Simple Finite Element Solutions for Eddy Current Losses in Pipe-Type Cables

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Abstract

This paper outlines a simple finite element approach for computing eddy current losses in pipe-type cables. Essentially, what is solved for is the eddy current loss in a metallic shell that encompasses one or more power frequency currents, flowing parallel to the longitudinal axis of the shell. The traditional cross sectional model for this type of problem involves an open boundary, which can present difficulties for conventional finite element methods, although some success has recently been reported with asymptotic boundary conditions [1]. For many practical cases, the simplest way to bypass this obstacle is to impose an approximate boundary condition, $\mathbf{H} = 0$, at the outer surface of the metal shell. It is demonstrated in this paper that this approximate boundary condition is quite accurate if the shell is, as a rule of thumb, at least three skin depths thick.

Introduction

In the field of power engineering, much attention has been devoted to the problem of computing eddy current losses in various devices associated with the transmission and distribution of electrical power. One particular area of research is devoted to the calculation of eddy current losses in pipe-type cable and similar structures [2]. Basically, pipe-type cable consists of a set of three power cables situated inside a metal pipe which is often filled with insulating oil. The alternating currents in the cables produce an electromagnetic field which, in turn, induces eddy currents in the pipe. This pipe and cable arrangement is primarily used as a means of transmission in underground power systems. The losses in the metal pipe due to eddy currents are an important consideration in pipe-type cable design.

Much of the research into eddy current losses in pipe-type cable is also applicable to certain electrical engineering problems that arise in the relatively new technology of in-situ electrical heating of heavy oil [3]. One aspect of this technology involves transmitting electrical power down a well which has been drilled into an underground, oil-bearing formation. Power cables in the well conduct a low frequency alternating current from a power source at the surface to electrodes embedded in the formation. The well is usually lined with a steel pipe, called a wellbore casing, to maintain its structural integrity. The alternating currents carried by the power cable(s) inside the well induce eddy currents in the surrounding steel casing [4]. This is essentially the same mechanism for eddy current loss that occurs in pipe-type cables.

One of the more common approaches for calculating eddy current losses is to model a cross section of the pipe and cable(s) and to use classical electromagnetic theory to solve for the eddy current density or electric field in the pipe [5 to 13]. An example of this cross sectional model for a system composed of a metallic pipe and single power cable is illustrated in Figure 1.

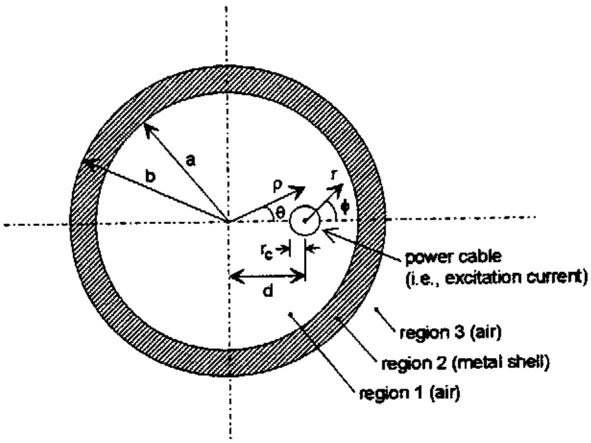

Figure 1. Cross sectional model of a pipe and cable eddy current problem.

In Figure 1, a power cable, carrying a power frequency alternating current, is situated inside a homogeneous, cylindrical metal shell. In some models, the power cable is replaced by a thin conductor or filament which carries the same current as the cable. The homogeneous metal shell (region 2) models the pipe. In the analysis of the eddy current problem in two dimensions (as opposed to analysis in three dimensions), both the cable and shell are assumed to be infinitely long.

The alternating current in the cable is responsible for exciting the electromagnetic fields in the model. The current produces an electric field in the z direction (i.e., perpendicular to the page) and a magnetic field with components in the r and ϕ directions (or alternately, in the ρ and θ directions). These fields exist in all three regions denoted in Figure 1.

