TM Electromagnetic Scattering from 2D Multilayered Dielectric Bodies – Numerical Solution

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Abstract

An integral equation approach is derived for an electromagnetic scattering from an M arbitrary multilayered dielectric domain. The integral equation is valid for the $2D$ and $3D$ Helmholtz equation. Here we show the numerical solution for the $2D$ case by using the Nyström method. For validating the method we develop a mode matching method for the case when the domains are multilayered circular cylinders and give numerical results for illustrating the algorithm.

Introduction

Problems of electromagnetic scattering in layered media are of significant importance in many areas of technology such as optics, geophysical probing, communication, etc. (see [6] and the references therein). In this paper we discuss some analytical and computational results for the problem of approximating the scattered electromagnetic field from M layered two-dimensional scatterer. The scatterer is a nested body consisting of a finite number of homogeneous layers (annular regions) with boundary conditions on the interfaces. For the case when the boundaries are circular, closed form solutions can be obtained via a mode matching approach (see [9], [16] and [6], Chapter 6). For boundaries of arbitrary shapes, one of the most efficient techniques to tackle the problem is using (volume or surface) integral equation methods. There are also other type of methods such as the domain decomposition methods [12] and k-space methods (Cf. [3] and [4]). In this paper we choose the surface integral equation method since the inhomogeneities are piecewise constants in each region. The problem can thus be solved (via a boundary element method) on surfaces. It has an advantage over the volume integral equation method, where the whole multilayered domain has to be discretized and the unknowns are in a volume rather than on a surface (see [13]). The straightforward way for solving this type of problems via boundary element methods is by using Green's theorem in each domain [6]. Another alternative is to consider the use of single and/or double layer potentials [7]. In the case of one interface, both methods yield a single integral equation for a single unknown if the interface is impenetrable (e.g., impedance core). However, when the body is penetrable with one interface (e.g., dielectric core), they lead to a pair of integral equations for a pair of unknowns [7]. We deduce that, by using these approaches in the multilayered dielectric domain, for N interfaces we have $2N$ unknown functions to determine. From a computational point of view, it is highly desirable to obtain less equations and less unknowns. In the case of one interface, the so called transmission problem, one integral equation involving one unknown was obtained by a

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few authors (see [10], [14] and the references therein). In [10] the single integral equation for one unknown was obtained for the transmission problem by using a hybrid of Green's theorem and layer potentials. In [6], Chapter 8.3, single integral equations are obtained for multilayered domains by using the extended boundary condition method. But this method suffers from ill-conditioned equations and is mainly convenient for a scatterer where the fields around it are expandable to cylindrical harmonics. The purpose of this paper is to obtain Fredholm type single integral equations on each interface for the multilayered domain case. To this end, we alternate the layer potentials and Green's theorems in the multilayered domain and implement numerical computations using the Nyström method. For a theoretical study of the problem, see [1] and [2]. Our results are validated by developing a mode matching approach for the case of a multilayered circular cylinder and comparing the two algorithms.

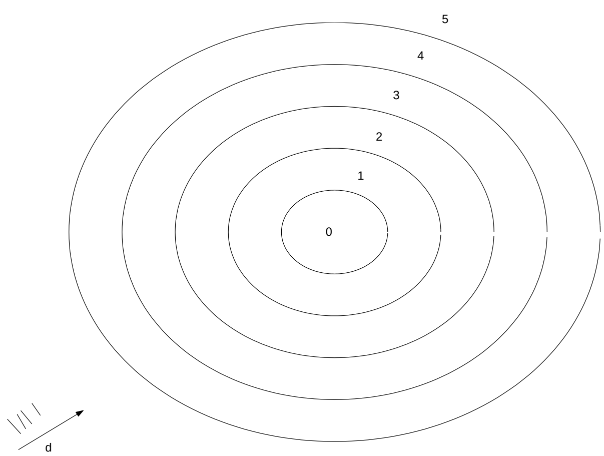

Figure 1: The geometry for the case of five concentric layered cylinder. The incident field is a plane wave propagating in a direction $\mathbf{d}$.

