Parallel SAI Preconditioned Adaptive Integral Method For Analysis of Large Planar Microstrip Antennas

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Abstract **─** An efficient parallel sparse approximate inverse (PSAI) preconditioning of the adaptive integral method (AIM) is proposed to analyze the large-scale planar microstrip antennas. The PSAI preconditioner is based on the parallelized Frobenius-norm minimization, and is used to speed up the convergence rate of the loose generalized minimal residual method (LGMRES) iterative solver. The parallel AIM is used to accelerate the required matrix vector product operations. Numerical results demonstrate that the PSAI preconditioner is effective with the AIM and can increase the parallel efficiency significantly when analyzing the large planar microstrip antennas.

Index Terms **─** Adaptive integral method, microstrip antennas, parallel sparse approximate inverse.

I. INTRODUCTION

In electromagnetic (EM) calculations, the calculation of the currents generated on the surface of an object when illuminated by a given incident plane wave or fed by a microstrip line is essential for the simulation of microstrip structures. For the microstrip structures, the finite element method (FEM), and the finite-difference time-domain (FDTD) method often have a large number of unknowns due to the volumetric discretization. The method of moments (MoM) is a preferred method to solve this problem, since by using the integral equation (IE) and the layered media Green's functions, it only discretizes the metallic surface, which leads to a relative small number of unknowns. The implementation of the MoM requires $O(N^3)$ operations and $O(N^2)$ memory storage [1], where N is the number of unknowns. The size of the MoM matrix increases so rapidly with the increase of the number of unknowns that the computation will be intractable for the computational capacity. The difficulty can be overcome by use of Krylov iterative methods, and the required matrix-vector product operations can be accelerated by AIM [2]. The application of AIM reduces the memory requirement to $O(N)$ and the computational complexity to $O(NlogN)$.

With the increase of the dimensions of the object to be solved, the impedance matrix associated with the linear systems becomes larger. The computation will be time consuming for a single processor. Fortunately, the parallel technique is applied in many fast EM methods to circumvent the above difficulty. The parallel multilevel fast multipole method (MLFMA) is proposed in [3, 4] for the solution of scattering from large-scale objects, the mpi-based parallel precorrected fast Fourier transform (FFT) algorithm is proposed in [5] for analyzing scattering from arbitrary shaped three-dimensional objects, a methodology for designing a high performance parallel 3-D finite element method (FEM) is proposed in [6]. All of these techniques provide an efficient parallel scheme to deal with the large-scale problems.

Although the computational complexity and memory requirement is decreased in the fast EM methods and the computational capacity is increased by use of the parallel technique, the number of iterations needed to achieve the desired precision remains the same as the original MoM. It is natural to use preconditioning techniques [7-9] to improve the convergence of the linear systems. There are many preconditioning techniques. Simple diagonal or diagonal blocks of the impedance matrix can be parallelized easily, while they are effective only when the matrix has some degree of diagonal dominance [10]. Block diagonal preconditioner is generally more robust, which requires matrix permutations or renumbering of the grid points to cluster the large entries close to the diagonal. Incomplete LU (ILU) has been used for solving nonsysmmetric dense systems [11] and the threshold-based incomplete LU (ILUT) has been applied in the AIM [12], while the factorization is often ill conditioned that leads to the triangular solvers unstable. Shifted symmetric successive over-relaxation (SSOR) [13] and spectral two-step preconditioning techniques [14] are efficient while they are difficult to be parallelized. The SAI preconditioner based on Frobenius-norm minimization is chosen in this paper because it allows the decoupling of the constrained minimization problem into independent linear least-squares problems for each rows of the preconditioner. This is convenient to be used by parallelization. Recently, PSAI is proposed to be combined with MLFMA for solving scattering problems with a large number of unknowns [15, 16]. For layered media spatial domain Green's function, using MLFMA is much more involved than in the surface scattering problems where the free space Green's would be employed [17]. Therefore, in this paper a synthesis of the AIM and PSAI preconditioning technique is proposed for analyzing large-scale planar microstrip antennas. To the best of our knowledge, it is the first time that the parallel SAI preconditioning technique was applied into the AIM in our work. As a result, the parallel efficiency is greatly enhanced.

The paper is organized as follows. Section 2 describes the essential algorithms for the analysis of the planar microstrip antennas. The workflow of the parallel AIM is described in section 2.1; the construction of the PSAI preconditioner in the parallel AIM is described in section 2.2. Numerical results in section 3 demonstrate the efficiency of the proposed method. Finally, a brief conclusion is given in Section 4.

