The Spherical Harmonic Interface Procedure for MM and UTD Codes

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Abstract – This paper describes the use of a spherical harmonic expansion as an efficient interface between a low frequency method of moments (MM) code and a high frequency uniform geometrical theory of diffraction (UTD) code. It is shown that the method can save significant CPU time in the UTD code provided that the number of MM current filaments per cubic wavelength is large.

I. INTRODUCTION

In computational electromagnetics, when a problem involves both an electrically small part and an electrically large part, the standard approach is a hybrid solution [7, 12, 5, 11]. For example, consider the problem of an antenna radiating in the presence of an electrically large structure such as a ship or a building at X band. Typically, a low frequency method of moments (MM) code is used to find the currents on the antenna in free space, or in the presence of a small part of the structure closest to the antenna. These antenna currents are then input to a high frequency uniform geometrical theory of diffraction (UTD) code which determines their fields in the presence of the electrically large support structure.

The purpose of this paper is to describe the spherical harmonic interface procedure (SHIP) [9] in which a spherical harmonic expansion [10] of the free space antenna fields is used as an interface to the UTD code, rather than the MM antenna currents. The primary advantage of the SHIP is a result of the fact that in UTD ray tracing codes, the CPU time is proportional to the number of ray origins from which rays must be traced. Since for a complex antenna or large array, there could be thousands of MM currents, the UTD code would have to trace rays from thousands of origins. By contrast, a spherical harmonic expansion of the antenna fields has a single origin, and thus the UTD code need only trace rays from a single point. The CPU time for the SHIP is proportional to the number of spherical harmonics needed to accurately represent the antenna fields, which in turn is proportional to the electrical size of the antenna.

A secondary advantage of the SHIP is that the UTD code can be written independent of the details of the low frequency code. The low frequency code can be based upon the MM, FEM, FDTD, etc., and can employ any basis functions. Providing that the UTD code is given the coefficients in the spherical harmonic expansion of the antenna fields, these details are not relevant to the UTD code.

Section II will begin with a description of the basic single origin or cell SHIP. It is then pointed out that there are two instances when it is either necessary or desirable to employ a multiple origin or multiple cell SHIP. First, the use of the spherical harmonic expansion requires the SHIP cells to be in the far zone of points of diffraction on the scattering structure. If an electrically large SHIP cell is in the near zone of the scatterer, then it must be split into smaller cells so that the points of diffraction on the scattering body are in the far zone of each cell. Second, it will be shown that it is possible to reduce the CPU time of the UTD code by employing a multiple cell SHIP. The paper will conclude with examples illustrating the reduced CPU time for the SHIP versus the standard MM current approach. For all examples, the MM code will be *The Electromagnetic Surface Patch Code: Version V* (ESP5) [8], and the UTD code will be the *NEC Basic Scattering Code* (NEC-BSC) [6]. All UTD code CPU times will be for a far zone pattern at 360 angles. All CPU times are for a PC with a 1.7 GHz Intel(T) Pentium M processor.

II. DESCRIPTION OF THE SHIP

II. A. Single Cell SHIP

A hybrid MM/UTD solution for an antenna radiating in the presence of a large structure begins with the MM code determining the current on the antenna, and then the UTD code finds the fields produced by these currents in the presence of the large structure. Rather than have the UTD code trace rays from each MM current segment or current filament, one can construct a spherical wave expansion of the fields of the free space fields of the MM currents. In the far zone of a sphere of radius r_1 enclosing the currents, the spherical wave expansion for each component of the electric field is of the form

$$E(r,\theta,\varphi) = A\sum_{m=0}^{N_{\max 1}}\sum_{n=m}^{N_{\max 1}}[a_{mn}Y_{mn}^{e} + b_{mn}Y_{mn}^{o}]\frac{e^{-jkr}}{r}. \quad (1)$$

For simplicity we have assumed the enclosing sphere is centered at the coordinate origin, the subscript 1 emphasizes that this is for a single origin expansion, and the reader is referred to [1] for a detailed description of the various terms. To obtain reasonable accuracy from equation (1), the number of terms which must be kept in the summations is approximately [2-4]

$$N_{\max 1} = kr_1 + 3\ln(kr_1 + \pi). \quad (2)$$

In the SHIP, the MM code provides the UTD code with the a_{mn} and b_{mn} coefficients of equation (1), and thus the UTD code need only trace rays from a single coordinate origin. It is also is completely divorced from the details of the MM (or other) solution. The CPU time of the UTD code will be dependent upon the number of harmonics which must be summed in equation(1),

$$N_{H1} = N_{\max 1}^2 \approx (kr_1)^2 \quad \text{if } kr_1 \gg 1. \quad (3)$$

