Preconditioned MDA-SVD-MLFMA for Analysis of Multi-Scale Problems

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Abstract- Multilevel fast multipole algorithm (MLFMA) has been widely used to solve electromagnetic scattering problems from the electrically large size objects. However, it consumes very large memory to store near the interaction matrix for the object with fine structures because the “low frequency breakdown” phenomenon would happen when the finest level box’s size is below 0.2 wavelengths. The matrix decomposition algorithm - singular value decomposition (MDA-SVD) is one remedy to alleviate this pressure because it has no limit of the box’s size. However, the matrix assembly time of MDA-SVD is much longer than that of the MLFMA. In this paper, a hybrid method called MDA-SVD-MLFMA is proposed to analyze multi-scale problems, which uses the main framework of MLFMA but adopts the MDA-SVD to deal with the near interaction of MLFMA. This method takes advantage of the virtues of both MLFMA and MDA-SVD and is more efficient than either conventional MLFMA or conventional MDA-SVD. An efficient preconditioning technique is combined into the inner-outer flexible generalized minimal residual (FGMRES) solver to speed up the convergence rate. Numerical results are presented to demonstrate the accuracy and efficiency of the proposed method.

Index Terms- Flexible generalized minimal residual (FGMRES), matrix decomposition algorithm - singular value decomposition (MDA-SVD), multilevel fast multipole algorithm (MLFMA).

I. INTRODUCTION

In electromagnetic wave scattering calculations, a classical problem is to compute the equivalent surface currents induced by a given incident plane wave. Such calculations, relying on the Maxwell equations, are required in the simulation of many industrial processes ranging from antenna design, electromagnetic compatibility, computation of back-scattered fields, and so on. All these simulations require fast and efficient numerical methods to compute an approximate solution of Maxwell’s equations. The method of moments (MoM) [1-2] is one of the most widely used techniques for electromagnetic problems. It is basically impractical to analyze electrically large problems using MoM because its memory requirement and computational complexity both are $O(N^2)$, where N refers to the number of unknowns. Recently, a wide range of fast methods have been developed for accelerating the iterative solution of the electromagnetic integral equations discretized by MoM. One of the most popular techniques is MLFMA [3-6], which has $O(N\log N)$ complexity for a given accuracy.

The increased power and availability of

computational resources and acceleration schemes have enabled the solution of problems with a very large number of unknowns, varying from a few thousands to a few millions [3-6]. Another class of problems arises when analyzing structures which require a high local density of unknowns to accurately capture geometric features. This class of problems is referred to as multi-scale problems exhibit multiple scales in length. For example, small length scale discretizations are required to capture sharp or fine geometric features that are embedded within large and smooth geometries discretized at a coarser length scale. Generally, the characteristic of a multi-scale problem is the concentration of large number of unknowns in electrically small domains. However, when the finest level box's size is below 0.2 λ (λ indicates the incident wavelength), MLFMA will suffer from the "low frequency breakdown" phenomenon [4].

MDA-SVD is another popular technique used to analyze the scattering/radiation [7-8], which has no limit of the box's size. However, the matrix assembly time of MDA-SVD is much larger than that of MLFMA. In this paper, a hybrid method called MDA-SVD-MLFMA is proposed, which uses the main framework of MLFMA but adopts the MDA-SVD to deal with the near interaction of MLFMA. This method takes advantage of both MLFMA and MDA-SVD and is more efficient than either conventional MLFMA or conventional MDA-SVD for analyzing the multi-scale problems.

It is well known that the matrix condition number of the electric field integral equation (EFIE) for an electrically large problem is large. Furthermore, for multi-scale problems, the matrix condition number is even larger due to the mixed discretization. Therefore, the system has poor convergence history and requires urgently a good solver or preconditioner. In this paper, an efficient preconditioning technique is combined into the inner-outer flexible generalized minimal residual (FGMRES) solver to improve the property of EFIE [9-12].

