# Accelerating the Multilevel Fast Multipole Method with Parallel Preconditioner for Large-Scale Scattering Problems

**Ming Chen, Rushan Chen, Zhenhong Fan, and Dazhi Ding**

Department of Communication Engineering
Nanjing University of Science and Technology
Nanjing, P.R. China, 210094
eechenrs@mail.njust.edu.cn

*Abstract*─ A novel parallel framework is proposed for the iterative solution of the multilevel fast multipole method (MLFMM). The inversion of the near-field impedance matrix is used as the preconditioner matrix to improve the convergence history of the ill-conditioned linear system formulated by electric field integral equation. In order to accelerate the inversion of the near field impedance matrix with huge number of unknowns, the parallel technique is used to construct the preconditioner matrix. Our numerical experiments reveal that with an efficiently parallelized MLFMM and the effective parallel preconditioner, we are able to solve problems with millions of unknowns in a few hours. Both the number of iteration steps and the overall simulation time can be saved significantly. For closed-surface problems analyzed by the combined-field integral equation, the number of iterations can also be reduced significantly by the proposed method. Numerical results are presented to demonstrate the accuracy and efficiency of the proposed method.

*Index Terms*─ Multilevel fast multipole method, preconditioning, scattering problems.

## I.  INTRODUCTION

The method of moments (MoM) [1-4] has found widespread application in a variety of electromagnetic radiation and scattering problems. The matrix associated with the resulting linear systems is large and dense for electrically large objects in electromagnetic scattering. It is basically impractical to solve the electric-field integral equation (EFIE) matrix equation using the direct method due to the memory requirement of $O(N^2)$ and computational complexity of $O(N^3)$,

where $N$ is the number of unknowns. Making such solutions prohibitively expensive for large-scale problems, this difficulty can be circumvented by use of iterative solvers, and the matrix-vector product operation can be efficiently computed by the multilevel fast multipole method (MLFMM) [5-9]. The use of MLFMM reduces both the memory requirement and the computational complexity to $O(NlogN)$.

Although the MLFMM reduces the complexity of MoM from $O(N^2)$ to $O(N\log N)$, allowing for the solution of large problems with limited computational resources. However, accurate solutions of large problems require discretizations with millions of unknowns, which cannot easily be solved with sequential implementations of MLFMM running on a single processor. To solve such large problems, it is helpful to increase the computational resource by assembling parallel computing platforms and, at the same time, by paralleling MLFMM [10-14]. There are many efforts that have been done to improve the efficiency of the parallel MLFMM. Thanks to those efforts, it has become possible to solve millions of unknowns on relatively inexpensive platforms.

Among integral formulations, the application of boundary conditions for the electric field and the magnetic field on the surface of the object leads to the EFIE and the magnetic-field integral equation (MFIE), respectively. For closed object, EFIE and MFIE can be combined together to form the combined-field integral equation (CFIE), which provides better-conditioned matrix equations than EFIE and MFIE and is free of internal-resonance. However, for the open structure only EFIE can be used, the EFIE provides a first-kind Fredholm integral equation, which is usually ill-conditioned,

this is especially true when the higher-order basis functions are employed, which will drastically increase the condition number of the EFIE matrix. Generally, it requires a large number of iterations to reach convergence. This bottleneck severely limits the capability of the MLFMM for open structure. To break this bottleneck, many preconditioner techniques [15-20] have been developed to accelerate the convergence rate, such as the diagonal, block-diagonal, but they are not effective enough to yield a highly efficient solution.

We know that, in the MLFMM application, only the near-field impedance matrix is stored, which is composed of the interactions of the neighboring cubes in the lowest level of the octree structure. The Green's function used for the computation of the matrix elements decays with 1/R, where R is the distance between the pair of basis and testing functions. Due to this rapid decay of the Green's function, basis functions that are close to each other are expected to have strong electromagnetic coupling, resulting in matrix elements with larger magnitudes. Therefore, the near-field impedance matrix retains the most relevant contributions of the impedance matrix in MoM. In this paper, we use the exact inversion of the near-field matrix as a preconditioner matrix in MLFMM. From the numerical results, we can see that the number of iterations reduced significantly compared with that without the preconditioner. Moreover, for conducting geometries with closed surfaces, the CFIE should be considered. Even though EFIE can also be used in such problems, this has no practical use since CFIE can solve the closed-surface problems much faster. Furthermore, from the numerical results, we can see that the preconditioner for the CFIE systems can also get higher efficiency.