Often, the eddy current problem modeled by Figure 1 is framed as a boundary value problem. Over the years, various analytical solutions for the eddy current density in the metal shell have been published [6, 7, 8, 12, 13]. One frequently referenced analytical solution is that of Kawasaki, Inami and Ishikawa [13]. Their formula for eddy current density in the metal shell is valid for a *circular* shell of any thickness that encompasses an arbitrary arrangement of power cables (Kawasaki et al. model each cable with a filament that carries the same current as the cable). In their formulation, however, the power cable currents must constitute a balanced system, (i.e., the sum of all the cable currents must equal zero).

In this paper, the authors approach the boundary value problem using a new strategy which combines a basic *finite element method* with the judicious application of a homogeneous boundary condition at the outer surface of the metal shell. This leads to a solution for the eddy current density induced in the metal shell due to the current in a single power cable. For situations where the metal shell encompasses more than one power cable, a current density solution, attributable to the current in a single cable, is obtained for each cable. The individual solutions are then superimposed to obtain the current density in the original, multi-cable problem. However, unlike the formula derived by Kawasaki et al., the finite element approach presented in this paper is *not* restricted to situations where the power cables carry a balanced system of currents.

The finite element based approach to solving the boundary value problem has several advantages over the analytical approach. Chief among these benefits is the ability of the finite element method to solve problems that involve complex geometries. Although the scope of this paper is restricted to solutions for eddy currents in circular shells, the finite element approach is equally adept at solving for eddy current distributions in shells of any shape (triangular, rectangular, etc.).

Another advantage of the finite element approach over the analytic approach is the relative ease with which inhomogeneous problems can be solved. For example, finding an analytic solution for the electric fields in Figure 1 would become exceedingly difficult if one or more of the three regions were azimuthally or radially inhomogeneous. However, the degree of difficulty in obtaining a finite element solution for the same problem would be essentially unchanged by the presence of the inhomogeneity.

The finite element approach also has an advantage over the *method of moments* numerical approach used by Kriezis and Cangellaris [7] to solve the problem illustrated in Figure 1. The advantage comes into play when the relative permeability of the metal shell is greater than one (i.e., when the shell is composed of some ferromagnetic material). This situation poses no difficulty for the finite element method. However, a *proper* method of moments solution for eddy current density in a ferromagnetic shell involves a far more complex formulation than that used by Kriezis and Cangellaris to compute the current density in a non-magnetic shell.

There is, however, one major drawback to the application of the finite element method to this boundary value problem. Region 3, the region surrounding the shell, extends to infinity in the radial direction and therefore this problem has an open boundary. Unfortunately, the basic finite element method is unable to solve boundary value problems that have one or more open boundaries. Although there are various hybrid methods that can solve such problems, the simplest tactic is to apply an approximate, homogeneous boundary condition along some arbitrary boundary surrounding the shell. The intent is that the introduction of the approximate boundary condition should not *significantly* affect that portion of the finite element solution in the shell. Other common boundary conditions used in conjunction with artificially bounded solution spaces include absorbing [14] and radiation boundary conditions [14, 15].

For the homogeneous boundary condition, the solution space should extend far enough into region 3 to justify setting the electric or magnetic field to zero on the boundary. The main drawback to using this distant boundary, illustrated in Figure 2, is the extra elements required to model that portion of region 3 incorporated into the solution space.

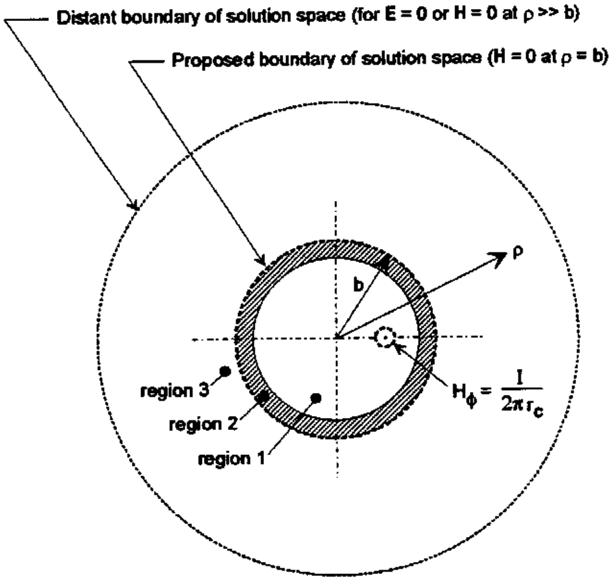

Figure 2. Boundaries of the solution space for the eddy current problem modeled in Figure 1.

