{S_4}-\frac{V_3}{S_3} & \cdots & \frac{V_{Q+2}}{S_{Q+2}}-\frac{V_{Q+1}}{S_{Q+1}} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{V_Q}{S_Q}-\frac{V_{Q-1}}{S_{Q-1}} & \frac{V_{Q+1}}{S_{Q+1}}-\frac{V_Q}{S_Q} & \cdots & \frac{V_N}{S_N}-\frac{V_{N-1}}{S_{N-1}} \\ \frac{V_N^*}{S_N^*}-\frac{V_{N-1}^*}{S_{N-1}^*} & \frac{V_{N-1}^*}{S_{N-1}^*}-\frac{V_{N-2}^*}{S_{N-2}^*} & \cdots & \frac{V_Q^*}{S_Q^*}-\frac{V_{Q-1}^*}{S_{Q-1}^*} \\ \frac{V_{N-1}^*}{S_{N-1}^*}-\frac{V_{N-2}^*}{S_{N-3}^*} & \frac{V_{N-2}^*}{S_{N-2}^*}-\frac{V_{N-3}^*}{S_{N-3}^*} & \cdots & \frac{V_{Q-1}^*}{S_{Q-1}^*}-\frac{V_{Q-2}^*}{S_{K-2}^*} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{V_{Q+1}^*}{S_{Q+1}^*}-\frac{V_Q^*}{S_Q^*} & \frac{V_Q^*}{S_Q^*}-\frac{V_{Q-1}^*}{S_{Q-1}^*} & \cdots & \frac{V_2^*}{S_2^*}-\frac{V_1^*}{S_1^*} \end{bmatrix} \times \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_Q \end{bmatrix}_{Q\times 1} = \begin{bmatrix} C \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{(2Q-1)\times 1} \tag{22}$$

or in matrix form as

$$[FB]\,[W] = [Y]. \tag{23}$$

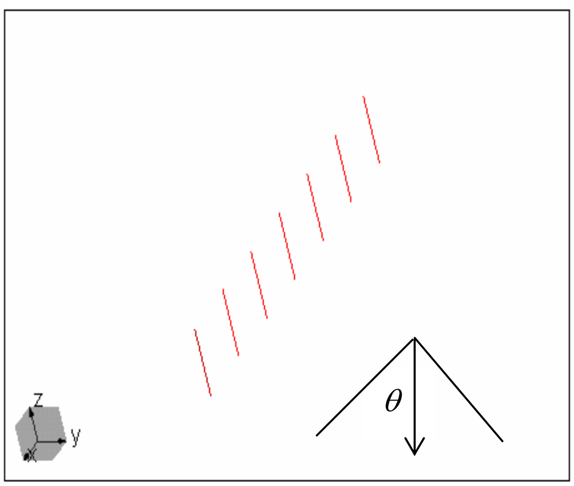

Fig. 1. A uniform linear array of dipoles.

III. SIMULATION RESULTS

As a first example, we consider a signal of unit amplitude arriving from θ_s = 95°. We form a 7-element dipole array of half wavelengths long and centrally loaded with 50 Ω as shown in Figure 1. The parameters for the dipole antenna array are given in Table I. We consider the mutual coupling in this array of dipoles and analyze the antenna array using WIPL-D [8]. The goal is to recover the complex amplitude of the SOI using the proposed method, in the presence of mutual coupling using the insitu measured element patterns. However, for numerical simulation we use the voltages measured at the loads of the antenna elements as the insitu element patterns are related to these voltages through reciprocity. The proposed algorithm tries to maintain the gain of the array along the direction of SOI while automatically placing nulls along the directions of the interferences. In this simulation we take the SOI of unit amplitude to be arriving from θ_s = 95°. Two jammers are present at 80° and 110°. The intensities of these jammers are varied from 1 to 30 [V/m]. We assume that we know the DOA of the signal but we need to estimate its complex amplitude. In addition, we do not know the complex amplitudes or the DOA of the interferers nor do we have any probabilistic description of the thermal noise. We assume that we have 20 dB of signal-to-noise ratio at the antenna elements. The output signal-to-interference-plus-noise ratio (Output SINR) is shown in Figure 2. The output SINR is an indicator of the accuracy of our estimate. It is defined as

$$\mathrm{SINR}_{\mathrm{out}} = 20\log\left|\frac{\alpha_s}{\alpha_s-\alpha_{est}}\right| \tag{25}$$

where, the numerator α_s provides the true value for the complex amplitude of the desired signal and α_{est} is the output providing the estimated complex amplitude. The denominator term $\alpha_s-\alpha_{est}$ then provides the residual interference plus noise error, which resulted from the processing. Results are shown for the three different methods (Forward method, Backward method and Forward-Backward method) in Figure 2. It is seen from Figure 2 that as the intensity of the interferer increases the results become inaccurate if we do not correct for the mutual coupling.

