A Green's Function Approach to Calculate Scattering Width for Cylindrical Cloaks

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Abstract**─**The anisotropic material properties of cylindrical cloaks can be simulated using thin, concentric layers of homogeneous, isotropic material. A Green's function for a line current in the presence of a layered PEC cylinder can be used to calculate the scattering width from a cloaked PEC cylinder with a significant improvement in computational efficiency compared to solutions obtained using the finite element method.

Index Terms**─**Electromagnetic cloaks, Finite Element Method (FEM), COMSOL Multiphysics Package, Green's functions.

I. INTRODUCTION

In 2006, Pendry *et al.* published results demonstrating it is theoretically possible to perfectly cloak an object thereby making it invisible to incident electromagnetic radiation [1]. Since the publication of this work, there has been a significant effort to show ideal cloaks result in a region of space into which no electromagnetic energy penetrates while preventing perturbation to the incident electromagnetic field outside of the cloaking mechanism [2]–[5].

In this paper, we will focus entirely on two-dimensional cylindrical cloaks. The material parameters for a cylindrical cloak can be determined using the coordinate transformation technique in [6] and are shown in equations (1) and (2). Note a and b are the inner and outer radial boundaries of the cloak. The material described by equations (1) and (2) is anisotropic and spatially varying. Manufacture of such a material would require detailed control of the six constitutive parameters which is quite difficult. In order to increase the manufacturability of a cylindrical cloak, the incident field can be decomposed into either transverse electric (TE) waves, in which the magnetic field is solely z-directed, or transverse magnetic (TM) waves, in which the electric field is solely z-directed. The decomposition is performed to reduce the number of constitutive parameters that require specificity from six to three. As an example, for a TE field incident on a cylindrical cloak, only the μ_z, ε_ρ, and ε_θ components of the constitutive parameter tensors impact the scattered field's behavior.

$$\mu_\rho = \frac{\rho - a}{\rho},\ \mu_\theta = \frac{\rho}{\rho - a},\ \mu_z = \frac{\rho - a}{\rho}\left(\frac{b}{b-a}\right)^2 \quad (1)$$

$$\varepsilon_\rho = \frac{\rho - a}{\rho},\ \varepsilon_\theta = \frac{\rho}{\rho - a},\ \varepsilon_z = \frac{\rho - a}{\rho}\left(\frac{b}{b-a}\right)^2 \quad (2)$$

In addition to field decomposition, simplified material parameter sets have been developed in order to increase manufacturability [7]–[10]. Note at $\rho = a$, ideal values for μ_θ and ε_θ are infinite. Simplified parameter sets typically eliminate the requirement for these infinite values. The penalty for this simplification is a reduction in cloaking effectiveness. However, these simplified cloaks do inherit many of the ideal cloak's energy bending properties.

There have been numerous simulations of cloaking geometries. The majority use the COMSOL Multiphysics package [11]–[16], a commercially available finite-element-method (FEM) based package capable of handling cloaks' anisotropic, inhomogeneous constitutive parameters. There have also been cloak simulations using the finite difference time domain method [17]–[19]. Simulation results confirm cloaks behave as theoretically predicted.

In this paper, we first present background theory on analytic solutions for the scattering width of a PEC cylinder, and then compare the results to those obtained using the COMSOL Multiphysics software. We then discuss a method to approximate cloak parameters using layered homogenous isotropic materials. This leads to the development of a Green's function which can be used to determine the scattering width of a cloaked PEC cylinder. The Green's function solution is shown to be significantly faster and less computationally intensive than the FEM-based solution.

II. BACKGROUND THEORY

In this section, we present basic results for the scattering width of a PEC cylinder using theoretical methods and the COMSOL Multiphysics software. We also discuss the method proposed in [20] that uses isotropic materials to simulate anisotropic cloaking media. This is the basis for our Green's function implementation proposed in Section III.

A. Scattering Width of a PEC Cylinder

The two-dimensional scattering width, σ_{2D}, of a PEC cylinder for an incident TE plane wave propagating in free space is known to be [21]

$$\sigma_{2D} = \frac{2\lambda}{\pi}\left|\sum_{m=0}^{\infty}\varepsilon_m \frac{J'_m(k_o a)}{H'^{(2)}_m(k_o a)}\cos(m\theta)\right|^2, \quad (3)$$

where k_o is the free space wave number, θ is the observation angle, a is the radius of the cylinder, and ε_m is 1 for m = 0 and 2 otherwise. Note that ' implies differentiation with respect to ρ. Equation (3) can be truncated based on the specified level of accuracy. In this paper, the summations in equation (3) and in equation (12) are truncated to $m = M$ such that