The remainder of this paper is organized as follows. Section II gives a brief introduction to the EFIE and MFIE, together with an introduction of the MLFMM. The efficient parallization of MLFMM and the construct preconditioner matrix are also investigated in this part in detail. Section III presents the numerical results to demonstrate the accuracy and efficiency of the proposed method. Finally, some conclusions are given in section IV.

## II. THEORY

The EFIE and MFIE formulation of electromagnetic scattering problems using planar Rao-Wilton-Glisson (RWG) basis functions for surface modeling is presented in [1]. The resulting linear systems from EFIE and MFIE formulation after Galerkin's testing are briefly outlined as follows

$$\sum_{n=1}^{N} Z_{mn} a_n = V_m, \quad m = 1, 2, ..., N, \quad (1)$$

where

$$Z_{mn}^{EFIE} = \iint_{S_m} ds \iint_{S_n} ds' [j\omega\mu \boldsymbol{J}_m \cdot \boldsymbol{J}_n - \frac{j}{\omega\mu}(\nabla\cdot\boldsymbol{J}_m)\nabla'\cdot\boldsymbol{J}_n)]g(\boldsymbol{r},\boldsymbol{r}'), \quad (2)$$

$$Z_{mn}^{MFIE} = \frac{1}{2}\iint_S \boldsymbol{J}_m \cdot \boldsymbol{J}_n dS - \iint_S \boldsymbol{J}_n \cdot \hat{n} \times \nabla \times \iint_{S'} G\boldsymbol{J}_m dS'dS, \quad (3)$$

and

$$V_m^{EFIE} = \iint_{S_m} \boldsymbol{J}_m \cdot \boldsymbol{E}^{inc} ds, \quad (4)$$

$$V_m^{MFIE} = \iint_{S_m} \boldsymbol{J}_m \cdot (n \times \boldsymbol{H}^{inc}) ds. \quad (5)$$

Here $g(\vec{r},\vec{r}') = e^{-jk|\vec{r}-\vec{r}'|} / 4\pi|\vec{r}-\vec{r}'|$ refers to the Green's function in free space and $a_n$ are the coefficients of the induced current expanded in RWG basis functions, $\overline{V}$ is the vector of incident field. $r$ and $r'$ denote the observation and source point locations. $\boldsymbol{E}^{inc}$ and $\boldsymbol{H}^{inc}$ are the incident plane wave, $\eta$ and $k$ denote the free space impedance and wave number, respectively.

For closed structure, the CFIE is simply a linear combination of EFIE and MFIE and is of the form

$$Z^{CFIE} = \frac{\alpha}{\eta} Z^{EFIE} + (1-\alpha)Z^{MFIE}. \quad (6)$$

The combination parameter $\alpha$ ranges from 0 to 1 and can be chosen to be any value within this range.

From above we can see that, if we have discretization of equation (2) and (3) by MoM, we end up with a dense linear matrix. The surface of the object is in general meshed with $0.1\lambda$ of the wavelength for accuracy. Hence, for high frequencies where the electrical size of the object becomes large in terms of the wavelength, the system matrix becomes also large. When iterative methods are used to solve such systems, they can at best provide $O(N^2)$ complexity, which severely limit the capacity of the MoM in dealing with

large objects. Fortunately, a solution has been proposed to accelerate the computation of the MVP using the MLFMM for the MoM, which reduces the memory requirement and the computational complexity to $O(N\log N)$.

In MLFMM, the impedance matrix $\overline{\overline{Z}}$ can be split into two parts as

$$\overline{\overline{Z}} \cdot \overline{a} = \overline{\overline{Z}}_{NF} \cdot \overline{a} + \overline{\overline{Z}}_{FF} \cdot \overline{a} \,, \tag{7}$$

where $\overline{\overline{Z}}_{NF}$ denotes the sparse matrix that corresponds to near-field interactions, while $\overline{\overline{Z}}_{FF}$ denotes the matrix that corresponds to far-field interactions. To calculate electromagnetic interactions by MLFMM in a multilevel scheme, a tree structure first needed to construct. The entire object is first enclosed into a large cube, and then the cube is partitioned into eight smaller cubes. Each subcube is recursively subdivided into eight smaller cubes until the finest cubes satisfy the termination criterion. After constructing an octree, we find nonempty cubes by sorting. Only nonempty cubes are recorded using tree-structured data at all levels. Thus, the tree structure is sparse. The computational cost depends on the nonempty cubes, not all cubes. Two cubes are well-separated if the ratio of the cube-center-distance to the cube size is greater than or equal to 2. The interaction between them can be computed in a group-by-group manner by MLFMM. Otherwise, they are near each other and share at least one edge point, the interaction between the two cubes can be calculated directly by MoM.