II. THEORY

A. The parallel AIM algorithm

For the sake of brevity, the summary, and sequence of the operations in the parallel AIM will be only described, the basic principles of the AIM can be found in [2]. It is known that the computation of the AIM technique mainly contains four parts: evaluate the near field impedance matrix, evaluate the expansion coefficients (i.e. projecting the RWG basis functions onto the rectangular grids), evaluate the matrix-vector product operations of the near field sparse matrix, and evaluate matrix-vector product operations of the far field. The AIM is able to be parallelized, since the above four operations are independent and there is independence in each operations. In this paper, the computation task is decomposed by distributing an equal number of unknowns to each processor for balancing the four operations simultaneously. The scheme is described below:

Step 1. The elements of the near field matrix are evaluated by MPIE and stored in a compressed sparse column format. The basis functions are distributed equally to every processor, and the relative elements of the near field impedance matrix are computed and stored, respectively. There is no inner-processor communications in this step.

Step 2. The basis functions are mapped onto a regular grid in order to apply the FFT to a Toeplitz matrix for speedup the matrix-vector product operations. The solution can be highly parallelized, since every basis function is independent during computation. As step1, the basis functions are distributed equally to every processor. Only part of the expansion coefficients just need to be computed for each processor.

Step 3. The direct matrix-vector product operations of the near field sparse matrix are parallelized as step 1 and step 2 by distributing the basis functions. However, frequent inner-processor communications are required during the iterative solution of the linear system, which leads to frequent inner-processor communications. The inner-processor communications will take the most of the computational time if the number of unknowns is small, and leads to a low parallel efficiency.

Step 4. The matrix-vector product operation of the Toeplitz matrix of the far field is accelerated by FFT. In the code, the two dimensional Toeplitz matrix is first extended to a one dimensional circulant matrix, and the one dimensional FFT [18] is implemented in the circulant matrix, thus the matrix-vector product operation of the far field can be parallelized by the parallel one dimensional FFT. It should be noted that the inner-processor communication time and the synchronization time should, also, be considered as step 3.

From the above steps, it is found that there is a balance between the computational time and inner-processor communication time. For objects with a small number of unknowns, the inner-processor communication takes the most of the total solution time which leads to a low efficiency of the parallel algorithm. For objects with a large number of unknowns, the computation takes the most of the total solution time which leads to a high efficiency of the parallel algorithm. Since a fast convergence solution of the liner systems plays an important role for the efficiency of the parallel scheme, the PSAI preconditioning technique will be proposed in detail in the next part.

B. The PSAI preconditioning technique

First, we consider the SAI preconditioner combined with the AIM. In the context of AIM, the $N \times N$ dense impedance matrix is decomposed as $\mathbf{Z}=\mathbf{Z}_s + \mathbf{Z}_{AIM}$ [2], where $\mathbf{Z}_s = \mathbf{Z}_{near} - \mathbf{Z}_{near\_AIM}$. $\mathbf{Z}_{near}$ is the MoM interaction between elements, $\mathbf{Z}_{near\_AIM}$ is the inaccurate contribution from grid and $\mathbf{Z}_{AIM}$ is the matrix related to interaction from far field. Since $\mathbf{Z}_s$ is already stored in the memory, $\mathbf{Z}_s$ is chosen instead of $\mathbf{Z}_{near}$ for the construction of the SAI preconditioner and the approximation is of the form $\mathbf{M} \approx \mathbf{Z}_s^{-1}$. It is found that the iterative steps of the LGMRES with SAI preconditioner constructed by $\mathbf{Z}_s$ are similar with constructed by $\mathbf{Z}_{near}$, and it is shown in section 3. The SAI preconditioner in this paper is based on a Frobenius-norm minimization. The approximate inverse of $\mathbf{Z}_s$ is computed by minimizing

$$\left\| \mathbf{I} - \mathbf{M}\mathbf{Z}_s \right\|_F . \tag{1}$$

The Frobenius norm is usually chosen because it allows the decoupling of the constrained minimization problem into n independent linear least-squares problems for each row of M:

$$\left\| \mathbf{I} - \mathbf{M}\mathbf{Z}_s \right\|_F^2 = \left\| \mathbf{I} - \mathbf{M}\mathbf{Z}_s^T \right\|_F^2 = \sum_{j=1}^{n} \left\| e_j - \mathbf{Z}_s m_j \right\|_2^2, \tag{2}$$

where e_j is the j th unit vector and m_j is the column vector representing the j th row of $\mathbf{M}$.