$$x = \max\left|F_m\right|,\ m \in [0, M], \qquad \forall\ m > M,\ \left|F_m\right| < 0.01x, \quad (4)$$

where $F_m = \frac{J'_m(k_o a)}{H'^{(2)}_m(k_o a)}$, $\quad F_m = B_m^{n+1}$,

in equation (3) and in equation (19) respectively. The validity of truncating using equation (4) can be verified by comparing the calculated scattering widths of a PEC cylinder of radius $a = \lambda$ for m = 10 and m = 50. The metric we used to compare the similarity between the solutions is the average difference in σ_{2D}. Mathematically, this is

$$\Delta = \frac{1}{M}\sum_{m=1}^{M}\left|\sigma_{2D}^{A} - \sigma_{2D}^{B}\right|, \quad (5)$$

where M is the total number of observation angles, and the σ_{2D}-terms are the scattering widths we wish to compare. Δ for $\sigma_{2D}|_{m=10}$ compared to $\sigma_{2D}|_{m=50}$ is 0.0063 m^2, which is negligible since σ_{2D} is on the order of 10 m^2 for the measurement set.

Unfortunately, most objects of interest for which scattering width data is desired do not have nice analytic solutions such as equation (3). Other computational methods can be used for such calculations. FEM is a useful approach, particularly in the field of cloaks due to the spatial dependence of the constitutive parameters. However, the computational burden of the finite element method can be significant.

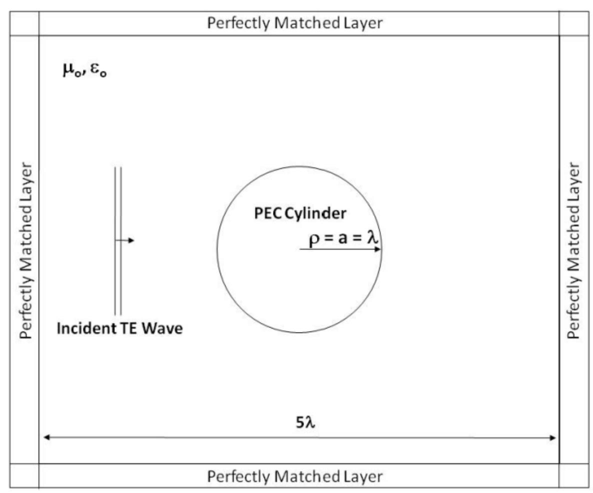

Fig. 1. Geometry for an FEM simulation of scattering from a PEC cylinder.

As an example, consider the geometry shown in Fig. 1. We wish to solve for the PEC cylinder's scattering width due to illumination by an incident TE plane wave. We can use the theoretical solution in equation (3) and compare with the FEM solution obtained using the COMSOL software. Note the computational boundary is only 5λ×5λ.

Therefore, Huygen's principle is used within the COMSOL software to transform the near field results to the far zone, from which scattering width data can be obtained.

Different uniform meshes were used in the COMSOL simulations, where a smaller maximum element length (MEL) corresponds to a finer, denser mesh. We simulated the geometry shown in Fig. 1 using seven different values of MEL and compared the results to the theoretical solution where m = 10. Results are shown in Table 1 with additional information on problem size and solution speed.

Table 1: Analytic and FEM Solution Comparison.

MEL	Unknowns	Time	$\Delta\sigma_{2D}\|_{m=10}$
0.5λ	2,056	0.17 s	0.171 m^2
0.25λ	3,832	0.23 s	0.125 m^2
0.1λ	23,146	1.04 s	0.039 m^2
0.075λ	40,654	1.67 s	0.039 m^2
0.05λ	92,168	4.02 s	0.041 m^2
0.025λ	366,024	16.5 s	0.043 m^2
0.01λ	2,292,872	1,493 s	0.043 m^2
m = 10	N/A	0.14 s	N/A
m = 50	N/A	0.25 s	0.006 m^2

Obviously there is good agreement between the analytic and FEM solutions. However, we needed a metric to define FEM solution accuracy. Therefore, based on the results in Table 1, we used a Δ on the order of 0.1 m^2 to be the threshold to define good agreement between the solutions. For the geometry in Fig. 1, an MEL of 0.1λ is sufficient to obtain good solution agreement. However, as will be seen shortly, there are cloak geometries where finer mesh densities are required due to the thinness of subdomains within the computational boundary. These geometries will require finer meshes, which will result in a larger number of unknowns. As shown in Table 1, solution time dramatically increases as the number of unknowns increases.