The remainder of this paper is organized as follows. Section II demonstrates the formulation of EFIE and the theory of MDA-SVD briefly. Section III describes the theory and implementation of MDA-SVD-MLFMA in more details and gives a brief introduction to the inner-outer flexible generalized minimal residual (FGMRES) method. Numerical experiments are presented to demonstrate the efficiency of this proposed method in Section IV. Conclusions are provided in Section V comments.

II. THE THEORY OF MDA-SVD

A. The formulation of EFIE

In this paper, the electric field integral equation (EFIE) is used to analyze electromagnetic scattering problems. The EFIE formulation of electromagnetic wave scattering problems using planar Rao-Wilton-Glisson (RWG) basis functions for surface modeling is presented in [2]. The resulting linear systems from EFIE formulation after Galerkin's testing are briefly outlined as follows

$$\sum_{n=1}^{N} Z_{mn} I_n = V_m, \qquad \mathrm{m} = 1, 2, \ldots, N \tag{1}$$

where

$$\begin{aligned} Z_{mn} = \frac{jk}{4\pi} (& \iint_s \Lambda_m(\mathbf{r}) \cdot \iint_s G(\mathbf{r},\mathbf{r}')\Lambda_n(\mathbf{r}')dS'dS \\ & - \frac{1}{k^2} \iint_s \nabla\cdot\Lambda_m(\mathbf{r}) \cdot \iint_s G(\mathbf{r},\mathbf{r}')\nabla\cdot\Lambda_n(\mathbf{r}')dS'dS) \end{aligned} \tag{2}$$

and

$$V_m = \iint_s \Lambda_m(\mathbf{r}) \cdot \left(\frac{1}{\eta} \mathbf{E}^i(\mathbf{r}) \right) dS ,$$

$$G(\mathbf{r},\mathbf{r}') = \frac{e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}$$

Here, $G(\mathbf{r},\mathbf{r}')$ refers to the Green's function in free space and $\{I_n\}$ is the column vector containing the unknown coefficients of the surface current expansion with RWG basis functions. Also, as usual, $\mathbf{r}$ and $\mathbf{r}'$ denote the observation and source point locations. $\boldsymbol{E}^i(\mathbf{r})$ is the incident excitation plane wave, and η and k denote the free space impedance and wave number, respectively. N is the number of unknowns used to discretize the object.

Once the matrix equation (1) is solved, the expansion coefficients $\{I_n\}$ can be used to calculate the scattered field and RCS. In the following, we use Z to denote the coefficient matrix in equation (1), $I = \{I_n\}$ and $V = \{V_n\}$ for simplicity. Then, the EFIE matrix equation (1) can be symbolically rewritten as

$$ZI = V \tag{3}$$

To solve the above matrix equation by an iterative method, the matrix-vector products are needed at each iteration. Traditionally, a matrix-vector production requires the operation cost $O(N^2)$.

B. The theory of MDA-SVD

The multilevel matrix decomposition algorithm (MLMDA) was originally proposed for two-dimensional geometries in [13], which utilizes the idea of equivalent point sources. The extension of this algorithm is presented in [14-19] for analyzing arbitrary three-dimensional geometries. However, it is efficient only for planar or piecewise planar objects and is inefficient for analyzing the general electrically large scatterer. MDA-SVD presented in [7-8] shows a better efficiency by recompressing the matrix of MDA using the SVD technique.

Consider, there exists two subdomains, the first one is an observation box i that contains m_1 basis functions; whereas, the second one is a source box j that contains m_2 test functions. When the two boxes are sufficiently separated, the impedance matrix associated with them can be expressed using low rank representations [20]. In MDA implementation, the impedance matrix which is gotten through the EFIE of two well-separated regions can be expressed as three small matrices

$$[Z_{mn}] = [U_{mr}][\omega_{rr}]^{-1}[V_{rn}] \tag{4}$$

where $[Z_{mn}]$ is the interaction matrix between observation and source subdomains, r denotes the number of equivalent RWG sources, which is much smaller than n and m. Therefore, the matrix-vector product operation of the three matrices is much smaller than the operation of the direct multiplication [16].