The main issue for the computation of the SAI preconditioner is the selection of the nonzero pattern of $\mathbf{M}$ that is the set of indices:

$$S = \{(i,j) \in [1, N^2] \text{ s.t.} m_{ij} \neq 0\}. \tag{3}$$

If the sparsity of $\mathbf{M}$ is known, the none zero structure for the j th column of $\mathbf{M}$ can be automatically determined and defined as

$$J = \{i \in [1, N] \text{ s.t.} (i,j) \in S\}. \tag{4}$$

The solution of (2) involves only the columns of $\mathbf{Z}_s$ indexed by J, which can be denoted by $\mathbf{Z}_s(:,J)$. Since $\mathbf{Z}_s$ is sparse, many rows in $\mathbf{Z}_s(:,J)$ are usually null, not affecting the solution of the least-square problems. Thus, if I is the set of indices corresponding to the nonzero rows in $\mathbf{Z}_s(:,J)$, and if we defined $\tilde{\mathbf{Z}} = \mathbf{Z}_s(I,J)$ by $\tilde{m}_j = m_j(J)$ and $\tilde{e}_j = e_j(J)$, the "reduced" least-square problems to solve are

$$\min \left\| \tilde{e}_j - \tilde{\mathbf{Z}}_s \tilde{m}_j \right\|_2^2, \qquad j = 1, \ldots N. \tag{5}$$

In general, the size of problems (5) is much smaller than problems (2). The above procedure is shown clearly in Figure 1.

As shown in Figure 1(a), "X" stands for none zero impedance matrix elements, and m_4 is chosen to be computed as an example. For row 4, since the none zero columns are 2, 4, 6, 8, the rows of 2, 4, 6, 8 in Figure 1(a) are chosen. Then, the submatrix in Figure 1(b) is obtained for constructing the preconditioner. Excluding the columns with null elements in the above submatrix, after which the final least matrix in Figure 1(c) will be obtained. It can be seen that the size of the "reduced" matrix in Figure 1(c) is much smaller than the original matrix in Figure 1(a). Although the null columns are excluded, the size of the "reduced" matrix in (5) is still very large for the targets with a large number of unknowns. The difficulty can be circumvented with a prior sparsity pattern selection strategy. In the code, a constant number $K_{\max}$ is set to select the most informative elements in each row of Zs for the construction of the preconditioner by checking the value of the elements [19, 20].

The core of PSAI preconditioner is to store the minimum near field impedance matrix in each processor. The PSAI preconditioner is constructed by three steps. First, the elements of the sparse near field impedance matrix are distributed evenly on the processors. Second, the required elements which are distributed on the other processors are appended on the present processor by the all-to-all communication of the rows and columns list and the value of the elements, and the process is shown in Figure 2 to Figure 4. Third, the least-squares minimizations are solved independently on each processor as the construction of the SAI preconditioner described above.

	1	2	3	4	5	6	7	8
1	X	X			X			
2	X	X		X				X
3			X					
4		X		X		X		X
5	X							
6				X				X
7								
8		X		X		X		X

(a)

	1	2	3	4	5	6	7	8
2	X	X		X				X
4		X		X		X		X
6				X				X
8		X		X		X		X

(b)

	1	2	4	6	8
2	X	X	X		X
4		X	X	X	X
6			X		X
8		X	X	X	X

(c)

Fig. 1. The configuration of the SAI, (a) the original impedance matrix, (b) the selected rows of the matrix for construction of the SAI preconditioner, (c) the reduced impedance matrix for construction of the SAI preconditioner.

The above algorithm is shown in Figure 5 clearly. Considering the impedance matrix of the near field, the elements of the matrix $\mathbf{Z}_s$ in rows 1-4 and 5-8 are distributed in two processors respectively. m_4 is to be solved as discussed above. The elements of rows 2, 4, 6, 8 of the impedance matrix should be selected, while rows 6 and 8 are in the other processor; thus, the necessary inner-processor communications are implemented to append rows 6 and 8 to the first processor as shown in Figure 2(c). After the procedure of the all-to-all communication, all the processors store the minimum elements of the matrix $\mathbf{Z}_s$. It should be noted that the inner-processor communication in the construction of PSAI preconditioner is small since there is a trade-off for controlling the selected number of elements in each row of Zs. The flow chart of the PSAI preconditioned AIM is shown in Figure 6 clearly. As shown in Figure 6, there is inner-processor communication in the process to evaluate the near field, project the unknowns onto the regular grid. And there is inner-processor communication in the process of construction of the SAI preconditioner and the matrix vector multiplication.

for each $z_{ij} \in \mathbf{Z}_k^s$

Do $i = 1, n_k$

if j is not in the local rank，then

P = findProcId(j)

Append j into $rowRecvList(P)$

endif

Endfor

Fig. 2. The process that finds and exchanges the rows list of the elements should be appended of each processor, where *rowSendList(P)*, *rowRecvList(P)*is the row list to be sent and received of processor *p*.