1 The mathematical formulation of the problem

Let $\mathbf{D}_l$, $l = 0, 1, \cdots M-1$ be M bounded domains in $\mathbf{R}^2$ such that $\overline{\mathbf{D}}_{l-1} \subset \mathbf{D}_l$, $l = 1, 2, \cdots, M-1$. Let Γ_l be the C^2 boundaries of $\mathbf{D}_{l-1}$, $l = 1, \cdots, M$. Now let $\Omega_1 = \mathbf{D}_0$, $\Omega_l = \mathbf{D}_l \backslash \overline{\mathbf{D}}_{l-1}$, $l = 1, \cdots, M-1$, and $\Omega_M = \mathbf{R}^2 \backslash \overline{\mathbf{D}}_{M-1}$. We assume that Ω_M is simply connected. See Figure 1 for Ω_l, $l = 0, 1, \cdots, 5$. This is a special case of the general geometry where we have the cross section of $(M = 5)$ concentric cylinders that are infinite in length and their axes are parallel to the z direction.

Each of the regions Ω_l is a dielectric material of constant complex permittivity and permeability ϵ_l and μ_l $(l = 0, \cdots, M)$, respectively. This geometry is illuminated by an incident field which is a plane wave with direction $\mathbf{d} = (\cos\phi_0, \sin\phi_0)$.

It can be shown that we have to solve the following type of boundary value problem for the Helmholtz equation.

$$(\Delta + \kappa_l^2)u_l = 0 \quad \text{in} \quad \Omega_l, \quad l = 0, \cdots, M,$$

where the wave numbers κ_l are given by $\kappa_l = \omega\sqrt{\epsilon_l \mu_l}$, ω is the frequency, with the following continuity conditions on the internal interfaces:

$$\frac{\partial}{\partial\nu}u_l = \rho_l \frac{\partial}{\partial\nu}u_{l-1} \quad \text{on} \quad \Gamma_l, \quad l = 1, \cdots, M-1,$$

$$u_l = u_{l-1} \quad \text{on} \quad \Gamma_l, \quad l = 1, \cdots, M-1,$$

with $\rho_l = \frac{\hat{\rho}_l}{\hat{\rho}_{l-l}}$, $l = 1, 2, \cdots, M$, where $\hat{\rho}_l = \sqrt{\frac{\mu_l}{\epsilon_l}}$ is the intrinsic impedance.

On the outermost interface we have

$$\frac{\partial u}{\partial\nu} = \rho_M \frac{\partial}{\partial\nu}u_{M-1} \quad \text{on} \quad \Gamma_M,$$

$$u = u_{M-1} \quad \text{on} \quad \Gamma_M,$$

where,

$$u = u_M + u^i \quad \text{in} \quad \Omega_M$$

and the given incident field, u^i, satisfies

$$\Delta u^i + \kappa_M^2 u^i = 0$$

everywhere. In addition, u_M must satisfy the Sommerfeld radiation condition, i.e.,

$$\lim_{|\mathbf{x}|\to\infty} |\mathbf{x}|^{1/2} \left(\frac{\partial u_M}{\partial|\mathbf{x}|} - i\kappa_M u_M \right) = 0.$$

The unit outward normal ν to Γ_l is assumed to be directed towards the exterior domain. The above problem is known as the TM mode. The TE mode is obtained by replacing ρ_l by $\frac{1}{\rho_l}$. We denote the fundamental solution to the Helmholtz equations (the free-space source) by

$$\Phi_k(\mathbf{x},\mathbf{y}) = -\frac{i}{2}H_0^{(1)}(\kappa_k|\mathbf{x}-\mathbf{y}|), \quad k=0,1,\cdots,M,$$

where $H_0^{(1)}$ is the Hankel function of the first kind and order zero. Throughout this paper i will denote the complex constant satisfying $i^2=-1$.