Although the MLFMM has reduced the complexity of the MoM from complexity $O(N^2)$ to $O(N\log N)$, allowing for the solution of large problems with limited computational resources. However, accurate solutions of large problems require discretizations with millions of unknowns, which cannot easily be solved with sequential implementations of MLFMM running on a single processor. Both the computation ability and the storage ability of a single computer can not afford. To solve such large problems, it is helpful to increase the computational resource by assembling parallel computing platforms and, at the same time, by paralleling MLFMM. Parallel MLFMM has received much attention in recent years with the development of the computer science; the bigger problems have been solved on the super computer. To improve the convergence rate of the

equation (1) for the open structure, the parallel preconditioner for the parallel MLFMM was also constructed.

## A. Efficient parallelization of MLFMM

Because of the complicated structure, parallelization of MLFMM is not a trivial. Simple parallelization schemes usually lead to inefficient solutions due to dense communications and unbalanced distribution of the workload among processors. For high efficiency parallelization, several issues must be carefully considered to obtain an efficient parallelization of MLFMM.

In the past few years, a series of implementation tricks have been developed for efficiently parallelizing the MLFMM, these tricks are different, but the most important thing in those tricks in parallelizing MLFMM is load-balancing and minimizes the communications between the processors. This is achieved by using different partitioning strategies for the lower and higher levels of the tree structure. In the lower levels of the tree structure, there are many clusters with a small number of samples for the radiated and incoming fields. The number of cubes is much larger than the number of processors. Therefore, it is natural to distribute the cubes equally among processors. However, it is difficult to achieve good load-balancing in higher levels with this parallel approach, since the number of cubes in the coarse levels is small and the electric size of the cube is large, the far-field patterns is large. Therefore, in the coarse level, we adopt another parallel approach in the coarse levels; we replicate the cubes in every processor, but partition the far-field patterns equally among all processors as paper. Using this approach for the parallel MLFMM in the far-field, good load balancing can be achieved.

The interaction in MLFMM is classified into a near-field interaction and the far-field interaction. After distribute the cubes to each processor, the near-field interaction lists and the far-field interaction lists can be set up in a parallel way. The near-field interaction are calculated directly and stored in memory without any communication. For the far-field interaction in the MLFMM is transformed into three phases: called the aggregation, translation, and disaggregation stage.

*Aggregation stage*: The far-field interaction begins with aggregating basis functions at the finest level to obtain the radiation pattern. Each processor

calculates and stores the radiation and receiving patterns of the basis and testing functions included in its local box. Then each processor shifting the radiation pattern to the center of the box in the second finest level, and finally interpolating the deficient radiation pattern to obtain the radiation pattern of the box in the second-finest level. This procedure repeats until the shared levels. In the shared levels, each box is assigned to the same processor. The far-field pattern of each box is distributed equally among processors. In the distributed levels, even though a local interpolation is used, some of the far-field patterns may locate in other processors. Therefore one-to-one communications are needed to get the required data. **_Translation stage:_** The translation stage is one of the most important stages in the parallelization MLFMM; since the cubes are distributed among the processors; one to one communications are required between the processors for the translation stage. To eliminate this overhead, each processor is loaded with extra cubes called the ghost cubes. For example, if cube $i$ at processor $a$ needed the far-field samples of cube $j$ at processor $b$ , maybe another cube at processor $a$ also needed the far-field samples of cube $j$ at processor $b$ , to reduce the communication between the processors; we allocate space for the cube $j$ at processor $a$. When the far-field samples of the cube $j$ is received by processor $a$ , we store it at processor $a$, this ensure that the same data is not transferred more than once. In the shared levels, the far-field samples of each cube are distributed equally among the processors. Therefore, there is need no communication between the processors at the translation stage in the shared levels.
**_Disaggregation stage:_** The disaggregation stage is generally the inverse of the aggregation stage; the incoming fields are calculated at the centre of each cube from the top of the tree structure to the lowest level using the interpolation and shift operations. Some of the samples obtained from the interpolation operation should be sent to other processors, this is because interpolation during the aggregation stage are performed using the inflated data prepared by one-to-one communications among the processors. Following an interpolation operation, some of the resulting data are used locally, while the rest are sent to other processors,

similar to the communications during the translation stage.