For $i \in rowSendList$, do

Append column indices of row i to *sendColIndices*

Endfor

Send *sendColIndices*, receive *recColIndices* at the same time ! All-to-All communication

Fig. 3. The process that finds and exchanges the columns list of the elements should be appended of each processor, where *sendColIndices*, *recColIndices* is the column list to be sent and received.

For $i \in rowSendList$, do

Append the matrix element z_{ij} of row i to $sendColValues$

Endfor

Send $sendColValues$, receive $recvColValues$ at the same time !

All-to-All communication

Fig. 4. The process that finds and exchanges the values of the elements to be sent and received by processor *p*, where *sendcolIndices*, *reccolIndices* is the value of the elements to be sent and received.

	1	2	3	4	5	6	7	8
1	X	X			X			
2	X	X		X				X
3			X					
4		X		X		X		X
5	X							
6				X				X
7								
8		X		X		X		X

(a)

	1	2	3	4	5	6	7	8
1	X	X			X			
2	X	X		X				X
3			X					
4		X		X		X		X

(b)

	1	2	3	4	5	6	7	8
1	X	X			X			
2	X	X		X				X
3			X					
4		X		X		X		X
6				X				X
8		X		X		X		X

(c)

Fig. 5. The configuration of the parallel SAI, (a) the original impedance matrix, (b) the rows of the impedance matrix stored in one processor, (c) the appended minimum impedance matrix been stored in one processor for construction of the PSAI preconditioner.

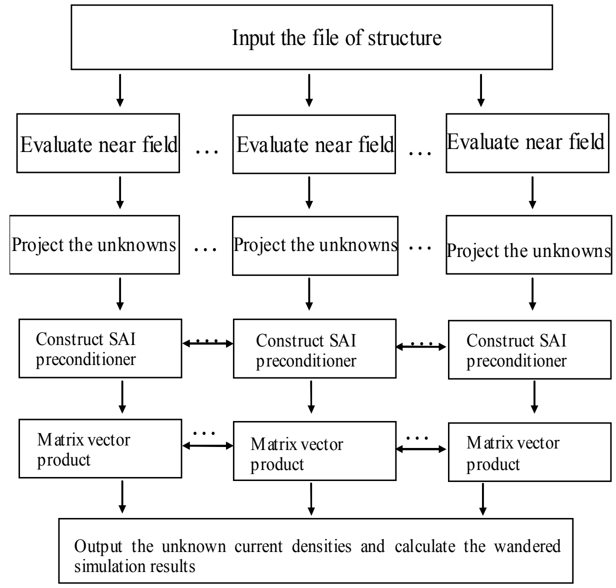

Fig. 6. The flow chart for the PSAI preconditioned AIM.

III. RESULTS AND DISCUSSIONS

In this section, some microstrip antennas are analyzed by the PSAI preconditioned AIM. The resultant linear systems are solved by the LGMRES solver [21] and its tolerance is 10^{-4}. The results presented here are all computed on 2-node clusters connected with an infiniband network. Each node includes a quad-core Intel processor and 8 GB of RAM.

First, an 8×8 microstrip corporate-fed planar antenna is considered, the parameters are depicted in Figure 6. It is discretized with 61, 345 RWG unknowns. As shown in Figure 7, the reflection coefficients versus frequency simulated by the proposed method and the Ansoft Designer® are plotted. It can be seen the results agree well which demonstrate the accuracy of the proposed method. The total simulation time of one frequency point for the PSAI preconditioned AIM is 2, 512 s, and the time for the Ansoft Designer® is 12, 560s. We, also, compare the H-plane far field pattern of the 8×8 microstrip antenna at the frequency of 9.42 GHz with the CGFFT approach [22] in Figure 8 to verify the proposed method, where reasonable agreements are observed. The time for constructing the PSAI preconditioner is 613 s and the solution time is 1, 576 s. Figure 9 shows the residual norm histories for the 8×8 microstrip corporate-fed planar antenna at 9.42 GHz simulated by AIM with and without the PSAI preconditioner. The label "PSAI-0" represents the PSAI preconditioner constructed by Znear, and "PSAI-1" represents the PSAI preconditioner constructed by $\mathbf{Z}_S$. It is found that the LGMRES PSAI-0 and PSAI-1 have the similar iterative steps which can greatly improve the convergence by a factor of 3.8. In order to describe the proposed method clearly, the label "PSAI" shown below denotes the PSAI preconditioner constructed by $\mathbf{Z}_S$. The advantage of the PSAI preconditioner can be found significantly when solving large dense linear systems with multiple right-hand sides arising in monostatic RCS, since the PSAI preconditioner needs to be constructed only once.