The authors of this paper have established that, in many practical cases, the solution space boundary need extend no further than the outer surface of the metal shell. More precisely, one can impose the simple, Dirichlet boundary condition $\mathbf{H}=0$ along the outer surface of the shell without significantly affecting the accuracy of the finite element solution within the shell itself (note that $\mathbf{H}$ is the total magnetic field intensity, $\mathbf{H}=H_r\hat{\mathbf{a}}_r+H_\phi\hat{\mathbf{a}}_\phi$). The obvious advantage of this new solution space boundary (shown in Figure 2) is the reduction in the number of elements required to solve the problem.

To demonstrate that the finite element method, in conjunction with the $\mathbf{H}=0$ boundary condition applied along the outer surface of the shell, will yield a correct solution for the eddy current loss in the shell, the *analytical solution* published by Kawasaki et al. is used for comparison. From the numerical examples presented in this paper, it will be seen that the finite element solutions compare favorably with analytic solutions.

The finite element solutions to be presented in this paper were calculated using software based on a computer program called UNAFEM, written by W. J. Denkmann, in collaboration with D. S. Burnett [16]. The code for UNAFEM is presented in Burnett's book entitled *Finite Element Analysis: From Concepts to Applications*.

The Governing Partial Differential Equation and Boundary Conditions

In preparation for the finite element method, a governing partial differential equation is constructed for the boundary value problem modeled in Figure 1. Beginning with Maxwell's equations in phasor form,

$$\nabla\times\mathbf{E}=-j\omega\mu\mathbf{H}, \tag{1}$$

and

$$\nabla\times\mathbf{H}=(\sigma+j\omega\varepsilon)\mathbf{E}. \tag{2}$$

Substituting (1) into (2) yields

$$\nabla\times\left(\frac{-1}{j\omega\mu}(\nabla\times\mathbf{E})\right)-(\sigma+j\omega\varepsilon)\mathbf{E}=0. \tag{3}$$

Since power losses in cable systems are generally computed for electrically short cable lengths, wavelength effects can be neglected. Consequently, the only existing field components, E_z, H_ϕ and H_r, are assumed to exhibit no variation with respect to the z coordinate. Thus, equation (3) may be written more explicitly as

$$-\frac{\partial}{\partial r}\left(\alpha_r(r)\frac{\partial E_z}{\partial r}\right)-\frac{\partial}{\partial\phi}\left(\alpha_\phi(r)\frac{\partial E_z}{\partial\phi}\right)+\beta(r)E_z=0, \tag{4}$$

where

$$\alpha_r(r)=\frac{-r}{j\omega\mu}, \tag{5}$$

$$\alpha_\phi(r)=\frac{-1}{j\omega\mu}\cdot\frac{1}{r}, \tag{6}$$

and

$$\beta(r)=-r(\sigma+j\omega\varepsilon). \tag{7}$$

The boundary conditions for this problem are specified at the surface of the power cable in region 1 and, as discussed earlier, at the outer surface of the metal shell. At the surface of the power cable, $H_\phi=I/(2\pi r_c)$, where I is the total current in the cable and r_c is the radius of the power cable. At the outer surface of the shell (at $\rho = b$), the approximate boundary condition $\mathbf{H} = 0$ is imposed. The justification for this approximate boundary condition is discussed later in this paper.

The magnetic field boundary conditions are related to the Neumann boundary conditions on E_z by the following relations:

$$H_r = \frac{-1}{j\omega\mu} \cdot \frac{1}{r} \frac{\partial E_z}{\partial \phi}, \tag{8}$$

and

$$H_\phi = \frac{1}{j\omega\mu} \frac{\partial E_z}{\partial r}. \tag{9}$$

These two expressions can be derived from equation (2).

Synopsis of the Procedure for Obtaining the Finite Element Solution

By employing the Galerkin method of weighted residuals, along with the desired element trial solutions and appropriate shape functions, a system of equations can be assembled. The solution to this system of equations is the finite element solution for the electric field, E_z.