For the NEC-BSC code, Figure 1 shows a log-log plot of the CPU time versus N_{H1} for the scattering body being free space (i.e., no scatterer) and for the wedge of Figure 2. Free space and the wedge represent extremes in terms of the complexity of the ray trace. Noting that for large N_{H1}, these two extremes produce straight lines with essentially the same slope, the single cell CPU time must be of the form

$$T_1 = CN_{H1}^{\beta} = C(kr_1)^{2\beta} \quad (4)$$

where from the slope of the lines $\beta \approx 1.45$. C is a constant dependent upon the complexity of the ray trace, and a reasonable fit to the data of Figure 1 is

$$C_{FSp} \approx 6.82\ \mu\text{sec}, \quad C_{Wedge} \approx 85.5\ \mu\text{sec}. \quad (5)$$

Note that one should always choose the coordinate origin for the spherical wave expansion near the center of the MM currents, since this will minimize r_1 and thus the number of required harmonics, N_{H1}.

II. B. Multiple Cell SHIP

By a P cell SHIP it is meant that the single cell of radius r_1 is segmented into P smaller cells, and the

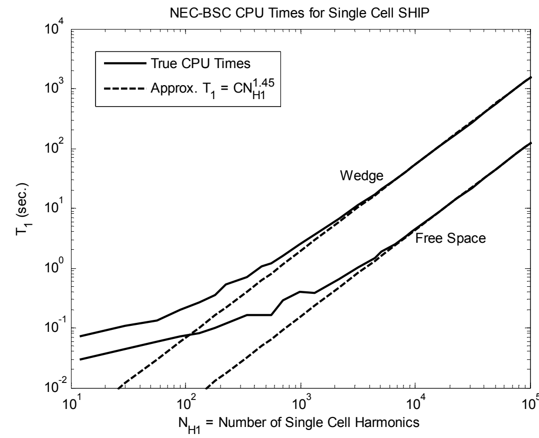

Fig. 1. CPU times for the single cell SHIP in free space and for the wedge.

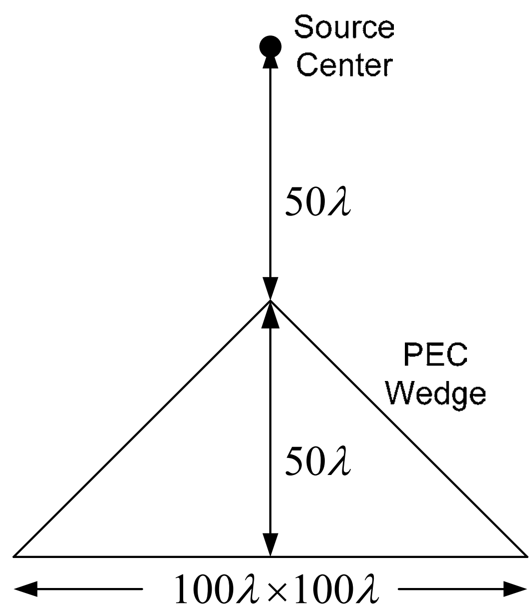

Fig. 2. Geometry for a PEC square base wedge.

total fields are expressed as the sum of the spherical wave expansions for the fields of the currents located in each of the P cells. If the single cell of radius r_1 is not in the far zone of a point of diffraction on the scatterer, then it must be segmented into smaller cells which are in the far zone. The cells may be in the near zone of a point of reflection since the NEC-BSC (and we assume most UTD codes) treat this via image theory. The P cell SHIP tends to increase the CPU time since the UTD code must trace rays from P origins, however, each ray trace is faster since the cells are smaller and thus require fewer harmonics. As shown below, it is possible to reduce the overall UTD code CPU time by employing a multiple cell SHIP.

Referring to Figure 3, for simplicity we will assume that the radiating body can be classified, based upon its overall dimensions, as one of the following:

- 1D Body: 1 large and 2 small dimensions (linear array),
- 2D Body: 2 large and 1 small dimensions (square array),
- 3D Body: 3 large dimensions (cubic array).

In this case, the cell radius for P cells is simply related to that for the single cell by

$$r_P = \frac{r_1}{P^{\alpha}} \tag{6}$$

where $\alpha_{1D} = 1$, $\alpha_{2D} = 1/2$, and $\alpha_{3D} = 1/3$ for the three cases.