2 The integral equation approach

First, for non-zero functions ϕ_l, $l=1,2,\cdots,M$, define the single and double layer potentials as

$$S_k^l\phi_l(\mathbf{x}) = \int_{\Gamma_l}\Phi_k(\mathbf{x},\mathbf{y})\phi_l(\mathbf{y})\,ds(\mathbf{y}), \qquad \mathbf{x}\in\mathbf{R}^2\backslash\Gamma_l,$$

and

$$D_k^l\phi_l(\mathbf{x}) = \int_{\Gamma_l}\frac{\partial}{\partial\nu_l(\mathbf{y})}\Phi_k(\mathbf{x},\mathbf{y})\phi_l(\mathbf{y})\,ds(\mathbf{y}), \quad \mathbf{x}\in\mathbf{R}^2\backslash\Gamma_l,$$

respectively, for $k=0,1,\cdots,M$. Their normal derivatives are denoted by P_k^l and Q_k^l, respectively, for $k=0,1,\cdots,M$.

We have the continuity relations

$$S_k^l = \hat{S}_k^l, \quad Q_k^l = \hat{Q}_k^l,$$

and the jump relations

$$D_k^l = \mp I + \hat{D}_k^l \quad \text{and} \quad P_k^l = \pm I + \hat{P}_k^l,$$

where, the upper (lower) sign corresponds to the limit when $\mathbf{x}$ approaches Γ_l from the outside (inside). The hat on each operator represents it on the boundary Γ_l.

To arrive at the desired integral equation we define a layer ansatz by $E_k^l := D_k^l - i\eta_l S_k^l$ for $l\neq 0$ and $E_k^0=0$ (with normal derivative $H_k^l := \partial E_k^l/\partial\nu$) in Ω_k, where η_ls are nonzero complex constants chosen to obtain well-posedness, $k=0,2,4,\cdots$, and Green's theorem in $\Omega_{k'}$, $k'=1,3,5,\cdots$. In particular, let us assume that M is odd. Then, in the core region Ω_0 we define

$$u_0(\mathbf{x}) = E_0^1\phi_1(\mathbf{x}), \quad \mathbf{x}\in\Omega_0. \tag{2.1}$$

In the outermost domain, we use Green's theorem ([7] pp. 68-70) to obtain

$$\begin{cases} 2u_M(\mathbf{x}) = S_M^M\frac{\partial}{\partial\nu}u(\mathbf{x}) - D_M^M u(\mathbf{x}), \quad \mathbf{x}\in\Omega_M, \\ -2u^i(\mathbf{x}) = S_M^M\frac{\partial}{\partial\nu}u(\mathbf{x}) - D_M^M u(\mathbf{x}), \quad \mathbf{x}\in\mathbf{R}^2\backslash\overline{\Omega}_M. \end{cases} \tag{2.2}$$

In the other domains, for $l=2,4,\cdots,M-1$, we define

$$u_l(\mathbf{x}) = E_l^l\phi_l(\mathbf{x}) + E_l^{l+1}\phi_{l+1}(\mathbf{x}), \quad \mathbf{x}\in\Omega_l, \tag{2.3}$$

and, using Green's theorem for $l=1,3,\cdots,M-2$ we have

$$\begin{cases} 2u_l(\mathbf{x}) = S_l^l\frac{\partial}{\partial\nu}u_l(\mathbf{x}) - S_l^{l+1}\frac{\partial}{\partial\nu}u_l(\mathbf{x}) - \\ \quad (D_l^l - D_l^{l+1})u_l(\mathbf{x}), \quad \mathbf{x}\in\Omega_l, \\ 0 = S_l^l\frac{\partial}{\partial\nu}u_l(\mathbf{x}) - S_l^{l+1}\frac{\partial}{\partial\nu}u_l(\mathbf{x}) - \\ \quad (D_l^l - D_l^{l+1})u_l(\mathbf{x}), \quad \mathbf{x}\in\mathbf{R}^2\backslash\overline{\Omega}_l \end{cases} \tag{2.4}$$