Terms corresponding to the Neumann boundary conditions on E_z arise naturally in the system of equations assembled from the element equations. The Neumann boundary condition terms may be substituted with the magnetic field boundary conditions, in accordance with equations (8) and (9).

A Discussion of the Approximate Boundary Condition, H = 0

As a "rule of thumb," the approximate boundary condition, $\mathbf{H} = 0$, is valid if the thickness of the metal shell, $b - a$, is at least 3 skin depths, where one skin depth equals $\sqrt{2/(\omega\mu_2\sigma_2)}$; μ_2 and σ_2 are the permeability and conductivity of the metal shell.

The rationale behind this rule is fairly obvious. The magnetic field (as well as the electric field) can be viewed in terms of inward and outward propagating, cylindrical waves. These waves, which emanate from the power cable, penetrate the lossy metal shell and attenuate as they propagate through the shell in the outward radial direction. For a shell 3 skin depths thick, an incident wave originating at the inner surface of the shell will have decayed by a factor of approximately e^3 by the time it reaches the shell's outer surface. Thus the magnitude of the magnetic field at the outer surface of the shell is negligibly small compared to the magnitude at the inner surface.

It should be noted, however, that this rule of thumb is highly conservative because it does not account for the effects of reflection at the air-metal interface at the outer surface of the shell. When the incident waves of the magnetic field reach the outer surface of the shell, the waves can, depending on the angle of incidence, undergo near total reflection. In many practical cases, the combined effects of attenuation and reflection will diminish the magnitude of $\mathbf{H}$ at the shell's outer surface to such an extent that the $\mathbf{H} = 0$ approximation is valid, *even when the shell thickness is significantly less than a single skin depth.*

For the problem illustrated in Figure 1, it is difficult to make a general statement about the effects of reflection because the cylindrical waves emanating from the power cable are obliquely incident to the surfaces of the shell. However, for a somewhat simplified problem where the power cable is situated *at the center of the shell*, the waves are normally incident to the surfaces of the shell. In this situation, the degree to which the magnetic field at the shell's outer surface is diminished by reflection and attenuation can be illustrated mathematically and physically.

For the case where the power cable is located at the center of the shell, the magnetic field in the shell, H_{ϕ_2}, can be expressed by the following formula, derived in reference [17]:

$$H_{\phi_2}(r) = \frac{I}{2\pi\sqrt{ar}} \frac{\left[e^{\alpha(b-r)}e^{j\beta(b-r)} - \Gamma e^{-\alpha(b-r)}e^{-j\beta(b-r)}\right]}{\left[e^{\alpha(b-a)}e^{j\beta(b-a)} - \Gamma e^{-\alpha(b-a)}e^{-j\beta(b-a)}\right]}, \tag{10}$$

where

$$\alpha = \beta = \sqrt{\frac{\omega\mu_2\sigma_2}{2}}, \tag{10a}$$

$$\Gamma = \frac{Z_3(b) - \eta_2}{Z_3(b) + \eta_2}; \tag{10b}$$

and where

$$\eta_2 = \sqrt{\frac{j\omega\mu_2}{\sigma_2}}, \tag{10c}$$

$$Z_3(b) = j\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{H_0^{(2)}\left(\omega\sqrt{\varepsilon_0\mu_0}b\right)}{H_1^{(2)}\left(\omega\sqrt{\varepsilon_0\mu_0}b\right)}. \qquad (10d)$$

Equation (10) satisfies Maxwell's equations and the *exact* boundary conditions at the outer surface of the shell (i.e., the tangential electric and magnetic fields are continuous across the air-metal interface; and the fields outside the shell, in region 3, are subject to the radiation boundary condition). The terms α and β are, respectively, attenuation and phase constants. The significance of Γ will be discussed shortly. η_2 is the *intrinsic impedance* of the metal shell. $Z_3(b)$ is the *wave impedance* in region 3 at $r = b$; it is derived from the definition $Z_3(b) = -E_Z(b)/H_\phi(b)$. The $H_n^{(2)}(z)$ term in the expression for $Z_3(b)$ is a Hankel function of the second kind of order n.