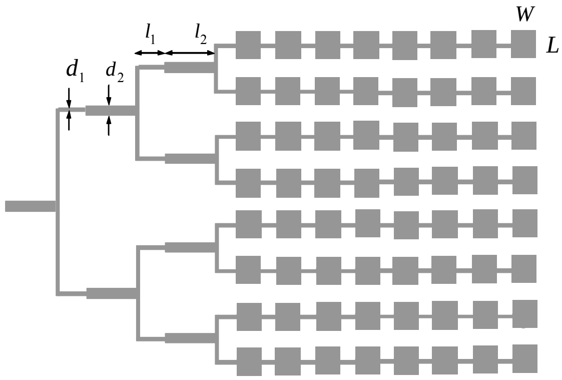

Fig. 7. The geometry of the 8 × 8 microstrip corporate-fed planar antenna, L_1=12.32mm, L_2 =18.48mm, W=10.08mm, L=11.79mm, d_1=1.3mm, d_2=3.93mm,the thickness of substrate h=1.59mm, $\varepsilon_r = 2.2$.

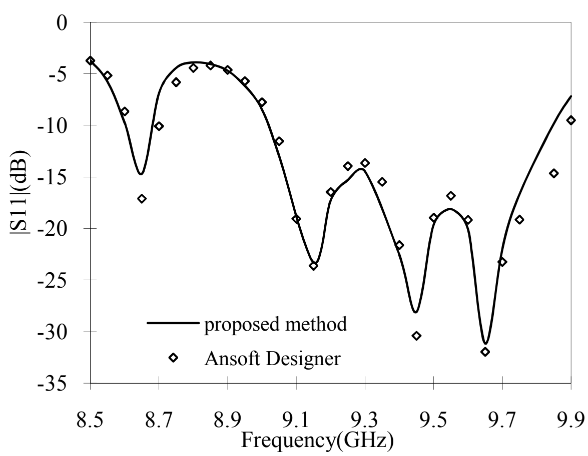

Fig. 8. The reflection coefficients versus frequency for the 8 × 8 corporate-fed planar antenna.

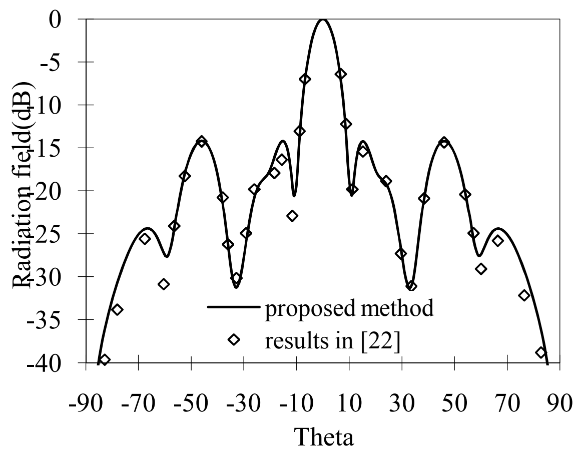

Fig. 9. The H-plane far field pattern of the 8×8 microstrip corporate-fed planar antenna compared with [22].

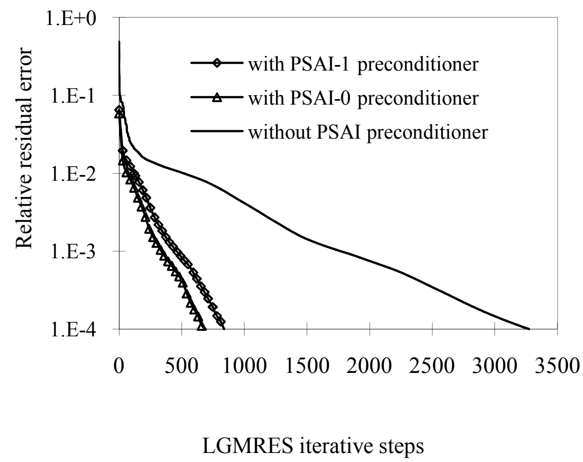

Fig. 10. Residual norm histories for the 8 × 8 microstrip corporate-fed planar antenna at 9.42 GHz simulated by LGMRES with and without the PSAI preconditioner.