B. Cloak Approximation Using Homogenous Layers of Isotropic Material

Huang *et al.* implemented a simplified cloak using layered homogeneous materials to approximate a cloak's anisotropic material [20]. They use the fact a two-layered structure of homogeneous isotropic material can be treated as a single anisotropic medium provided the layers are small compared to wavelength. For a given set of two layers of material that are sufficiently thin, the effective permittivity values are [22]

$$\varepsilon_\theta = \frac{\varepsilon_1 + \eta\varepsilon_2}{1+\eta}, \tag{6}$$

$$\frac{1}{\varepsilon_\rho} = \frac{1}{1+\eta}\left(\frac{1}{\varepsilon_1} + \frac{\eta}{\varepsilon_2}\right), \tag{7}$$

where ε_1 and ε_2 are the relative permittivity of the two layers; and $\eta = d_2/d_1$ where d_i are the layer thicknesses. For this work, $\eta = 1$. They used equations (6) and (7) to derive the appropriate material parameters and thicknesses for a cylindrical cloak made out of homogeneous materials. They simulated a cloak which had the effective material properties similar to those of the reduced cloak put forth by Schurig *et al.* in [7] and obtained similar results.

Huang *et al.* did not use their method to approximate the functioning of an ideal cylindrical cloak, although they gave no reason why this could not be done. As a verification of their work, we performed a COMSOL simulation of an approximated ideal cloak. The spatial variation in the ideal cloak is first approximated using ten layers of homogeneous, anisotropic material, with the parameters ε_ρ and ε_θ evaluated at $\rho = \rho_n$ where ρ_n is the radial location of the n^{th} layer. Each homogeneous anisotropic layer is then approximated

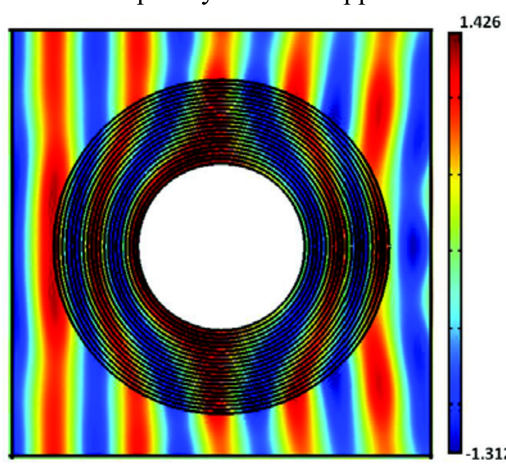

Fig. 2. Total magnetic field for an ideal cloak approximated using 20 layers of isotropic homogeneous material.

using two layers of homogeneous isotropic material with permittivity values defined by equations (6) and (7). Hence, 20 layers were used in this approximation. Layer values of μ_z were calculated for each layer by evaluating μ_z in equation (1) at $\rho = \rho_n$.

As shown in Fig. 2, small homogeneous layers can be used to adequately approximate the bulk anisotropic material parameters required for a cylindrical cloak. No comparison of this cloak's scattering width performance is done in this subsection, as the main point was to confirm Huang *et al.*'s method can be used to approximate an ideal cloak. Scattering width performance will be analyzed in a later section. We have confirmed a cloak can be realized using layers of homogeneous isotropic media, which leads directly to our Green's function implementation.

III. GREEN'S FUNCTION

Green's functions are commonly used in electromagnetic scattering problems. However, they have not been applied to solve radiation problems involving cloaks, likely due to the difficulty in their derivation due to the anisotropic nature of a cloak's material parameters. Methods similar to what we propose here has been used to study near and far field solutions for a PEC cylinder covered by an isotropic lossless layers [23], [24]. However, these analyses focused on the scattering properties of PEC cylinders layered with double-negative materials.

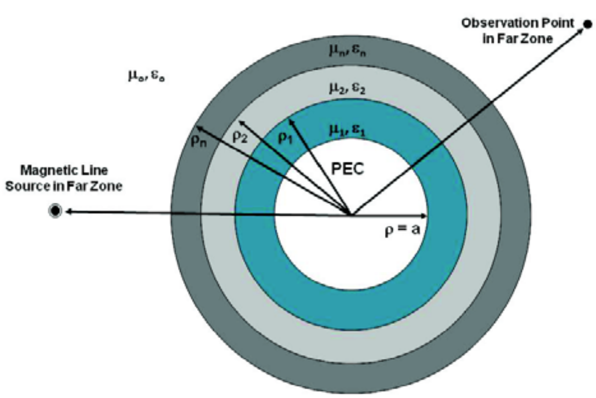

Fig. 3. Problem geometry for Green's function derivation.