-The number of harmonics in each of the P cells is

$$N_{HP} = N_{\max P}^2 = (kr_P)^2 = \left(k\frac{r_1}{P^{\alpha}}\right)^2 = \frac{N_{H1}}{P^{2\alpha}}. \tag{7}$$

Since the UTD code must trace rays from each of the P origins, the total CPU time for the P cell SHIP is

$$T_P = P[CN_{HP}^{\beta}] = PC\left(\frac{N_{H1}}{P^{2\alpha}}\right)^{\beta} = P^{1-2\alpha\beta}T_1 = F_P T_1 \tag{8}$$

where

$$F_P = P^{1-2\alpha\beta} \tag{9}$$

is the factor to convert the single cell to the P cell CPU time. For the three cases above

- 1D Body: $F_P = P^{1-2(1)(1.45)} = P^{-1.9}$,
- 2D Body: $F_P = P^{1-2(1/2)(1.45)} = P^{-0.45}$,
- 3D Body: $F_P = P^{1-2(1/3)(1.45)} = P^{+0.033}$.

It follows that for 1D and 2D bodies, one should use as many cells as possible, while for 3D bodies there is always a slight disadvantage to segmentation.

There are three important caveats to the above statement. First, one must always segment so that the SHIP cells are small enough that they are in the far zone of the points of diffraction on the scattering body. Second, equation (8) will accurately predict T_P only if $N_{HP} = N_{H1}/P^{2\alpha}$ is large enough to be on the linear portion of Figure 1. For smaller values of N_{HP}, equation (8) should be considered as qualitative only. This problem could be removed by making the slope, β a function of the number of the number of harmonics in a single cell; however, for simplicity it was not done here. Finally, it is assumed that as the radiating body is segmented, each smaller cell contains at least 1 MM current, so that rays must be traced from each SHIP cell.

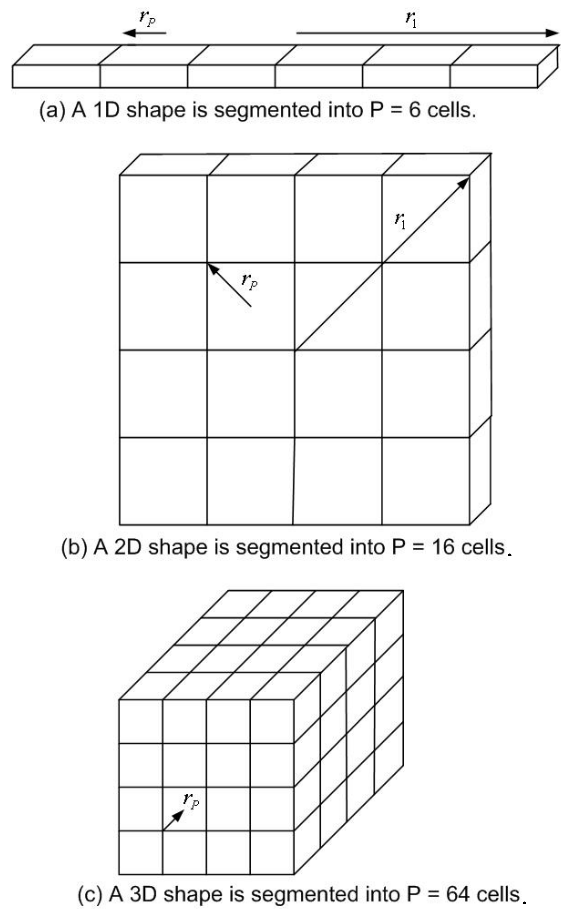

Fig. 2. A 1D, 2D, and 3D source is segmented in P cells.

For a 1D body radiating in the presence of the wedge, Figure 4 shows NEC-BSC CPU times for a P = 1, 2, and 4 cell SHIP versus N_{H1} = the number of single cell harmonics. According to equation (7), the number

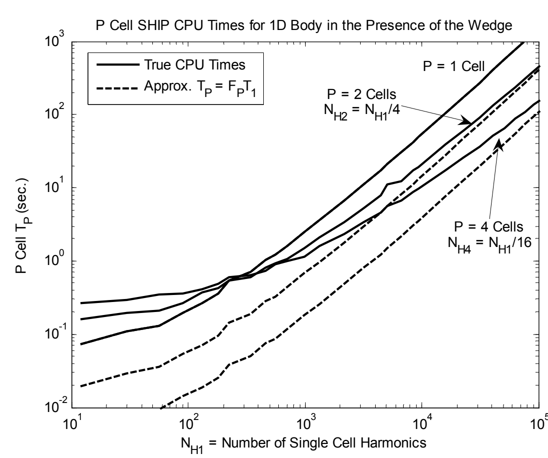

Fig. 3. CPU times for the 1D P cell SHIP in the presence of the wedge.

of harmonics *in each cell* of the P cell SHIP is $N_{H2} = N_{H1}/4$ and $N_{H4} = N_{H1}/16$. The solid lines are the actual NEC-BSC times, while the dashed lines are obtained by applying the F_P factor of equation (9). The reason that the approximation for the P = 2 curve is better than that for P = 4, is that $N_{H2} = 4N_{H4}$, and thus is more on the linear portion of the curves in Figure 1.