To examine the parallel efficiency of the proposed method, the monostatic RCS of a series of microstrip antennas [23] are simulated. The layout of the microstrip antennas are shown in Figure 10, the configuration of the unit of the arrays is $L = 3.66\text{cm}$, $W = 2.60\text{cm}$, $a = b = 5.517\text{cm}$, the dielectric constant and the thickness of the substrates is $\varepsilon_r = 2.17$, $d = 0.158\text{cm}$. The microstrip antennas with 7 × 7, 20 × 20 arrays are simulated. The number of unknowns is 8, 428 and 84, 000, respectively. Table 1 and Table 2 list the CPU time of the above antennas simulated by AIM with and without the PSAI preconditioner at the frequency of 3.7 GHz. The notations used in the tables are denoted below.

- *near* denotes the time used for filling the near field impedance matrix.
- *cof* denotes the time used for computing the expansion coefficients.
- *set* denotes the time used for constructing the PSAI preconditioner.
- *steps* denotes the average number of iterative steps of the LGMRES for $\varphi = 0^\circ$ and θ varying from 0° to 85°.
- *sol* denotes the time used for solving the liner systems.
- *tol* denotes the total CPU time for simulation.
- *mem* denotes the memory usage for one processor.
- *ef* denotes the efficiency of the parallelization which is defined as $\frac{T_{Total}}{nT_{nTotal}}$, where T_{Total} is the total CPU time computed by one processor, T_{nTotal} is the total CPU time computed by n processors.

As shown in Table 1, the time for filling near field matrix and solving the expansion coefficients in columns 3, 4 is almost a linear reduction with the increase of the number of processors. In column 5, little solving time of the linear system is saved when the number of unknowns is small (i.e. the 7 × 7 arrays), since the inner-processor communication takes the most of the CPU time. Much solving time of the linear systems is saved when the number of unknowns is large (i.e. the 20 × 20 arrays), since the computation takes the most of the CPU time. Similarly, as shown in Table 2, the time for filling the near field matrix, solving the expansion coefficients and construction of the preconditioner in columns 3, 4, and 5 is also linear reduction with the increase of the number of processors.

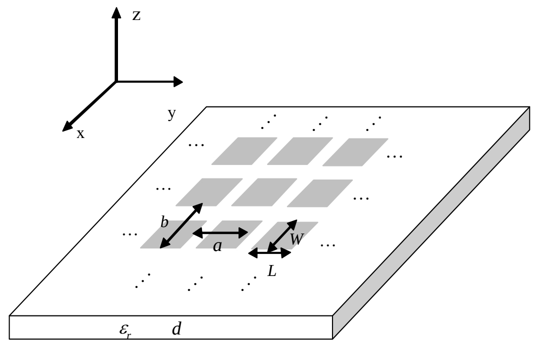

Fig. 11. The layout of the microstrip antenna.

Little solving time of the linear system in columns 6 is saved, when the number of unknowns is small (i.e. the 7 × 7), and much solving time of the linear systems is saved when the number of unknowns is large (i.e. the 20 × 20 arrays). Comparing the memory storage, the total solving time, and the parallel efficiency of Table 1 and Table 2, it can be found that by using the PSAI preconditioner only increases small memory usage while the total solving time and the parallel efficiency are improved significantly due to the decrease of the number of iterative steps of the LGMRES.

Finally, we verify the proposed method by a 30 × 30 microstrip antenna with 231, 300 RWG basis functions. The monostatic RCS computed by the AIM with and without PSAI preconditioner is plotted in Figure 11, where reasonable agreements are observed. The time for constructing the PSAI preconditioner and per iteration is 1, 304, and 5 seconds, respectively. Figure 12 shows the iterative steps of the LGMRES with and without a PSAI preconditioner for the 30 × 30 microstrip arrays. Where $\varphi = 0^\circ$ and θ is varying from 0° to 85°. It can be found that the PSAI preconditioning technique can greatly improve the convergence by at least a factor of 4.1 compared with no preconditioned LGMRES.

Table 1: The CPU time of the 7× 7 and 20× 20 arrays simulated by the AIM without PSAI preconditioner

	processors	near(sec)	cof(sec)	steps	sol(sec)	tol(sec)	mem(Mb)	ef
7 × 7	1	312	144	243	8, 319	8, 790	43	-
	4	78	36		8, 094	8, 215	30	26%
	8	39	18		7, 710	7, 770	25	14%
20 × 20	1	12, 724	5, 842	257	102, 017	120, 684	410	-
	4	3, 862	1, 367		47, 355	52, 685	171	57%
	8	1, 977	723		43, 771	46, 572	149	32%

Table 2: The CPU time of the 7×7 and 20×20 arrays simulated by the AIM with PSAI preconditioner