According to (10), the magnetic field at the outer edge of the shell, $r = b$, is

$$H_{\phi_2}(b) = \frac{I}{2\pi\sqrt{ab}}\frac{[1-\Gamma]}{\left[e^{\alpha(b-a)}e^{j\beta(b-a)} - \Gamma e^{-\alpha(b-a)}e^{-j\beta(b-a)}\right]}. \qquad (11)$$

The term Γ is key to understanding the effects of reflection at the outer surface of the shell. Γ is the ratio of the reflected portion of the *electric field* to the incident portion of the field (at the shell's outer surface). This term is completely analogous to the *reflection coefficient* for uniform plane waves as discussed in reference [18].

When $|Z_3(b)| >> |\eta_2|$, Γ (as defined in equation 10b) approaches unity and as a result, the numerator in (11), and $H_{\phi_2}(b)$ itself, approach zero. Physically, this implies that the incident and reflected magnetic fields at $r = b$ are nearly equal in magnitude, but are almost 180° out of phase; hence the incident and reflected portions of the field cancel each other out. This behavior of the magnetic field is highly analogous to that of current at the end of an open circuited transmission line. It is of interest to note that $|Z_3(b)| >> |\eta_2|$ is more likely to occur for non-magnetic shells than for magnetic shells. It should also be noted that regardless of the relative magnitude of Γ, as the shell thickness approaches two to three skin depths, the denominator of (11) becomes large and reduces $H_{\phi_2}(b)$ to effectively zero.

Comparison of Finite Element and Analytic Solutions for Eddy Current Loss

In this section, it will be shown by example that the finite element method, in concert with the $\mathbf{H} = 0$ boundary condition, yields solutions that are consistent with analytic solutions for eddy current losses in pipe-type cables and similar structures.

Consider the example illustrated in Figure 3: three power cables, arranged in a triangular configuration, are surrounded by a steel pipe (or steel wellbore casing). Each cable carries a 100 A rms alternating current at a frequency of 60 Hz. The current in any one cable is 120° out of phase with the currents in the other two; thus the currents constitute a balanced, three phase system.

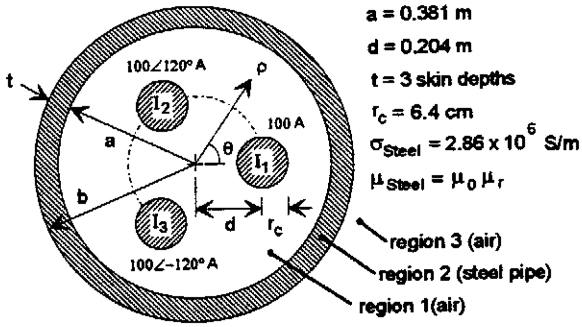

Figure 3. Cross sectional model of a pipe-type cable (three power cables, spaced 120° apart, encompassed by steel pipe).

Using the finite element method and the $\mathbf{H} = 0$ boundary condition, values for the eddy current loss in the steel pipe are computed for a range of different magnetic permeabilities. Eddy current loss as a function of the relative permeability of the steel pipe is plotted in Figure 4 and compared with the analytical solution provided by Kawasaki, et al. in equation 6 of reference [13]. As the permeability of the pipe is varied, the outer radius of the pipe, b, is adjusted so as to maintain a wall thickness of three skin depths. Thus, the rule of thumb discussed earlier, which stipulates whether or not $\mathbf{H} = 0$ is an accurate approximation for the magnetic field at the outer surface of the pipe, is satisfied.

Unlike the boundary value problem illustrated in Figure 1, the problem in this example involves three excitation currents instead of one (i.e., three power cables are situated in the pipe as opposed to a single power cable). Essentially the problem illustrated in Figure 3 is decomposed into three, single cable problems. Each "single-cable problem" consists of only the steel pipe and one power cable.

For the first single-cable problem, a finite element solution is obtained for the electric field that arises from I_1, the current in the first cable. The governing partial differential equation for the electric field, E_z, is given by equation (4). For this example, the authors modeled the solution space of the problem (regions 1 and 2) with linear quadrilateral elements. Using the Galerkin method of weighted residuals, along with element trial solutions and shape functions appropriate for linear quadrilateral elements, a system of equations is assembled from the element equations. Neumann boundary conditions for E_z, which can be related via (8) and (9) to the magnetic field boundary conditions, can be readily substituted into the right hand side of the resulting system of equations. Once again, the magnetic field boundary conditions are $\mathbf{H} = 0$ at the outer surface of the pipe, and, for this first problem, $H_\phi = I_1/(2\pi r_c)$ at the surface of the cable. This yields a solution for that part of the total E_z attributable to the current in the first power cable, I_1.