Since the matrices $[U_{mr}]$ and $[V_{rn}]$ generated by MDA are usually not orthogonal, they may contain redundancies, which can be removed by the following algebraic recompression technique. This method may be regarded as the singular value decomposition optimized for rank-k matrices. Utilize QR and SVD to reorthonormalize $[U_{mr}]$ and $[V_{rn}]$ and the equation (4) can be obtained as

$$[Z_{mn}] = [U][\omega][V]^T \tag{5}$$

where $[U]$ and $[V]$ are both orthogonal. These techniques can reduce the required amount of storage of MDA, while the asymptotic complexity of the approximation remains the same.

MDA-SVD is one of the most popular methods for analyzing three-dimensional electromagnetic problems, but the far-field matrix assembly time of MDA-SVD is much longer than that of MLFMA. In order to reduce the matrix assembly time of MDA-SVD, a new hybrid method is proposed in the following.

III. FORMULATION

According to [3-6], MLFMA has been widely used to solve the scattering from the electrically-large size objects. When it is applied into analyzing the scattering from the multi-scale objects where dense discretization is necessary to capture geometric features accurately, the memory usage of MLFMA is very large. MDA-SVD is another popular technique for solving the three dimensional problems in [7-8], but the far-field matrix assembly time of MDA-SVD is much longer than that of MLFMA. In this section, a hybrid method called MDA-SVD-MLFMA is proposed.

Take three dimensional problems into account, both MLFMA and MDA-SVD are based on the data structure of the octree. In Fig.1, the box enclosing the object is subdivided into smaller boxes at multiple levels, in the form of an octal tree. The largest boxes not touching each other are at level 2, while the smallest boxes are at level *L*. The subdivision process runs recursively until the finest level *L*.

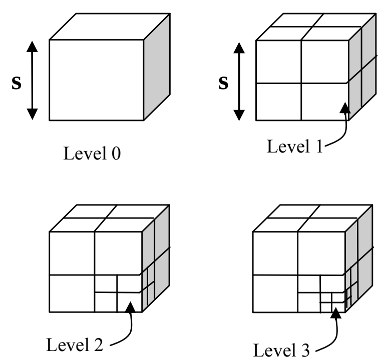

Fig. 1. The sketch of the octree structure.

It is well known that when the box size is less than 0.2 λ, MLFMA will suffer from the "low frequency breakdown" phenomenon. The relative error of MLFMA and MDA-SVD corresponding to the size of the finest level box is analyzed. The formulation of the relative error is described by

$$\text{Relative error } = \|T - M\| / \|M\|,$$

where *M* denotes the bistatic scattering from PEC sphere computed by Mie series while *T* is the bistatic scattering computed by MLFMA or MDA-SVD. The incident direction is $\theta_i = 0^\circ, \phi_i = 0^\circ$ and the scattered angles vary from 0° to 180° in azimuth direction when pitch angle is fixed at 0°. The number of unknowns is 15918 and the MDA-SVD truncating tolerance is 10^{-3} relative to the largest singular value. Figure 2 shows the relative error of MLFMA will increase greatly when the size of the finest level box is less than 0.2 λ. It can be seen that MDA-SVD has an acceptable precision even when the finest level box size is below 0.2 λ.

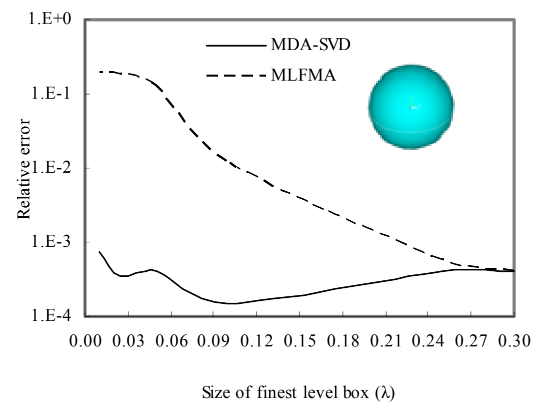

Fig. 2. Relative error of MLFMA and MDA-SVD for a sphere corresponding to the size of finest level box for bistatic scattering.