Now, using the jump and continuity relations we obtain the second equation in (2.2) on Γ_M and the second equation in (2.4) on Γ_l and Γ_{l+1} ($l=1,3,5,\cdots M-2$). Using the boundary conditions, jump properties for the single and double layer potentials together with their derivatives, and replacing u_0 (given in (2.1)) and u_l (given in (2.3)) into these equations we arrive at a set of M integral equations with M unknowns ϕ_l on Γ_l, $l=1,2\cdots,M$. In particular, on Γ_M we have

$$\begin{aligned} -2u^i = \quad & (\rho_M\hat{S}_M^M\hat{H}_{M-1}^M - (\hat{D}_M^M+I)\hat{E}_{M-1}^M)\phi_M + \\ & \left(\rho_M\hat{S}_M^M H_{M-1}^{M-1,M} - (\hat{D}_M^M+I)E_{M-1}^{M-1,M})\right) \\ & \times\phi_{M-1}, \end{aligned}$$

and for $l=1,3,5,\cdots M-2$, we have

$$\begin{aligned} 0 = \quad & \left(\rho_l\hat{S}_l^l H_{l-1}^{l-1,l} - (\hat{D}_l^l+I)E_{l-1}^{l-1,l}\right)\phi_{l-1} + \\ & \left(\rho_l\hat{S}_l^l\hat{H}_{l-1}^l - (\hat{D}_l^l+I)\hat{E}_{l-1}^l\right)\phi_l - \\ & \left(\frac{1}{\rho_{l+1}}S_l^{l+1,l}\hat{H}_{l+1}^{l+1} - D_l^{l+1,l}\hat{E}_{l+1}^{l+1}\right)\phi_{l+1} - \\ & \left(\frac{1}{\rho_{l+1}}S_l^{l+1,l}H_{l+1}^{l+2,l+1} - D_l^{l+1,l}E_{l+1}^{l+2,l+1}\right)\phi_{l+2} \end{aligned}$$

on Γ_l and

$$\begin{aligned}0 = \quad & \left(\tfrac{1}{\rho_{l+1}}\hat{S}_l^{l+1}\hat{H}_{l+1}^{l+1} - (\hat{D}_l^{l+1} - I)\hat{E}_{l+1}^{l+1}\right)\phi_{l+1} + \\ & \left(\tfrac{1}{\rho_{l+1}}\hat{S}_l^{l+1}H_{l+1}^{l+2,l+1} - (\hat{D}_l^{l+1} - I)E_{l+1}^{l+2,l+1}\right)\phi_{l+2} \\ & - \left(\rho_l S_l^{l,l+1}H_{l-1}^{l-1,l} - D_l^{l,l+1}E_{l-1}^{l-1,l}\right)\phi_{l-1} - \\ & \left(\rho_l S_l^{l,l+1}\hat{H}_{l-1}^{l} - D_l^{l,l+1}\hat{E}_{l-1}^{l}\right)\phi_l \quad \text{on } \Gamma_{l+1},\end{aligned}$$

where $\hat{E}_k^m = \mp I + \hat{D}_k^m - i\eta_m \hat{S}_k^m$, $\hat{H}_k^m = \hat{Q}_k^m - i\eta_m(\pm I + \hat{P}_k^m)$, and by $T_k^{m,n}$ (T is for S, D, E or H) we mean that T_k^m is evaluated on Γ_n when $n \neq m$. Numerically the above system has to be discretized and solved to obtain an approximation of the unknowns ϕ_l, $l = 1, 2 \cdots, M$. Then the solution of the layered problem can be constructed for the discretized forms of (2.1)-(2.4).

Remark: The above system is also valid for the $3D$ Helmholtz equation. The only difference is that the fundamental solution is

$$\Phi_k(\mathbf{x}, \mathbf{y}) = -\frac{e^{i\kappa_k|\mathbf{x}-\mathbf{y}|}}{2\pi|\mathbf{x}-\mathbf{y}|}.$$

3 Numerical validation and results

This section is devoted to the numerical solution of the above system and its validation for the $2D$ case. To this end, we use the Nyström method for the numerical solution and Bessel function expansion for the validation. Then we show the numerical results.