## B. Novel preconditioner in MLFMM

For equation (1), there are two ways to solve it. One is the direct solver, and the other is the iterative solver. For large-scale problems, it is impractical to solve the matrix equation with a large number of unknowns using the direct solver because it has a memory requirement of $O(N^2)$. This difficulty can be circumvented by solving the matrix equation using the Krylov-sub-space iterative method, which requires the matrix-vector product (MVP) in each iteration step. In the past, several iterative methods, including the conjugate gradient (CG), the biconjugate gradient (BCG), the stabilized biconjugate gradient (BCGS), and the generalized minimal residual (GMRES) have been employed. The use of those methods reduce the memory requirement to $O(N)$. Iterative solutions of linear systems using Krylov-subspace methods make it possible to solve large-scale scientific problems with modest computing requirements. Effective parallelization of both the matrix-vector multiplication and the iterative solvers are possible, allowing even larger systems to be solved with cost-effective parallel computers.

Unfortunately, the MoM matrix based on EFIE is usually ill-conditioned and requires a large number of iterations to reach convergence. In order to speed up the convergence rate of the GMRES solution, preconditioning techniques are often used, such as the block-diagonal. The block-diagonal techniques can help to partially alleviate this difficulty, but they are not effective enough to yield a highly efficient solution. In this paper, we use the exact inverse of the near-field matrix as the precondition for the MLFMM, The preconditioner matrix is very easy to construct, which can be obtained from the near-field interaction matrix. Number results are presented to show the high effectiveness of the proposed preconditioner and fast convergence of the iterative solution.

From above, we have known that the impedance matrix can be split into two parts, where $\overline{\overline{Z}}_{NF}$ denotes the near-field interactions and $\overline{\overline{Z}}_{FF}$ denotes the far-field interactions, since $\overline{\overline{Z}}_{FF}$ is not readily available and $\overline{\overline{Z}}_{NF}$ retains the most relevant contributions of the impedance matrix, it is

customary to construct preconditioners from $\overline{\overline{Z}}_{NF}$ assuming it to be a good approximation to $\overline{\overline{Z}}$. Therefore, in this paper, we choose the preconditioning matrix $M = \overline{\overline{Z}}_{NF}$. We call this proposed preconditioner the NF_LU preconditioner method.

In order to speed up the convergence rate of the Kryloy iterative methods, the preconditioning matrix $M$ are employed to transform (5) into an equivalent form

$$M^{-1}Ax = M^{-1}b, \qquad (8)$$

with $\tilde{A} = M^{-1}A$ and $\tilde{b} = M^{-1}b$, equation (8) can be written as the following form

$$\tilde{A}x = \tilde{b}. \qquad (9)$$

$M^{-1}$ is a matrix for preconditioning the matrix $A$ from the left. The purpose of preconditioning is to make the condition number of the matrix $\tilde{A}$ better than the original matrix $A$. So the Kryloy iterative methods for the equation (9) can get a fast convergence.

For sequential implementations of MLFMM running on a single processor, the calculation of $M^{-1}$ can be obtained by the Umfpack strategy, However, for large-scale problems, the direct solver $M^{-1}$ may require prohibitive memory and the time used to construct the inverse matrix will be very long. Fortunately this cost can be alleviated by parallelization. With an efficient parallelization to compute the $M^{-1}$, problems that are discretized with tens of millions of unknowns are easily solved on a cluster of computers. In this paper, we use the parallel LU factorization to construct the preconditioner matrix $M^{-1}$, after decomposing the matrix $M$ in the form of $M = LU$, preconditioning operation is performed in each step by solving $LUv = w$, the preconditioning operation $v = M^{-1}w$ is computed by solving the linear system $LUv = w$ is performed in two distinct steps: solve $Lx = w$ and $Uv = x$ successively.