When the multi-scale problems are analyzed, MDA-SVD is adopted for the level with the box size smaller than 0.2 λ while MLFMA is adopted for other cases. The details of the hybrid method are shown in algorithm I.

Algorithm I: MDA-SVD-MLFMA

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1) Grouping on the target to achieve multilevel structures with the largest level *L*. When the number of basis functions of the box is less

than or equal to the number of equivalent RWG sources of the box, the finest level L is gained. At each level, the sizes of the boxes are same.

2) For each level, determine which algorithm is applied according to the box size. MDA-SVD is adopted for the level with the box size smaller than 0.2 λ while MLFMA is adopted for other cases. L_{MLFMA} is the level beginning of the MLFMA.

3) Calculation of the impedance matrix

   a) MoM is used to calculate the near interaction impedance matrix.

   b) From $l = L : L_{\text{MLFMA}}+1$
      MDA-SVD is applied to calculate the impedance matrix
      End

   c) From $l = L_{\text{MLFMA}}$: 2
      MLFMA is used to calculate the impedance matrix
      End

4) Iterative solution of the matrix equation

   d) The direct matrix-vector production is used to the near interaction impedance matrix.

   e) From $l = L : L_{\text{MLFMA}}+1$
      MDA-SVD is applied to speed up matrix-vector production
      End

   f) From $l = L_{\text{MLFMA}}$: 2
      MLFMA is used to speed up matrix-vector production
      End

This new method takes advantages of the virtues of both MLFMA and MDA-SVD, which uses MLFMA to reduce the matrix assembly time of MDA-SVD and utilizes MDA-SVD to alleviate the near field burden of MLFMA. The efficiency of the method is demonstrated by the numerical results.

In this paper, the FGMRES is used as the iterative solver for the EFIE to further accelerate the convergence [9-12]. Consider the iterative solution of equations of the form $Ax=b$. The GMRES algorithm with right preconditioning solves the modified system $AM^{-1}(Mx)=b$, where the preconditioner M is constant. However, in FGMRES, the preconditioner is allowed to vary from one step to another in the outer iteration. We have GMRES for the inner iterations whose preconditioner is chosen as the near interaction of MDA-SVD-MLFMA.

IV. NUMERICAL RESULTS

To validate and demonstrate the accuracy and efficiency of the proposed MDA-SVD-MLFMA, some numerical results are presented in this section. All the computations are carried out on a personal computer with 1.86 GHz CPU and 1.96GB RAM in single precision and the MDA-SVD truncating tolerance is 10^{-3} relative to the largest singular value. The restart number of GMRES is set to be 30 and the stop precision for restarted GMRES is denoted to be 10^{-3}. Both the inner and outer restart numbers of FGMRES are 30. The stop precision for the inner and outer iteration in the FGMRES algorithm is 10^{-2} and 10^{-3}, respectively. The normalized RCS is defined as

$$\frac{RCS(\theta,\phi)}{\lambda^2} = \frac{1}{\pi}\lim_{r\to\infty}\left\{(kr)^2\frac{\left|E^s(r,\theta,\phi)\right|^2}{\left|E^i(r,\theta,\phi)\right|^2}\right\} , \qquad (6)$$

for any direction (θ,ϕ), where E^s and E^i represent the scattered and incident electric fields.

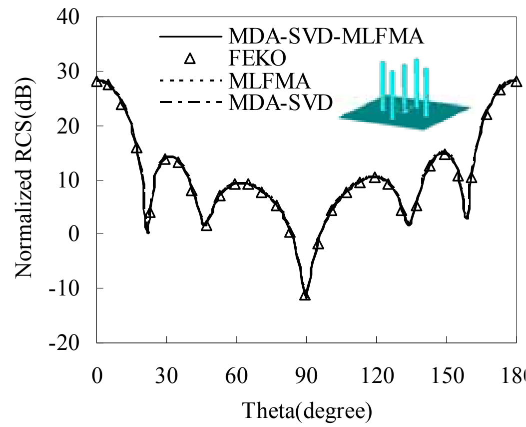
A. The plane-cylinder geometry

The multi-scale examples are analyzed in the following. The first multi-scale example is plane-cylinder geometry. The edge length of the square plane is 4 m, the radius of the small column is 0.1 m, and the height of small column is