	Processors	near (sec)	cof(sec)	set(sec)	steps	sol(sec)	tol(sec)	mem(Mb)	ef
7×7	1	312	144	886	13	891	2, 241	69	-
	4	78	36	222		655	996	42	56%
	8	39	18	111		608	782	30	36%
20×20	1	12, 724	5, 842	10, 251	19	7, 413	36, 588	510	-
	4	3, 862	1, 367	2, 447		6, 314	11, 737	210	78%
	8	1, 977	723	1, 310		4, 830	8, 959	158	51%

IV. CONCLUSION

In this paper, the PSAI preconditioned AIM method is proposed for analyzing the large scale microstrip antennas. The parallel AIM is used to accelerate the matrix vector multiplication. The PSAI is used to improve the iterative convergence of the LGMRES. The PSAI is based on the parallelized Frobenius-norm minimization, and the construction time of the preconditioner is further saved by selecting the most informative elements of the sparse near field impedance matrix. Numerical results prove that by using the PSAI preconditioner, the parallel efficiency is significantly improved.

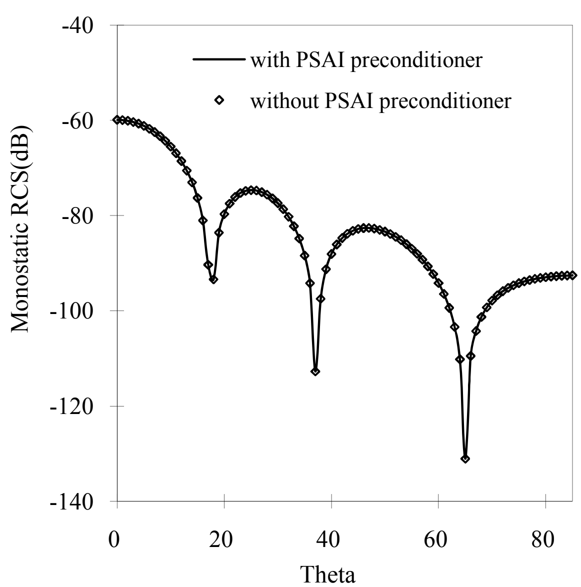

Fig. 12. The monostatic RCS of the 30×30 microstrip arrays simulated by the AIM with and without PSAI preconditioner, $\varphi = 0^{\circ}$ and θ is varying from 0° to 85°, and the frequency of the incident plane wave is 300MHz.

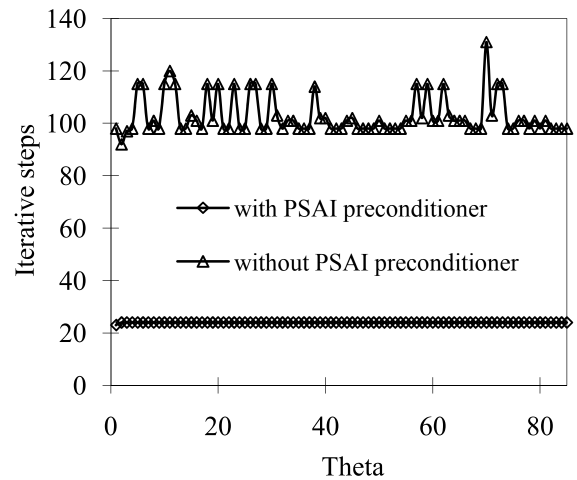

Fig. 13. The iterative steps of the LGMRES for the 30×30 microstrip arrays simulated by the AIM with and without PSAI preconditioner, $\varphi = 0^{\circ}$ and θ is varying from 0° to 85°, and the frequency of the incident plane wave is 300MHz. And the tolerance of LGMRES is 10^{-4}.

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REFERENCES

[1] R. F. Harringlon, *Field computation by Moment Methods*, New York. MacMillan, 1968.

[2] F. Ling, C .F. Wang, and J. M. Jin, "An efficient algorithm for analyzing large-scale microstrip structures using adaptive integral method combined with discrete complex-image method,"

IEEE Trans. Antennas Propag., vol. 48, no.5, pp. 832－839, 2000.

[3] X. M. Pan, and X. Q. Sheng, "A high performance parallel MLFMA for scattering by extremely large targets," *Microwave Conference.*, pp. 16-20, 2008.

[4] Ö. Ergül and L. Gürel, "Efficient parallelization of the multilevel fast multipole algorithm for the solution of large-scale scattering problems," *IEEE Trans. Antennas Propag.*, vol. 56, no. 8, pp. 2335-2345, 2008.

[5] L. -W. Li, Y. J. Wang, and E.-P. Li, "Mpi-based parallelized precorrected FFT algorithm for analyzing scattering by arbitrary shaped three-dimensional objects," *Progress In Electromagnetic Research.*, PIER 42, pp. 247-259, 2003.