Table I. Parameters defining the elements of the dipole array.

Number of elements in array	7
Length of z-directed wires	$\lambda/2$
Radius of wires	$\lambda/200$
Spacing between wires	$\lambda/2$
Loading at the center	50 Ω

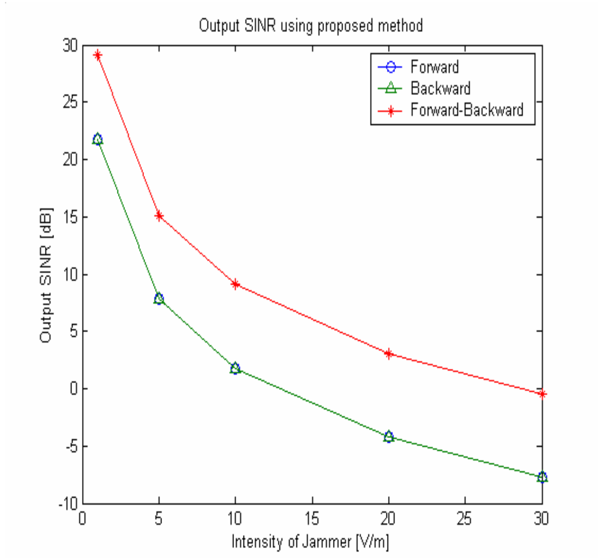

Fig. 2. Output SINR as a function of the intensity of the jammer for a 7-element ULA of half-wave dipoles without correcting for the mutual coupling.

Figure 3a presents the results for the D3LS method which does account for the mutual coupling between the antenna elements using the transformation matrix and forming a uniform linear virtual array as illustrated in [3]. Figure 3a shows the improvement in the performance of the output SINR for the Forward method and Figure 3b for the Forward-Backward method. As seen in Figures 3, the proposed method deteriorates gracefully as the intensity of the jammers increase and a more accurate analysis, through the use of a uniform linear virtual array to compensate for the mutual coupling between the antenna elements.

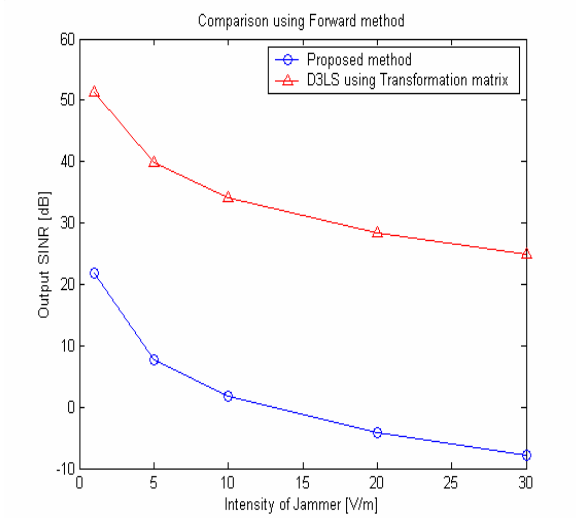

Fig. 3(a). Comparison of the output SINR using the Forward processor between the proposed method and the use of a more accurate treatment of mutual coupling through the transformation matrix.