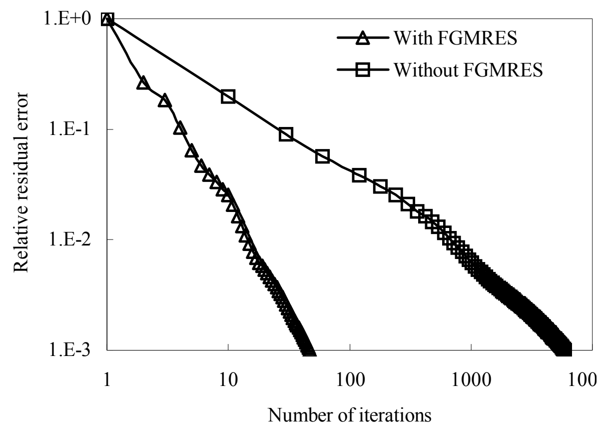
2 m. The rotation axis is *z*-axis. Here, the small column is densely discretized in comparison to the plane part of the structure. The incident and observed angles are ($\theta_i = 0^\circ, \phi_i = 0^\circ$) and ($0^\circ \le \phi_s \le 180^\circ, \theta_s = 90^\circ$), respectively. The size of the lowest-level box of the MDA-SVD-MLFMA is 0.16 λ , while the size of the lowest-level box of the MLFMA is 0.33 λ . Figure 3(a) shows that the result of MDA-SVD-MLFMA agrees very well with the FEKO [21].

Table 1 summarizes the matrix assembly time and the memory storages of the MDA-SVD-MLFMA, MDA-SVD, and MLFMA. "MVP time" in the table indicates the time of one matrix-vector production. It can be observed that the matrix assembly time of the MDA-SVD-MLFMA is half less than that of MDA-SVD and is, also, less than that of MLFMA. The total memory consumption of MDA-SVD-MLFMA is half less than that of MLFMA and is less than that of MDA-SVD. The MVP time of MDA-SVD-MLFMA is, also, much less than that of either MLFMA or MDA-SVD.

Figure 3(b) gives the convergence history curves of MDA-SVD-MLFMA solved with GMRES and FGMRES. In this numerical experiment, GMRES requires 6190 s with 5896 iterative steps, while FGMRES requires only 667 s with 46 outer iterative steps. The solving time of GMRES is 9 times longer than that of FGMRES in this example.



(a)



(b)

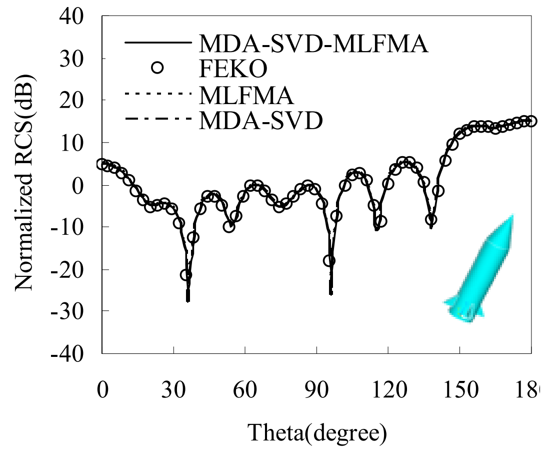
Fig. 3. (a) Bistatic scattering cross section of plane-cylinder geometry. (b) Convergence histories of MDA-SVD-MLFMA solved with GMRES and FGMRES.