3.1 Discretization and numerical solution

The system is discretized using the Nyström method [11]. The resulting matrix equation, that involves matrix multiplications resulted from the multiplications of layer potentials and/or their derivatives, is solved by a standard LU decomposition approach. Let us note that the assumption that M is odd is not a loss of generality. In fact, for an even M we can use the same method, but for $M+1$ regions, Γ_{M+1} encloses the scatterer, with $\kappa_{M+1} = \kappa_M$ and $\rho_{M+1} = 1$. This way has the advantage of keeping the same system of equations and the disadvantage of adding another equation and an unknown function ϕ_{M+1}. This may be overcome by starting with Green's theorem in the core region, alternate with layer ansatz and obtain the Green's theorem in Ω_M, which gives a different system than the previous argument.

3.2 The Mode Matching Approach

This method is studied in detail in the literature (see e.g., [9]). Consider the case when the regions $\mathbf{D}_l$'s are circular cylinders with radii r_{l+1} and origins $\mathbf{O}_{l+1}$, $l = 0, 1, 2, \cdots, M-1$; then we have the following expansions [6]: For the outermost region

$$\begin{aligned}u(\tilde{r}_M, \phi_M) = \quad & \textstyle\sum_{n=-\infty}^{\infty}\left(b_n^M H_n^{(1)}(\kappa_M \tilde{r}_M) + J_n(\kappa_M \tilde{r}_M)\right) \\ & \times e^{-in(\phi_M - \phi_o)}\end{aligned}$$

and for other regions we have

$$\begin{aligned}u_l(\tilde{r}_1, \phi_1) = \quad & \textstyle\sum_{n=-\infty}^{\infty}\left(b_n^l H_n^{(1)}(\kappa_l \tilde{r}_1) + a_n^l J_n(\kappa_M \tilde{r}_1)\right) \\ & \times e^{-in(\phi_1 - \phi_o)}, \quad l = 0, 1, 2, \cdots M-1,\end{aligned}$$

where $b_n^0 = 0$.

To enforce the boundary conditions we need the addition formula for u_l, $l = 1, \cdots, M-1$ which means that the fields expressed in terms of $X_1O_1Y_1$ be translated to $X_lO_lY_l$ coordinates. This yields, by the addition theorem (cf. [5] pp. 30-31),

$$\begin{aligned}u_l(\tilde{r}_l, \phi_l) = \quad & \textstyle\sum_{n=-\infty}^{\infty}\sum_{i=-\infty}^{\infty} J_{i-n}(\kappa_l d_{l_1}) \\ & \times \left[b_n^l H_i^{(1)}(\kappa_l \tilde{r}_l) + a_n^l J_i(\kappa_l \tilde{r}_l)\right] \\ & \times e^{i(\phi_0 - (i-n)\phi_{l_1})},\end{aligned}$$

where d_{l_1} is the distance between $\mathbf{O}_1$ and $\mathbf{O}_l$, and ϕ_{l_1} is the angle between $\mathbf{O}_1\mathbf{O}_l$ and the x axis.

The sums in the above equations have to be truncated, at some number, N_0, to obtain a finite system. Now we can use the expansions on the boundary together with their derivatives and the boundary conditions to obtain a linear system in the unknowns a_n^l and b_n^l. This system is also solved via LU decomposition approach.

3.3 Numerical Results

In this section, numerical solution obtained by using the integral equation (IE) and mode matching (MM)

methods are presented. We have conducted several nu-
merical experiments.

First we try to validate the MM method by analyz
the physical properties of the waves, by plotting
absolute value of the waves against the boundaries.
this end, we consider a cylinder with radius $r = 3$
that we only have the inner and outermost doma
and keep the angle of the incident field $\phi_0 = 90$
and $\rho_1 = 1$ fixed. We would expect, for real κ_1
complex κ_2 with negative imaginary part, the wav
diverge at the boundary. If, on the other hand,
have that the two wave numbers are real and eq
the absolute value of the wave should be unity. Fina
for the case when κ_1 is real and κ_2 is complex v
a positive imaginary part, because of absorption,
absolute value of the wave must decay at the bounc
and the bigger the imaginary part, the faster the w
should decay. Our numerical computations show t
all theses properties are satisfied, and the results are
summarized in Figure 2.