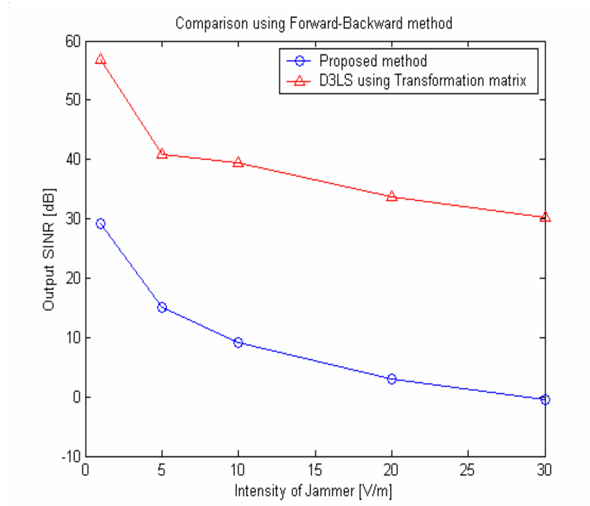

Fig. 3(b). Comparison of the output SINR using the Forward-Backward processor between the proposed method and the use of a more accurate treatment of mutual coupling through the transformation matrix.

For the second example, we study the performance of the proposed method as we increase the number of antenna elements both with and without an accurate compensation for the mutual coupling between the elements of the array. In this simulation we use the same geometry of the antenna array as in the first simulation. The DOA of the SOI and the jammers are also the same. The intensities of the jammers are 30 [V/m]. The number of antenna elements in the array is increased from 7 to 31. In Figure 4 we present the output SINR as a function of the number of antenna elements using the proposed method. It is seen that if we significantly increase the number of antenna elements then it is not necessary to perform an accurate compensation for the mutual coupling.

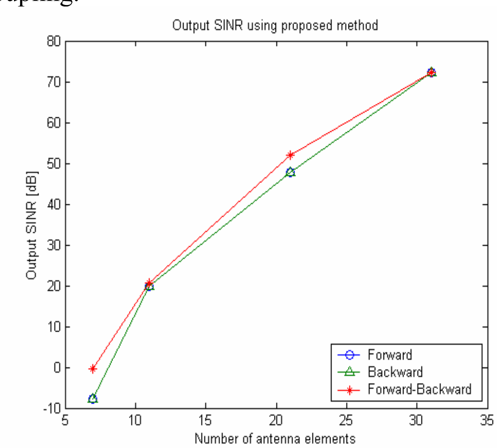

Fig. 4. Output SINR as a function of the number of antenna elements in the array using the proposed methods.

Figures 5a and 5b present the output SINR when we do and do not perform an accurate compensation for the mutual coupling between the elements in the array. It is seen that accurately compensating for the mutual coupling provides a more accurate estimate for the SOI.

For the third example, we simulate the same array but put some additional dummy antenna elements at the ends of the antenna array. In this case, the measured element pattern will be approximately the same for all the elements of the array including the ones at the end. We receive the same signaland the interferers using this modified array, which has dummy elements at the end, and apply the proposed adaptive algorithm to the voltages received by this modified array. In this simulation we take the SOI of unit amplitude to be arriving from θ_s = 90°. Two jammers are present at 70° and 120°. And the intensity of the SOI is 1 [V/m] and the intensity of the interferers is varied from 30 [V/m] to 1000 [V/m]. Simulation results are shown in Tables II through V.

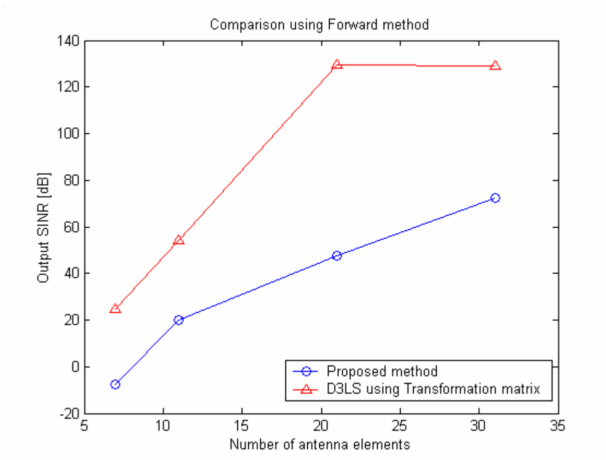

Fig. 5(a). Comparison of the output SINR between the proposed method and a more accurate compensation for mutual coupling using the Forward processor.