Table 1: The total memory, the matrix assembly time, and one matrix-vector multiplication time of MLFMA, MDA-SVD, and MDA-SVD-MLFMA of plane-cylinder geometry

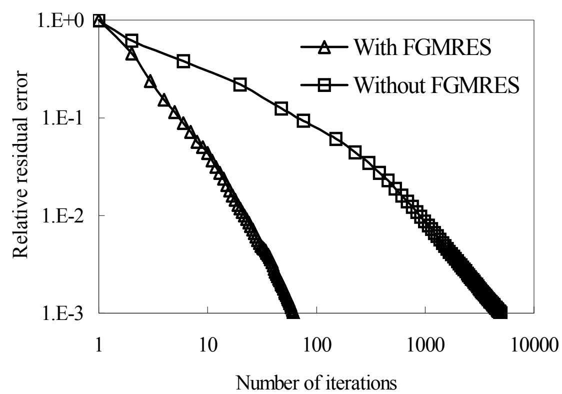
Frequency (MHz)	Unknowns	Algorithms	Matrix assembly time (s)	Memory (MB)	MVP time (s)
200	28756	MDA-SVD-MLFMA	400	458	1.05
		MDA-SVD	1101	666	1.53
		MLFMA	415	968	1.70

B. The missile geometry

The second multi-scale example is missile geometry. The height of the cylinder is 4.7 m, and the radius of the cylinder is 0.5 m. The rotation axis of missile geometry is *z*-axis. The incident and scattered angles are ($\theta_i = 0^\circ, \phi_i = 0^\circ$) and ($0^\circ \le \phi_s \le 180^\circ, \theta_s = 90^\circ$), respectively. The size of the lowest-level box of the MDA-SVD-MLFMA is 0.14 λ , while the size of the lowest-level box of MLFMA is 0.29 λ . Table 2 shows the matrix assembly time, the memory storages, and MVP time of MDA-SVD-MLFMA, MDA-SVD, and MLFMA. Again, the matrix assembly time of MDA-SVD-MLFMA is half less than that of MDA-SVD and is, also, less than that of MLFMA. The memory usage of MDA-SVD-MLFMA is half less than that of MLFMA and is less than that of MDA-SVD. The MVP time of MDA-SVD-MLFMA is also much less than that of MLFMA and MDA-SVD. The bistatic RCS by use of MDA-SVD-MLFMA is shown in Fig. 4(a), and is agreed well with FEKO. The convergence curves are plotted for MDA-SVD-MLFMA solved with GMRES and FGMRES in Fig. 4(b). GMRES requires 7964 s with 4958 iterative steps, while FGMRES requires only 1095 s with 65 outer iterative steps. The solving time of GMRES is 7 times longer than that of FGMRES, in this example.



(a)



(b)

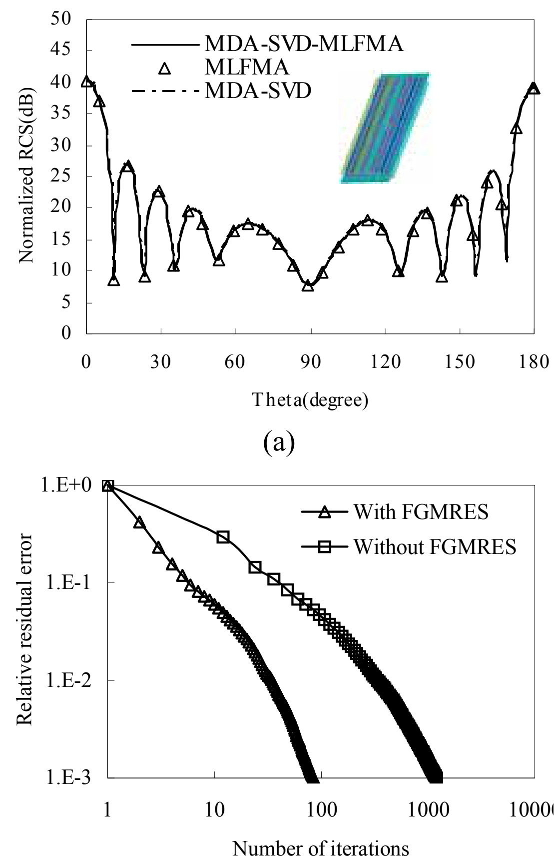
Fig. 4. (a) Bistatic scattering cross section of missile geometry. (b) Convergence histories of MDA-SVD-MLFMA solved with GMRES and FGMRES.