## III.    NUMERICAL RESULTS

In this section, several numerical examples are presented to demonstrate the efficiency of the proposed method. We calculate the RCS of three conducting geometries, which are shown in Fig. 1-3. The first two geometries are open structures analyzed by EFIE while the third structure is a closed structure computed by CFIE. All

experiments are performed on a 2-node cluster connected with an Infiniband network. Each node includes 8 cores and 48 GB of RAM. The resulting linear systems are solved by the restarted version of the GMRES solver with dimension of 30 and tolerance residual of $10^{-3}$.

First of all, the proposed method is used to the analysis scattering from a square PEC plane with length of 16m, the incident plane wave direction is fixed at $\theta_{inc} = 0°$ and $\phi_{inc} = 0°$, the scattering angle is fixed at $\theta_s = 0° \sim 180°$ and $\phi_s = 0°$ are shown in Fig. 1. The computational information (the electric size of the plane; the number of unknowns; the number of iterations without NF_LU preconditioner; the number of iterations with NF_LU preconditioner) for this example is given in Table I. From this table, we can conclude that without preconditioner, EFIE solutions converge fast only for a small number of unknowns. The number of iteration increases quickly to reach convergence without a preconditioner when the number of unknown increases. Convergence fails at the 1000th iterations without preconditioner when the electric size of the plane is $128\lambda$; it can be found from Table I that the NF_LU solve all problems within reasonable iteration counts. Compared with iterative solution without preconditioner, the NF_LU preconditioned method decrease the number of iterations about 12 times.
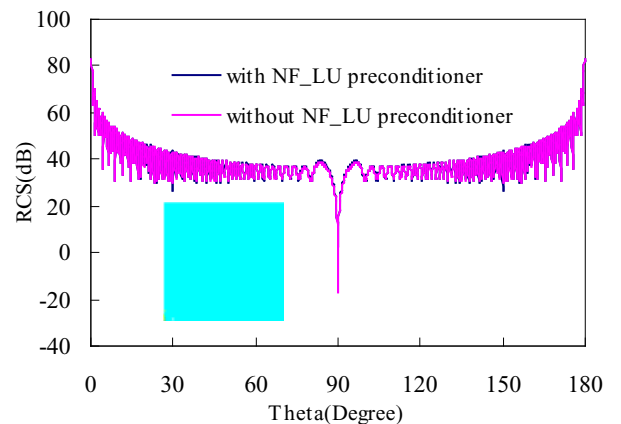


Fig. 1. Bistatic RCS of the conduct plane at 1.2 GHz.

The second example is the analysis of scattering from a half PEC sphere, the radius of the sphere is 8m. The incident direction of the plane wave are $\theta_{inc} = 0°$ and $\phi_{inc} = 0°$. The

scattering angle is observed at $\theta_s = 0° \sim 180°$, when $\phi_s = 0°$ are shown in Fig. 2. The computational information for this example is given in Table II. As the electrical size of the half sphere is bigger than $32\lambda$, it can be seen from Table II that convergence fails without preconditioner since the number of iterations is larger than 1000. In contrast, with a NF_LU preconditioner, it is able to achieve convergence in 200 iterations when the electric size of the half sphere is $128\lambda$. Compared with traditional method without preconditioner, the NF_LU preconditioned method performs high efficiency in this example.

problems confronted in CEM involve complicated structures enclosing a volume. Due to its favorable properties, CFIE is the preferred integral-equation formulation for those targets with closed surfaces. The incident direction is fixed at $\theta_{inc} = 90$ and $\phi_{inc} = 90$. The scattering angles are $\theta_s = 90$ at $\phi_s = 0 \sim 360$ are shown in Fig. 3. The electric current on the surface of the plane is shown in Fig. 4. The computational information for this example is given in Table III. It can be seen from Table III that even for the CFIE matrix, the proposed preconditioner also performs a better efficiency.
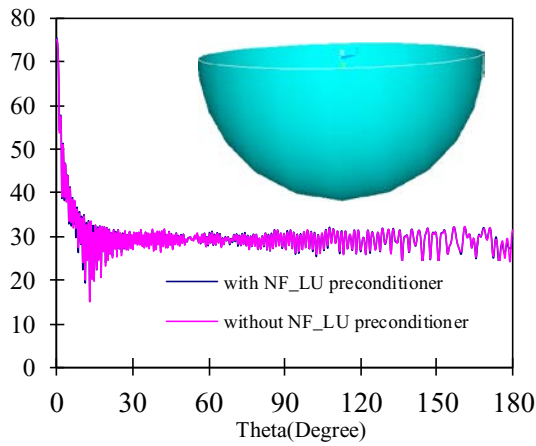


Fig. 2. Bistatic RCS in dB of the conduct half sphere at 1.2 GHz.