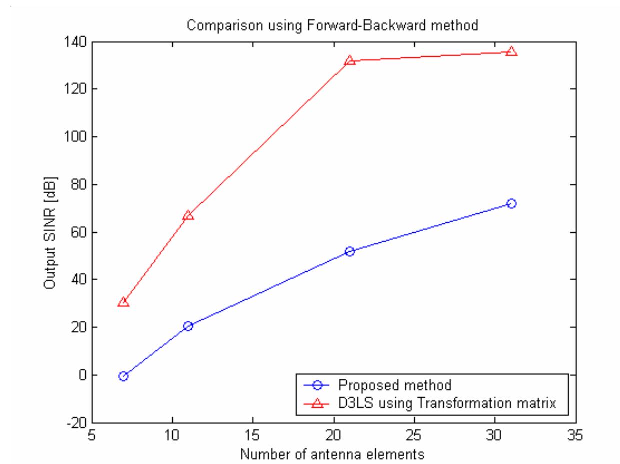

Fig. 5(b). Comparison of the output SINR between the proposed method and a more accurate compensation for mutual coupling using the Forward-Backward processor.

Table II. Output SINR as a function of the number of dummy elements at the two ends of the array (Intensity of SOI: 1 [V/m], Intensity of jammers: 30[V/m])

# of elements		Number of dummy elements (each side)			
		0	1	2	3
7	α ($SINR_{out}$)	1.8105 – 1.1179i (–2.8029 dB)	1.2645 – 0.2704i (8.4445 dB)	0.8695 + 0.2312i (11.5191 dB)	1.0054 + 0.0340i (29.2558 dB)
11		1.0565 + 0.3828i (8.2464 dB)	1.0416 + 0.0528i (23.4450 dB)	0.9950 – 0.0132i (37.0187 dB)	1.0120 + 0.0078i (36.8500 dB)
21		0.9976 + 0.0009i (51.9185 dB)	0.9983 + 0.0005i (55.2192 dB)	0.9986 + 0.0005i (56.5733 dB)	1.0000 – 0.0006i (64.9739 dB)

Table III. Output SINR as a function of the number of dummy elements at the two ends of the array (Intensity of SOI: 1 [V/m], Intensity of jammers: 100 [V/m])

# of elements		Number of dummy elements (each side)			
		0	1	2	3
7	α ($SINR_{out}$)	3.7017 – 3.7265i (–13.2605 dB)	1.8816 – 0.9013i (–2.0131 dB)	0.5649 + 0.7706i (1.0616 dB)	1.0181 + 0.1134i (18.7982 dB)
11		1.1882 + 1.2761i (–2.2112 dB)	1.1388 + 0.1760i (12.9874 dB)	0.9834 - 0.0439i (26.5611 dB)	1.0401 + 0.0262i (26.3925 dB)
21		0.9920 + 0.0029i (41.4609 dB)	0.9945 + 0.0017i (44.7613 dB)	0.9953 + 0.0017i (46.1159 dB)	1.0000 – 0.0019i (54.5161 dB)

Table IV. Output SINR as a function of the number of dummy elements at the two ends of the array (Intensity of SOI: 1 [V/m], Intensity of jammers: 500 [V/m])

# of elements		Number of dummy elements (each side)			
		0	1	2	3
7	α ($SINR_{out}$)	14.5086 –8.6324i (–27.2399 dB)	5.4082 – 4.5066i (–15.9925 dB)	–1.1753 + 3.8532i (–12.9178 dB)	1.0905 + 0.5670i (4.8188dB)
11		1.9409 + 6.3806i (–16.1906 dB)	1.6941 + 0.8802i (–0.9920 dB)	0.9168 - 0.2197i (12.5817 dB)	0.9168 – 0.2197i (12.4131 dB)
21		0.9602 + 0.0143i (27.4818 dB)	0.9724 + 0.0086i (30.7822 dB)	0.9767 + 0.0084i (32.1365 dB)	1.0001 – 0.0094i (40.5368 dB)

As shown in the tables the estimation for the SOI can be improved significantly when we use additional dummy elements at the end of the array.

For the final example, we consider a semi-circular array (SCA). The SCA is analyzed using WIPL-D. The signal intensity is then recovered using the proposed method, by using the voltages induced in the elements of the array. The antenna elements have the same dimension as presented in Table I. In this simulation we consider the SOI of unit amplitude is arriving from θ_s = 100°. And there are two jammers which are coming from 60° and 120°. The signal intensity is set to 1 [V/m] and the intensities of these jammers are varied from 1 to 50 [V/m]. In this case we observe the performance of this method as a function of the number of antenna elements in the circular array. As the number of elements increases the array dimension also increase as shown in Table VI. Table VII provides the output SINR for the circular array as a function of the number of antenna elements. Again, it is clear that a more accurate compensation for the mutual coupling is necessary when the intensity of the jammer increases.