[6] D. Q. Ren, Park. T., Mirican. B., McFee. S., and Giannacopoulos. D. D, "A methodology for performance modeling and simulation validation of parallel 3-D finite element mesh refinement with tetrahedral," *IEEE Trans. Magn.*, vol. 44, no. 6, pp. 1406-1409, 2008.

[7] Y. Saad, *Iterative methods for sparse linear systems*, PWS Publishing Company: New York, 1996.

[8] W. -B. Ewe, L. -W .Li, Q. Wu, and M.-S. Leong, "Preconditioners for Adaptive Integral Method Implementation," *IEEE Trans. Antennas Propag.*, vol. 53, no. 7, pp. 2346-2350, July 2005.

[9] J. Lee, J. Zhang, and C. Lu, "Performance of Preconditioned Krylov Iterative Methods for Solving Hybrid Integral Equations in Electromagnetics," *ACES Journal*, vol. 18, no. 3, pp. 54-61, 2003.

[10] J. M. Song, C. C. Lu, and W. C. Chew, "Multilevel fast multipole algorithm for electromagnetic scattering by large complex objects," *IEEE Trans. Antennas Propag.*, Vol. 45, No. 10, pp. 1488-1493, 1997.

[11] K. Sertel and J. L. Volakis, "Incomplete LU preconditioner for FMM implementation," *Microw. Opt. Technol. Lett.,* vol. 26, no. 7, pp. 265–267, 2000.

[12] M. Zhang, T. S. Yeo, and L. -W .Li, "Threshold-based Incomplete LU Factorization Preconditioner for Adaptive Integral Method," *Proc of 2007 Asia-Pacific Microwave Conference*, Bangkok, Thailand, pp. 913-916, December 11-14, 2007.

[13] J. Q. Chen, Z. W. Liu, K. Xu, D. Z. Ding, Z. H. Fan, and R. S. Chen, "Shifted SSOR preconditioning technique for electromagnetic wave scattering problems," *Microw. Opt. Technol. Lett.*, vol. 51, no. 4, pp. 1035-1039, 2009.

[14] P. L. Rui, R. S. Chen, D.X. Wang, and E.K.N. Yung, "Spectral two-step preconditioning of multilevel fast multipole algorithm for the fast monostatic rcs calculation," *IEEE Trans. Antennas Propag.*, vol. 55, no. 8, pp. 2268-2275,2007.

[15] M. Tahir, G. Levent, "Accelerating the multilevel fast multipole algorithm with the sparse-approximate-inverse (SAI) preconditioning," *SIAM. J. Sci. Comput.*, Vol. 3, No. 3, pp. 1969-1984, 2009.

[16] T. Malas, Ö. Ergül, and L. Gürel, "Parallel preconditioners for solutions of dense linear systems with tens of millions of unknowns," *22nd International Symposium on Computer and Information Sciences (ISCIS 2007)***,** 1-4, 2007.

[17] J.-S. Zhao, W. C. Chew, C.-C. Lu, E. Michielssen, and J. Song, " Thin-stratified medium fast-multipole algorithm for solving microstrip structures," *IEEE Trans. Microwave Theory Tech.*, vol. 46, no.4, pp. 395-403, Apr. 1998..

[18] D. Takahashi. Graduate School of Systems and Information Engineering University of Tsukuba. http://www.ffte.jp/s. 2004.

[19] P. L. Rui, R. S. Chen, "Sparse approximate inverse preconditioning of deflated block-GMRES algorithm for the fast monostatic RCS calculation," *International Journal of Numerical Modelling: Electronic Networks, Devices and Fields.*, vol. 21, pp. 297-307, 2008.

[20] D. Z. Ding, R. S. Chen and Z, H. Fan, "An efficient SAI preconditioning technique for higher order hierarchical MLFMM implementation," *Progress in Electromagnetics Research*, PIER 88, pp. 255-273, 2008.

[21] W. Zhuang, Z.H. Fan, and Y.Q. Hu, "Adaptive Integral Method (AIM) Combined with the Loose GMRES Algorithm for Planar Structures Analysis," *International Journal of RF and Microwave Computer-Aided Engineering.*, vol. 19, pp. 24-32, 2009.

[22] C. F. Wang, F. Ling, and J. M. Jin, "A fast full-wave analysis of scattering and radiation from large finite arrays of microstrip antennas," *IEEE Trans. Antennas Propagat.*, vol. 46, no.10, pp. 1467–1474, 1998.

[23] A. S. King and W. J. Bow, "Scattering from a finite array of microstrip patches," *IEEE Trans. Antennas Propagat .*, vol. 40, no. 2, pp. 770-774, 1992.

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