Next we validated the IE method for one interface, centered at $\mathbf{O} = (-0.2, 0.7)$, by plotting the absolute value of the far field pattern (measured at a fixed observation point $\hat{x}$) against the incidence angle for two different wave numbers using the IE and MM methods. See [8], page 20, for the definition of the far field pattern f. The result is given in Figure 3, which shows a very good agreement of the two methods. Unless otherwise stated we use 32 grid points for the Nyström solver.

Our next examples are for the two and three-layered circular cases. First we plot the absolute value of the far field against the incidence angle for the two-layered case and then for the three-layered case. The results are shown in Figure 4 and Figure 5, respectively. In these cases as well we have very good agreements of the two methods.

For the case of more circular layers we have the same conclusions, except that more grid points are needed, which is due to the errors in the computation of the layer potentials.

Our last example is for the case of three boundaries of kite type where MM method can not be performed (Figure 6). Here we investigate the convergence as well as the boundary conditions. For the former we compute the far field pattern for different wave numbers.

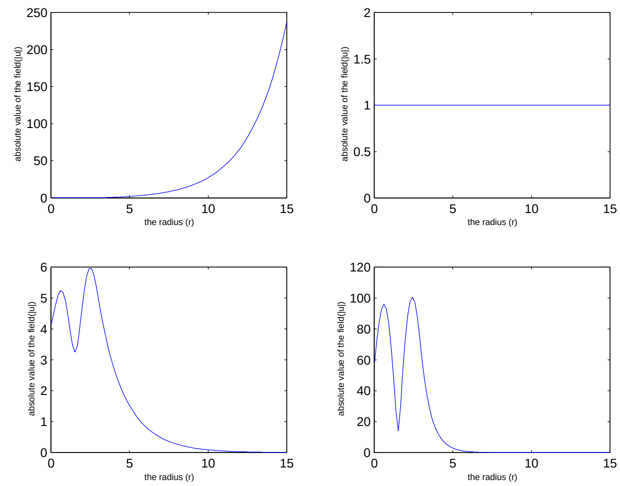

Figure 2: The case of one circular boundary ($r = 3$). The absolute value of the wave plotted against the radius. We have used $\kappa_1 = 2$ and $\kappa_2 = 2 - 0.5i$ (top left) $\kappa_1 = 2$ and $\kappa_2 = 2$ (top right), $\kappa_1 = 2$ and $\kappa_2 = 2 + 0.5i$ (bottom left), and $\kappa_1 = 2$ and $\kappa_2 = 2 + 1.5i$ (bottom right).

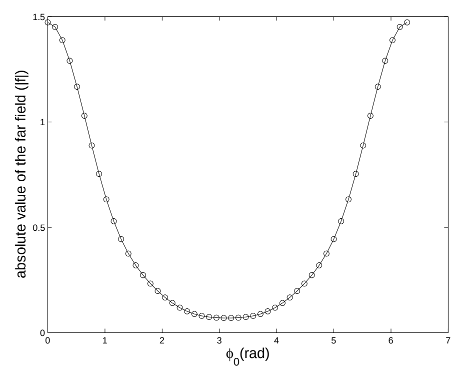

Figure 3: The absolute value of the far field pattern plotted against the incidence angle using the MM ('o') and IE (solid line) methods. The case of one interface. We used $\kappa_0 = 2$, $\kappa_1 = 3$ and the radius is $r = 1$.

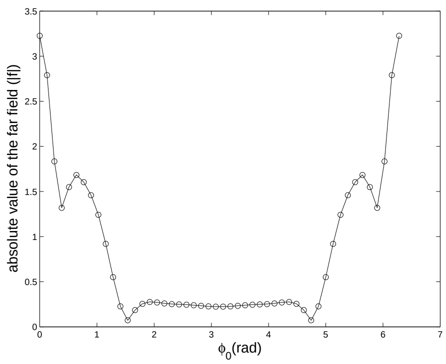

Figure 4: The absolute value of the far field pattern plotted against the incidence angle using the MM ('o') and IE (solid line) methods. The case of two-layered circular cylinders. Here $\kappa_0 = 2$, $\kappa_1 = 3$, $\kappa_2 = 4$,$r_1 = 1$ and $r_2 = 2$.