As shown in Section II-B, a cylindrical cloak can be approximated by using concentric layers of isotropic material with homogeneous permittivity and permeability. To implement such a solution, consider a cloaked PEC cylinder of radius $\rho = a$ where the cloak is approximated by n layers of homogeneous material as shown in Fig. 3.

We developed the Green's function for a magnetic line source radiating in the presence of a PEC cylinder covered by n layers of homogeneous, isotropic material using the method described in [25]. The result is

$$\bar{G} = -\frac{j}{4}\sum_{m=0}^{\infty}\varepsilon_m \cos\left[m(\theta-\theta')\right]$$

$$\left[J_m(k_o\rho_<) + \frac{B_m^{n+1}}{A_m^{n+1}}H_m^{(2)}(k_o\rho_<)\right]H_m^{(2)}(k_o\rho_>) \quad (8)$$

$$A_m^{n+1} = 1 \quad (9)$$

$$B_m^1 = -\frac{J_m'(k_1 a)}{H_m'^{(2)}(k_1 a)} \quad (10)$$

where $\rho_<$, $\rho_>$ are the lesser of ρ and ρ' respectively, and θ, θ' are the observation and source angular locations. The remaining coefficients will be defined shortly. Note equation (8) is valid when observing the field where $\rho > \rho_n$ i.e. in the free space region.

Equation (8) contains components for the incident field and the scattered fields. The incident field is represented by the $J_m(k_o\rho_<)$ components while the scattered field is represented by the $H_m(k_o\rho_<)$ terms. Thus, we can rewrite the Green's function as two separate functions. We do this because our ultimate goal is to compute σ_{2D}, which we find as follows.

Without loss of generality, we will assume the magnetic line source is positioned at $\theta' = 180^o$. Additionally, we assume both the source and observation points are in the far zone. Using the large argument approximation for the Hankel function of the second kind [26], and knowing that in general, scattering width is found by

$$\sigma_{2D} = \lim_{\rho\to\infty} 2\pi\rho \frac{\left|H^s\right|^2}{\left|H^i\right|^2}, \quad (11)$$

the scattering width for a layered PEC cylinder with an incident TE plane wave traveling in the positive x direction is

$$\sigma_{2D} = \frac{2\lambda}{\pi} \frac{\left| \sum_{m=0}^{\infty} \varepsilon_m \cos(m\theta) \frac{B_m^{n+1}}{A_m^{n+1}} \right|^2}{\left| \sum_{m=0}^{\infty} \varepsilon_m (-j)^m \cos(m\theta) J_m(k_o b) \right|^2} . \quad (12)$$

The only unknowns in equation (12) are the B_m coefficients in the n layered regions. These coefficients are determined based on the junction conditions at the radial boundaries which force continuity of tangential magnetic and electric fields. Due to the PEC boundary at $\rho = a$, the value of B_m in the first region ($n = 1$) is known, which allows the remaining values to be found by solving a system of $2n$ equations.

To ensure the accuracy of the derived Green's function, we compared σ_{2D} calculated using the Green's function in equation (12) to the scattering width results obtained using a COMSOL simulation for a simplified cloak with material parameters put forth by Yan *et al.* and shown in equation (13).

$$\varepsilon_\rho = \left(\frac{\rho - a}{\rho}\right)^2 \frac{b}{b-a}, \ \varepsilon_\theta = \frac{b}{b-a}, \ \mu_z = \frac{b}{b-a} . \quad (13)$$

In order for the Green's function to accurately approximate a radially varying cloak as described in equation (13), the number of layers used in the formulation must be large. The Green's function results were determined using 5,000 layers to

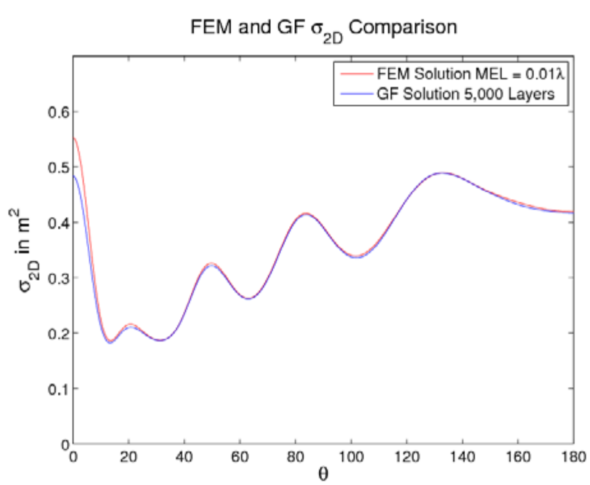

Fig. 4. Green's function and FEM results comparison.