Table I: Performance comparison of no preconditioner and NF_LU preconditioner

| Frequency (GHz) | Unknowns | No preconditioner | NF_LU preconditioner |
|---|---|---|---|
| 0.3 | 85840 | 255 | 21 |
| 0.6 | 344000 | 352 | 30 |
| 1.2 | 1377280 | 645 | 41 |
| 2.4 | 5509120 | >1000 | 60 |

Table II: Performance comparison of no preconditioner and NF_LU preconditioner

| Frequency (GHz) | Unknowns | No preconditioner | NF_LU preconditioner |
|---|---|---|---|
| 0.3 | 78004 | 420 | 44 |
| 0.6 | 312016 | >1000 | 73 |
| 1.2 | 1248064 | >1000 | 117 |
| 2.4 | 5001952 | >1000 | 198 |

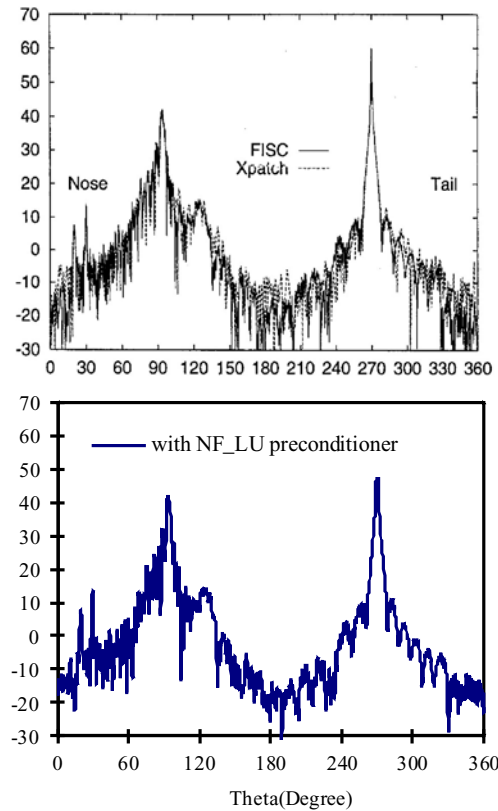The last example is used to analyze scattering from a VFY-218 plane, since many real-life



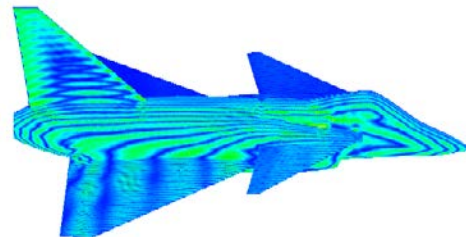Fig. 3. Bistatic RCS in dB of the VFY-218 plane at 3GHz.



Fig. 4. Current distribution on the VFY-218 at 3GHz.

Table III: Performance comparison of no preconditioner and NF_LU preconditioner

| Frequency (GHz) | Unknowns | No preconditioner | NF_LU preconditioner |
|---|---|---|---|
| 0.35 | 34242 | 82 | 12 |
| 0.75 | 136968 | 76 | 15 |
| 1.5 | 547872 | 164 | 44 |
| 3.0 | 2191488 | 223 | 67 |

## IV.    CONCLUSION

In this paper, we present the details of a hierarchical partitioning strategy for the efficient parallelization of MLFMM on distributed-memory architectures. The parallel NF_LU can accelerate the convergence of the GMRES iterative solver which is used for the solution of surface integral equations. From the numerical results, we can conclude that for the large open-surface problems that are analyzed by EFIE, the proposed preconditioner is more efficient and robust when compared with no preconditioner. For complex closed-surface problems that can make use of the well-conditioned CFIE, the proposed method is also very efficient.

## REFERENCES

[1] S. Rao, D. Wilton, and A. Glisson, "Electromagnetic Scattering by Surfaces of Arbitrary Shape," *IEEE Trans. Antennas and Propag.,* vol. 30, no. 3, pp. 409-418, May 1982.