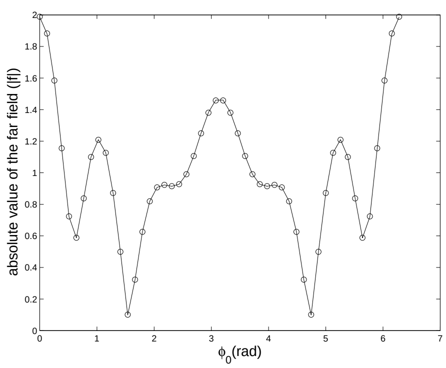

Figure 5: The absolute value of the far field pattern plotted against the incidence angle for the MM ('o') and IE (solid line) methods. The case of three-layered circular cylinder. Here $\kappa_0 = 2$, $\kappa_1 = 3$, $\kappa_2 = 4$, $\kappa_3 = 1$ $r_1 = 1$, $r_2 = 2$ and $r_3 = 3$.

The results are reported in the two tables below. We see clear convergence, and, as expected, it is fast. For the latter case we plot $|u_0|$ and $|u_1|$ on Γ_1, $|u_1|$ and $|u_2|$ on Γ_2, and $|u_2|$ and $|u_3 + u^i|$ on Γ_3, against the incidence angle. From the boundary conditions we know that they must coincide. This is shown in Figures 7. One way we have used to compare the IE and MM methods in the case of 3−layered kite is by enclosing the tree layers within a circular domain and choose all the inner layers to have the same wave numbers and the outer region to have a different wave number. Physically, this is a one-layered problem; but mathematically the four layers exist. By so doing we still obtain a figure similar to Figure 3.

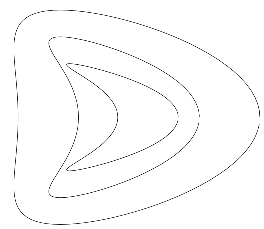

Figure 6: The geometry for the case of three boundaries of kite type.

Table 1: Parameter values and description for the geometry in Figure 6. D1, D2 and D3 are the first, the second and third data, respectively, for the numerical computation

Description	Symbol	D1	D2	D3
Wave numbers	κ_0	2	4	1+i
	κ_1	3	5	2
	κ_2	1.5	4.5	2+0.5i
	κ_3	2.5	5.5	3

4 Conclusion and future work

We have developed an integral equation approach for solving the M multilayered electromagnetic problem and used the Nyström method for the numerical computation. The algorithm was validated by a Fourier expansion method for circular (not necessarily concentric) cylinders. One may think as a disadvantage for our

Table 2: The numerical results using the integral equation (IE) approach for the geometry in Figure 6. The data are given in Table 1. The number N is the number of Nyström (grid) points.

	N	IE
D1	8	$-3.1863 + 0.4213i$
	16	$-3.5238 + 0.1952i$
	32	$-3.5215 + 0.1955i$
	64	$-3.5214 + 0.1954i$
D2	8	$-11.9843 + 15.8975i$
	16	$-3.5291 + 3.2993i$
	32	$-4.2103 + 3.5349i$
	64	$-4.2103 + 3.5344i$
D3	8	$-2.3889 + 1.6002i$
	16	$-2.5120 + 1.5746i$
	32	$-2.5126 + 1.5739i$
	64	$2.5126 + 1.5739i$

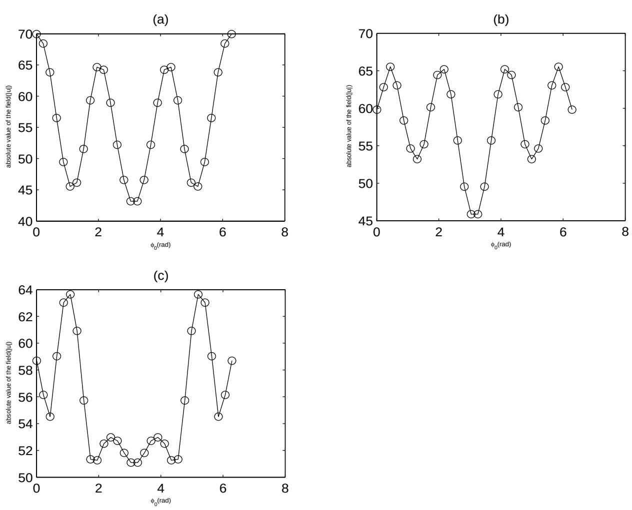

Figure 7: Here we plot $|u_j|$ ('o') and $|u_{j+1} + \frac{(j-1)(j-2)}{2} u^i|$ (solid line) on the boundary Γ_j against the incidence angle. In (a) we have the case j=1, in (b) j=2 and (c) j=3.