Except for the different phases of the excitation currents, the two remaining single-cable problems are identical to the first. Thus, to obtain a solution for the second single-cable problem, simply multiply the solution for E_z from the first problem by $\exp(j2\pi/3)$ to reflect the difference in phase between I_1 and I_2. Similarly, to obtain the solution for the third single-cable problem, multiply the solution for E_z from the first problem by $\exp(-j2\pi/3)$. The solutions are then superimposed with due regard to the original azimuthal position of the cables in Figure 3 (i.e., prior to superposition, the solution attributable to I_2 is rotated +120° about the center of the pipe and the solution attributable to I_3 is rotated -120°). The resultant solution for E_z can then be used to compute the eddy current loss in the steel pipe.

From Poynting's theorem, the eddy current loss per meter of pipe is given by

$$P_{eddy} = \sigma_{Steel} \int_0^{2\pi} \int_a^b |E_Z|^2 \rho \, d\rho \, d\theta \quad \mathrm{W/m}. \qquad (12)$$

Substituting the resultant solution for E_z into (12) and evaluating the integral numerically yields a value for P_{eddy}. Figure 4 is a plot of the eddy current loss in the steel pipe as a function of the relative permeability of the pipe. The solid line represents the finite element based solution and the dashed line denotes the analytical solution for eddy current loss, computed using equation 6 from reference [13].

Note that in Figure 4 the analytic and finite element solutions are in good agreement. The tendency of the eddy current loss to peak at a particular μ_r is not intuitively obvious and seems to be associated with very thick pipes (thickness >> one skin depth) which encompass a balanced, multi-phase system of currents. Similar behavior has been observed by Sikora, et al. in [9] and Poltz et al. in [19].

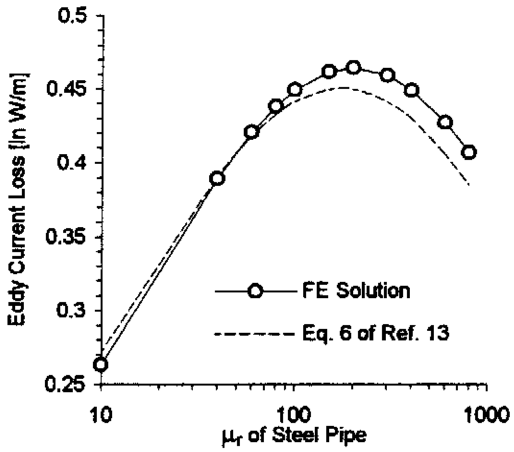

Figure 4. The eddy current loss in a steel pipe (depicted in Figure 3) as a function of the relative permeability of the pipe. The outer radius of the pipe is adjusted as the permeability varies, so as to maintain a wall thickness of 3 skin depths.

To demonstrate that the $\mathbf{H} = 0$ boundary condition is sometimes justified for pipes or metal shells less than 3 skin depths thick, consider a new problem, modeled in Figure 5, where the thickness of the metal shell varies between 20% and 150% of one skin depth.

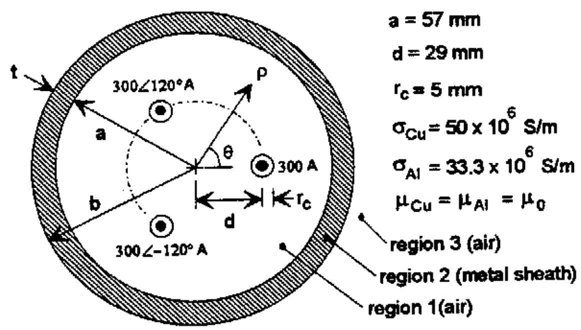

Figure 5. Cross sectional model of three current carrying conductors surrounded by a copper or aluminum sheath.