II. C. SHIP vs. Standard MM Current CPU Times

The CPU time for the standard approach, in which the UTD code superimposes the field of each MM current filament, is $T_{MMF} = DN_F$, where N_F is the number of MM filaments, and D is a constant dependent upon the complexity of the ray trace. For the NEC-BSC code, and the wedge of Figure 2,

$$D_{Wedge} \approx 87.8 \text{ msec.} \quad (10)$$

For the wedge, the ratio of the CPU time for the standard MM filament approach to that for the P cell SHIP is

$$R = \frac{T_{MMF}}{T_P} = \frac{D_{Wedge} N_F}{F_P (C_{Wedge} N_{H1}^{\beta})} \approx \frac{5}{F_P} \frac{N_F}{(r_1/\lambda)^{2.9}} . \quad (11)$$

This indicates that the P cell SHIP will be faster than the standard filament approach if

$$\frac{N_F}{F_P (r_1/\lambda)^{2.9}} \approx \frac{N_F}{F_P (r_1/\lambda)^{3}} > 0.2 , \quad (12)$$

i.e., if the number of MM filaments per cubic wavelength is large.

II. D. SHIP Examples

This section will present two examples illustrating the benefits of the SHIP. The first example will be a $50\lambda \times 50\lambda$ square array of dipoles over the PEC wedge of Figure 2. By making a series of runs with increasing density of dipoles within the fixed $50\lambda \times 50\lambda$ square, Figure 5 shows the UTD code CPU time versus the number of MM filaments. Since the electrical size of the source is fixed, the CPU times for the SHIP are independent of the number of MM filaments. Note that the P = 1 or 4 cell SHIP is faster than the standard filament approach when $N_f > 6500$ or 4200 filaments, respectively. Using $r_1 = 25\sqrt{2}\lambda$, equation (12) predicts that the P = 1 or 4 cell 2D SHIP will be faster than the standard filament approach when N_f > 6200 or 3300 filaments, respectively. The decrease in accuracy for the $P = 4$ cell SHIP is a result of not being on the linear portion of Figure 1.

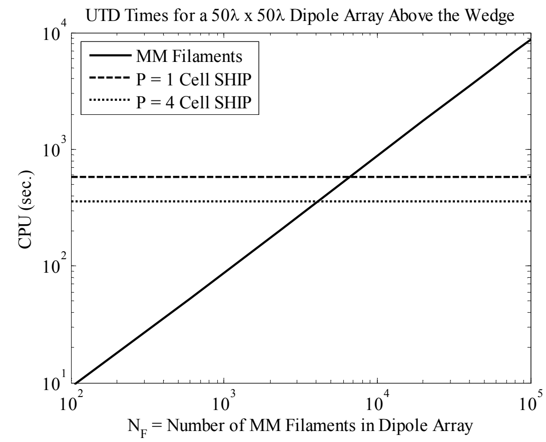

Fig. 4. UTD CPU times for a dipole array above the wedge.

Figure 6 shows a UTD plate model of a ship with a 30×30 array of $\lambda/2$ dipoles in a $15\lambda \times 15\lambda$ square. Using the standard filament approach, the UTD CPU time to compute an azimuth pattern was 351 min., but required only 261 sec. for a $P = 16$ cell 2D SHIP. This represents a reduction in CPU time by a factor of about 80. The CPU time to compute the a_{mn} and b_{mn} coefficients of equation (1) was only 61 sec.

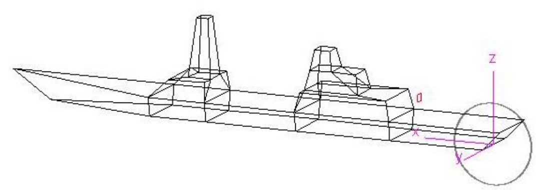

Fig. 5. A UTD model of a ship with a 30 x 30 array of dipoles in a $15\lambda \times 15\lambda$ square.

III. SUMMARY

This paper has described the use of a spherical harmonic expansion to reduce the UTD code CPU times in a hybrid MM/UTD solution. The advantage of the SHIP is that it reduces the number of origins from which the UTD code must trace rays. The SHIP is shown to be effective in reducing the UTD code CPU time when the number of MM filaments per cubic wavelength is large.

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