method the numerous matrix vector multiplications. This problem can be overcome by using fast multipole methods (see [15]) where these operations are done very quickly. Our results also show the (expected) fast convergence of the Nyström method for analytic boundaries. The natural expansion of our method is for the numerical solution of the three-dimensional electromagnetic problem, and to the case of multiple scatterers.

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References

[1] C. Athanastadis, “On the acoustic scattering amplitude for a multi-layered scatterer,” J. Austral. Math. Soc. Ser. B 39, pp. 431–448, 1998.

[2] C. Athanasiadis, A.G. Ramm and I.G. Stratis, “Inverse acoustic scattering by a layered obstacle,” in Inverse Problems, Tomography, and Image Processing (ed. A.G. Ramm), Plenum Press, New York, pp. 1–8, 1998.

[3] N. N. Bojarski, “The k-space formulation of the scattering problem in the time domain,” J. Opt. Soc. Amer., Vol. 72, pp. 570–584, 1982.

[4] O.P. Bruno and A. Sei, “A fast high-order solver for EM scattering from complex penetrable bodies: TE case,” IEEE Transactions on Antennas and Propagation, Vol. 48, 12, pp. 1862–1864, 2000.

[5] K. Chadan, D. Colton, L. Paivarinta and W. Rundell, An Introduction to Inverse Scattering and Inverse Spectral Problems, SIAM Monographs on Mathematical Modeling and Computation, 1997.

[6] W. C. Chew, Waves and Fields in Inhomogeneous Media, IEEE Press, 1995.

[7] D. Colton and R. Kress, Integral Equation in Scattering Theory, John Wiley & Sons, 1983.

[8] D. Colton and R. Kress, Inverse Acoustic and Electromagnetic Scattering Theory, Springer Verlag, 1992.

[9] A. Kishk, R. Parrikar and A. Elsherbeni, “Electromagnetic Scattering from an eccentric multilayered circular cylinder,” IEEE Trans. Antennas and Prop. Vol. 40, No3 pp. 295–303, 1992.

[10] R.E. Kleinmann and P.A. Martin, “On single integral equations for the transmission problem of acoustics,” SIAM J. Appl. Math. Vol. 48, No. 2, pp. 307–325, 1988.

[11] R. Kress, “On the numerical solution of a hypersingular integral equation in scattering theory,” J. Comp. Appl. Math. Vol. 61, pp. 345–360, 1995.

[12] E. Larsson, “A domain decomposition method for the Helmholtz equation in a multilayer domain,” SIAM J. Sci. Comput., Vol. 20, pp. 1713–1731, 1999.

[13] Olivier J. F. Martin and Nicolas B. Piller, “Electromagnetic scattering in polarizable backgrounds,” Phys. Rev. E 58, No. 3, pp. 3909–3915, 1998.

[14] P.A. Martin and P. Ola, “Boundary integral equations for the scattering of electromagnetic waves by a homogeneous dielectric obstacle,” J. Proc. R. Soc. Edinb., Sect. A 123, No. 1, pp. 185–208, 1993.

[15] J.M. Song and W.C. Chew, “FMM and MLFMA in 3D and Fast Illinois Solver Code, Chapter 3” in Fast and Efficient Algorithms in Computational Electromagnetics, edited by Chew, Jin, Michielssen, and Song, Artech House, 2001.

[16] Z. Wu and L. Guo, “Electromagnetic scattering from a multilayered cylinder arbitrarily located in a gaussian beam, a new recursive algorithms,” Progress In Electromagnetics Research, PIER 18, pp. 317–333, 1998.