[2] K. A. Michalski and D. L. Zheng, "Electromagnetic Scattering and Radiation by Surfaces of Arbitrary Shape in Layered media, Part I: Theory," *IEEE Trans. Antennas and Propag.*, vol. 38, no. 3, pp. 335-344, March 1990.

[3] K. A. Michalski and D. L. Zheng, "Electromagnetic Scattering and Radiation by Surfaces of Arbitrary Shape in Layered Media, Part II: Implementation and Results for Contiguous Half-Spaces," *IEEE Trans. Antennas and Propag.*, vol. 38, no. 3, pp. 345-352, 1990.

[4] D. Ding, J. Ge, and R. Chen, "Well-Conditioned CFIE for Scattering from Dielectric Coated Conducting Bodies above a Half-Space," *Applied Computational Electromagnetic Society (ACES) Journal*, vol. 25, no. 11, pp. 936 - 946, 2010.

[5] R. Coifman, V. Rokhlin, and S. M. Wandzura, "The Fast Multipole Method for the Wave Equation: A Pedestrian Prescription," *IEEE Trans. Antennas Propag.*, vol. 35, no. 3, pp. 7-12, 1993.

[6] J. M. Song, C. C. Lu., and W. C. Chew, "Multilevel Fast Multipole Algorithm for Electromagnetic Scattering by Large Complex Objects," *IEEE Trans. Antennas and Propag.*, vol. 45, no. 10, pp. 1488-1493, October 1997.

[7] C. C. Lu and W. C. Chew, "A Multilevel Algorithm for Solving Boundary Integral Equations of Wave Scattering," *Micro. Opt. Tech. Lett.*, vol. 7, pp. 466-470, July 1994.

[8] J. M. Song and W. C. Chew, "Multilevel Fast-Multipole Algorithm for Solving Combined Field Integral Equation of Electromagnetic Scattering," *Micro. Opt. Tech. Lett.,* vol. 10, pp. 14-19, September 1995.

[9] M. M. Li, H. Chen, C. Li, R. S. Chen, and C. Ong, "Hybrid UV/MLFMA Analysis of Scattering by PEC Targets above a Lossy Half-Space," *Applied Computational Electromagnetic Society (ACES) Journal*, vol. 26, no. 1, pp. 17-25, January 2011.

[10] H. Zhao, J. Hu, and Z. Nie, "Parallelization of MLFMA with Composite Load Partition Criteria and Asynchronous Communication," *Applied Computational Electromagnetic Society (ACES) Journal*, vol. 25, no. 2, pp. 167-173, 2010.

[11] H. Fangjing, N. Zaiping, and H. Jun, "An Efficient Parallel Multilevel Fast Multipole Algorithm for Large-scale Scattering Problems," *Applied Computational Electromagnetic Society (ACES) Journal*, vol. 25, no. 4, pp. 381-387, April 2010.

[12] K. C. Donepudi, J. M. Jin et al, "A Higher Order Parallelized Multilevel Fast Multipole Algorithm for 3-D Scattering," *IEEE Trans. Antennas and Propag.*, vol. 49, no. 7, pp. 1069-1078, July 2001.

[13] W. Rankin and J. Board. "A Potable Distributed Implementation of the Parallel Multipole Tree Algorithm," *IEEE Symp. on High Performance Distributed Computing*, pp. 17-22, 1995.

[14] X. M. Pan and X. Q. Sheng, "A Sophisticated Parallel MLFMM for Scattering by Extremely Large Targets," *IEEE Trans. Antennas and Propag.*, vol. 50, no. 3, pp. 129-138, June 2008.

[15] M. Grote and T. Huckle, "Parallel Preconditioning with Sparse Approximate Inverses," *SIAM J. Sci. Stat. Comput.,* vol. 18, pp. 838-853, 1997.

[16] E. Chow, "A Priori Sparsity Patterns for Parallel Sparse Approximate Inverse Preconditioners," *SIAM J. Sci. Comput.,* vol. 21, pp. 1804-1822, 2000.

[17] K. F. Tsang, R. S. Chen, M. Lei, and E. K. N. Yung, "Application of the Preconditioned Conjugate Gradient Algorithm to the Integral Equations for Microwave Circuits," *Micro. Opt. Tech. Lett.*, vol. 34 (4), pp. 266-270, 2002.