Again the performance of the adaptive algorithm improves as the number of elements in the array is increased. In addition, when the jammers are much stronger than the signal a more accurate method for treating mutual coupling between the antenna elements presented in [3] should be employed.

IV. REASON FOR A DECLINE IN THE PERFORMANCE OF THE ALGORITHM WHEN THE INTENSITY OF THE JAMMER IS INCREASED

The performance of the proposed method deteriorates when one increases the intensity of the jammers with respect to the SOI, is because when one is using the embedded element pattern, it is equivalent to use of the measured voltages at the loads of the

Table V. Output SINR as a function of the number of dummy elements at the two ends of the array (Intensity of SOI: 1 [V/m], Intensity of jammers: 1000 [V/m])

# of elements		Number of dummy elements (each side)			
		0	1	2	3
7	α ($SINR_{out}$)	28.0172 −7.2649i (−33.2605 dB)	9.8164 − 9.0133i (−22.0131 dB)	−3.3505 + 7.7064i (−18.9384 dB)	1.1809 + 1.1341i (-1.2018 dB)
11		2.8818 +12.7611i (−22.2112 dB)	2.3883 + 1.7604i (−7.0126 dB)	0.8335 − 0.4394i (6.5611 dB)	1.4013 + 0.2616i (6.3925 dB)
21		0.9204 + 0.0285i (21.4612 dB)	0.9448 + 0.0171i (24.7616 dB)	0.9535 + 0.0168i (26.1159 dB)	1.0003 − 0.0188i (34.5164 dB)

Table VI. Radius of the SCA

# of elements	Radius
7	1.115 m
11	1.752 m
21	3.344 m
31	4.936 m

Table VII. Output SINR as a function of the number of antenna elements in the SCA using the Forward method. (Intensity of signal: 1 [V/m])

# of elements		Intensities of the two Jammers		
		1 [V/m]	30 [V/m]	50 [V/m]
7	α ($SINR_{out}$)	1.6755 + 0.1147i (3.2846 dB)	21.2636 + 3.4414i (-26.2578 dB)	34.7727 + 5.7356i (-30.6948 dB)
11		1.3045 - 0.1666i (9.1918 dB)	10.1335 - 4.9987i (-20.3506 dB)	16.2226 - 8.3311i (-24.7876 dB)
21		0.9242 - 0.0528i (20.6891 dB)	-1.2743 - 1.5834i (-8.8534 dB)	-2.7905 - 2.6389i (-13.2903 dB)
31		0.9894 - 0.0284i (30.3767 dB)	0.6820 - 0.8509i (0.8342 dB)	0.4700 - 1.4182i (-3.6027 dB)

antenna elements due to the SOI only. This value is affected by the port admittance matrix of the array. Since when mutual coupling is present, the port admittance matrix is not diagonal. Because of these additional terms the equations used in (8), (19) and (22) does not exactly cancel the signal when we take the difference between the two ratios in the elements of the matrix. That is why when the jammer intensity increases it is necessary to introduce a more accurate compensation methodology using the transformation matrix. The use of a SCA provides worse results than a linear array as the influence of the port admittance matrix is more dominant in the circular array as the influence from neighboring elements are increased in a SCA over that of a linear array.

V. CONCLUSION

We have presented a new technique that compensates for the effect of mutual coupling among the elements of an array based on a direct data domain least squares algorithms using the embedded element patterns only. Since no statistical methodology is employed in the proposed adaptive algorithm, there is no need to compute a covariance matrix. Therefore, this procedure can be implemented on a general-purpose digital signal processor for real time implementations. As shown in the numerical examples, the proposed method provides a good estimate for the complex amplitude of the signal of interest when the jammer intensities are not high. We also investigated the relationship between the number of antenna elements in the array and the estimate of the signal estimation, both in the presence and absence of dummy elements at end of the array. When we increase the number of elements of the array we can get a higher output signal-to-interference plus noise ratio (output SINR), even for a non-linear array like a semi-circular array.

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