Table 2: The total memory, the matrix assembly time, and one matrix-vector multiplication time of MLFMA, MDA-SVD, and MDA-SVD-MLFMA of missile geometry

Frequency (MHz)	Unknowns	Algorithms	Matrix assembly time (s)	Memory (MB)	MVP time (s)
300	38145	MDA-SVD-MLFMA	600	638	1.60
		MDA-SVD	1757	949	2.01
		MLFMA	643	1272	2.22

C. The VIAS geometry

The third multi-scale example is the VIAS geometry [22]. The geometry fits within a cuboid with aspect radio $6:5:0.5$ and the maximum dimension is 6λ at 300 MHz. The incident and scattered angles are ($\theta_i = 0^\circ, \phi_i = 0^\circ$) and ($0^\circ \le \phi_s \le 180^\circ, \theta_s = 90^\circ$), respectively. The size of the lowest-level box of the MDA-SVD-MLFMA is 0.18 λ , while the size of the lowest-level box of MLFMA is 0.37 λ . The matrix assembly time, the memory storages and MVP time of MDA-SVD-MLFMA, MDA-SVD, and MLFMA are shown in Tab. 3. The matrix assembly time of MDA-SVD-MLFMA is much less than that of MDA-SVD and is, also, less than that of MLFMA, while the memory usage of MDA-SVD-MLFMA is much less than that of MLFMA and is less than that of MDA-SVD. The bistatic RCS by use of MDA-SVD-MLFMA is shown in Fig. 5(a), and is agreed well with that of MLFMA and MDA-SVD. The convergence curves are plotted for MDA-SVD-MLFMA solved with GMRES and FGMRES in Fig. 5(b). GMRES requires 2403 s with 1190 iterative steps, while FGMRES requires only 411 s with 84 outer iterative steps. The solving time of GMRES is 5 times longer than that of FGMRES in this example.



(a)

(b)

Fig. 5. (a) Bistatic scattering cross section of VIAS geometry. (b) Convergence histories of MDA-SVD-MLFMA solved with GMRES and FGMRES.

Table 3: The total memory, the matrix assembly time, and one matrix-vector multiplication time of MLFMA, MDA-SVD, and MDA-SVD-MLFMA of VIAS geometry

Frequency (MHz)	Unknowns	Algorithms	Matrix assembly time (s)	Memory (MB)	MVP time (s)
300	61099	MDA-SVD-MLFMA	811	661	2.02
		MDA-SVD	2631	922	2.43
		MLFMA	827	1284	2.90

In summary, the memory of MDA-SVD-MLFMA is much less than that of MLFMA and the matrix assembly time of MDA-SVD-MLFMA is much less than that of MDA-SVD. The matrix-vector multiplication of MDA-SVD-MLFMA is more efficient than that of either MLFMA or MDA-SVD. It can be applied to the monostatic RCS calculation of the complex object in future. The MDA-SVD-MLFMA is much more efficient than either MLFMA or MDA-SVD for the multi-scale problems. It is observed that the convergence rate of GMRES is remarkably accelerated by the application of FGMRES algorithm.

V. CONCLUSIONS

In this paper, a new efficient hybrid method named MDA-SVD-MLFMA is proposed. The MDA-SVD-MLFMA takes advantage of the virtues of both MLFMA and MDA-SVD, which uses MLFMA to reduce the matrix assembly time of MDA-SVD and utilizes MDA-SVD to alleviate the near field burden of MLFMA. The numerical results demonstrate that the memory of MDA-SVD-MLFMA is much less than that of MLFMA and the matrix assembly time of MDA-SVD-MLFMA is much less than that of MDA-SVD. It is observed that the convergence rate of GMRES is remarkably accelerated by the application of FGMRES algorithm. MDA-SVD-MLFMA combined with FGMRES is very efficient for analyzing the multi-scale problems.

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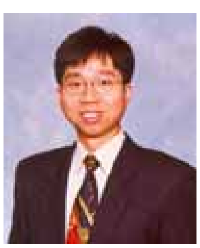

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