As illustrated in Figure 5, a non-magnetic metal sheath surrounds the three conductors. Each conductor carries a 300 A rms alternating current at a frequency of 50 Hz. In the first instance, the metal sheath is assumed to be composed of copper, which has a skin depth of 10 mm at 50 Hz. In the second instance, the metal sheath is assumed to be composed of aluminum, which has a skin depth of approximately 12 mm at 50 Hz.

Once again, a finite element solution is computed for the electric field E_z, and expression (12) is used to compute the eddy current losses in the sheaths. The eddy current losses in copper and aluminum sheaths are plotted in Figures 6a and 6b as functions of sheath thickness. The solid lines represent the finite element based solutions and the dashed lines denote the analytical solutions for eddy current loss, computed using equation 6 from reference [13].

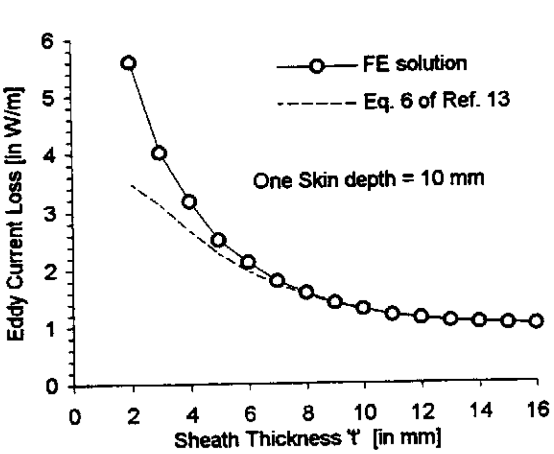

Figure 6a. Eddy current loss in a copper sheath as a function of the sheath's thickness.

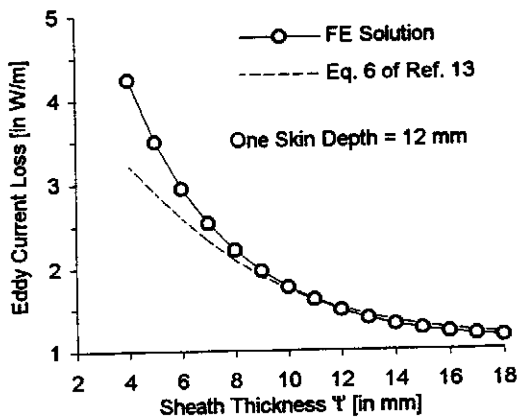

Figure 6b. Eddy current loss in an aluminum sheath as a function of the sheath's thickness.

For sheath thicknesses greater than about 0.8 skin depth, the finite element and analytic solutions are in good agreement. Consequently the approximate boundary condition, $\mathbf{H}=0$ at $\rho=b$, is *in this case* justified for sheaths as thin as one skin depth because of the aforementioned effects of reflection of the magnetic field at the outer surface of a metal shell surrounded by air. That is, the inward and outward traveling waves, which constitute the magnetic field, undergo almost complete cancellation at $\rho=b$.

As sheath thickness drops below 0.8 skin depth, the accuracy of approximate boundary condition becomes increasingly poor; the amount of attenuation experienced by the incident portion of magnetic field at the outer surface of the sheath is insufficient to justify setting $\mathbf{H}=0$. Thus the finite element solution, which relies on the $\mathbf{H}=0$ boundary condition, becomes increasingly inaccurate as the sheath thickness decreases. Although not shown in Figures 6a and 6b, the analytic solutions for eddy current loss rapidly decrease as sheath thickness drops below 0.2 skin depth.

Conclusion

For the problem of calculating the eddy current distribution and losses in pipe-type cables and similar structures, the finite element method can yield accurate solutions. One obstacle in applying the finite element method to these problems is the question of how to limit the solution space of, what is technically, an unbounded problem. The authors have demonstrated that, for metal shells greater than 3 skin depths thick, accurate finite element solutions can be obtained by imposing the approximate boundary condition, $\mathbf{H}=0$, along the outer surface of the shell. This rule of thumb is, however, somewhat conservative and in many practical cases the approximate boundary condition still holds for shells as thin as one skin depth. Finally, it should be noted that the finite element approach discussed in this paper lends itself quite easily to problems where the metal shell or pipe is not circular (i.e., triangular, rectangular, etc.). This is not necessarily true of the various analytical methods which are generally difficult to adapt to problems with non-circular shells.

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