[18] K. Wang, S. Kim, and J. Zhang, "A Comparative Study on Dynamic and Static Scarcity Patterns in Parallel Sparse Approximate Inverse Preconditioning," *Journal of Mathematical Modeling and Algorithms*, vol. 2, no. 3, pp. 203-215, 2003.

[19] Z. N. Jiang, Z. H. Fan, D. Z. Ding, R. S. Chen, and K. W. Leung, "Preconditioned MDA-SVD-MLFMA for Analysis of Multi-scale Problems," *Applied Computational Electromagnetic Society (ACES) Journal*, vol. 25, no. 11, November 2010.

[20] J. Lee, J. Zhang, and C. Lu, "Performance of Preconditioned Krylov Iterative Methods for Solving Hybrid Integral Equations in Electromagnetics," *Applied Computational Electromagnetic Society (ACES) Journal,* vol. 18, no. 3, pp. 54-61, November 2003.

**Ming Chen** was born in Anhui, China. He received the B.S. degree in Physics from Anhui University in 2006, and is currently working toward the Ph.D. degree at Nanjing University of Science and Technology (NJUST), Nanjing, China. His current research interests include computational electromagnetics, antennas, and electromagnetic scattering and propagation.

**Ru-Shan Chen** was born in Jiangsu, P. R. China. He received his B.Sc. and M.Sc. degrees from the Dept. of Radio Engineering, Southeast University, in 1987 and in 1990, respectively, and his Ph.D. from the Dept. of Electronic Engineering, City University of Hong Kong in 2001. He became a Teaching Assistant in 1990 and a Lecturer in 1992. Since September 1996, he has been a Visiting Scholar with the Department of Electronic Engineering, City University of Hong Kong, first as Research Associate, then as a Senior Research Associate in July 1997, a Research Fellow in April 1998, and a Senior Research Fellow in 1999. From June to September 1999, he was also a Visiting Scholar at Montreal University, Canada. In September 1999, he was promoted to Full Professor and Associate Director of the Microwave & Communication Research Center and in 2007, he was appointed Head of the Dept of Communication Engineering, Nanjing University of Science & Technology (NJUST). His research interests mainly include microwave/millimeter-wave systems, measurements, antenna, RF-integrated circuits, and computational electromagnetics. He is a Senior Member of the Chinese Institute of Electronics (CIE). He received the 1992 third-class science and technology advance prize given by the National Military Industry Department of China, the 1993 third class science and technology advance prize given by the National Education Committee of China, the 1996 second-class science and technology advance prize given by the National Education Committee of China, and the 1999 first-class science and technology advance prize given by JiangSu Province as well as the 2001 second-class science and technology advance prize. At NUST, he was awarded the Excellent Honor Prize for academic achievement in 1994, 1996, 1997, 1999, 2000, 2001, 2002, and 2003. He has authored or co-authored more than 200 papers, including over 140 papers in international journals. He is the recipient of the Foundation for China Distinguished Young Investigators presented by the National Science Foundation (NSF) of China in 2003. In 2008, he became a Chang-Jiang Professor under the Cheung Kong Scholar Program awarded by the Ministry of Education, China.

**Zhen-Hong Fan** was born in Jiangsu, the People's Republic of China in 1978. He received the M.Sc. and Ph.D. degrees in Electromagnetic Field and Microwave Technique from Nanjing University of Science and Technology (NJUST), Nanjing, China, in 2003 and 2007, respectively. During 2006, he was with the Center of Wireless Communication in the City University of Hong Kong, Kowloon, as a Research Assistant. He is currently an associated Professor with the Electronic Engineering of NJUST. He is the author or coauthor of over 20 technical papers. His current research interests include computational electromagnetics, electromagnetic scattering and radiation.

**Dazhi Ding** was born in Jiangsu, the People's Republic of China. He received the B.S. and Ph.D. degrees in Electromagnetic Field and Microwave Technique from Nanjing University of Science and Technology (NJUST), Nanjing, China, in 2002 and 2007, respectively. During 2005, he was with the Center of Wireless Communication in the City University of Hong Kong, Kowloon, as a Research Assistant. He is currently a Lecturer with the Electronic Engineering of NJUST. He is the author or coauthor of over 20 technical papers. His current research interests include computational electromagnetics, electromagnetic scattering and radiation.