approximate the anisotropic material. The FEM results were obtained with MEL = 0.01λ. The calculated scattering widths from the two methods were very similar, as shown in Fig. 4. The Δ for the results in Fig. 4 was 0.004 m^2, which is quite good. Based on these results, we concluded our Green's function is an accurate method to obtain scattering width from cloaked cylinders. Next, we simulate various cloak geometries and compare the solution times using the Green's function and COMSOL.

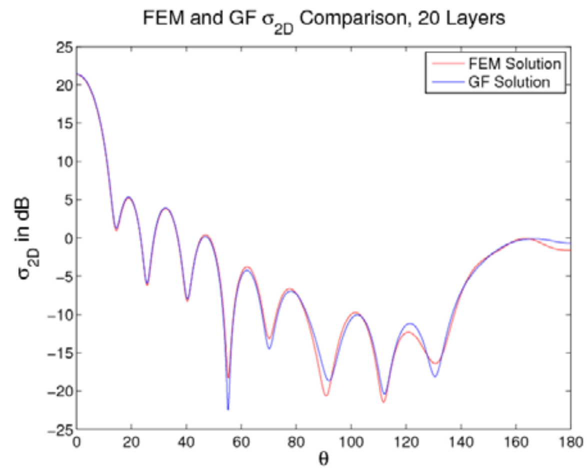

Fig. 5. Green's function and FEM results comparison for PEC with 20 layers.

IV. NUMERICAL BENEFIT

It is not yet possible to manufacture cloaks with the required spatially varying anisotropic parameters. Concentric rings of homogeneous anisotropic material can be used to approximate the spatial variation [7]. Such a realization can be simulated in COMSOL, but it is also well suited for our Green's function implementation. As an example, consider the COMSOL results shown in Fig. 2. Recall, this is a PEC surrounded by 20 layers of homogeneous material with material parameters chosen to approximate an ideal cloak. We simulated the same geometry using our Green's function and compared the scattering widths from the two methods. The results are shown in Fig. 5. We also compared the results from the two methods for a simulation using 40 layers of homogenous material to approximate an ideal cloak. These results are shown in Fig 6. Note these plots have σ_{2D} shown in dB rather than m^2 due to the large forward scattering.

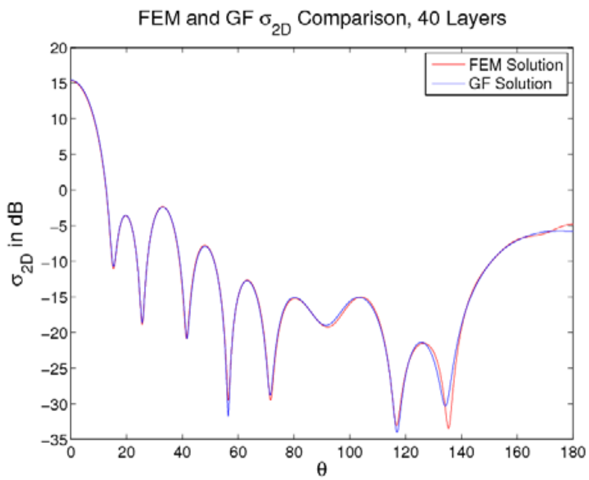

Fig. 6. Green's function and FEM results comparison for PEC with 40 layers.

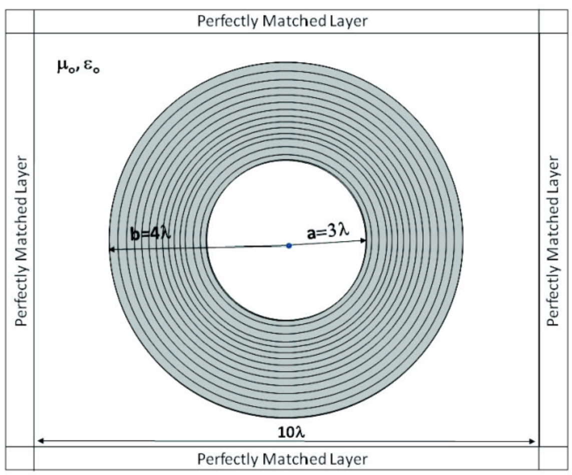

Fig. 7. Computational domain geometry for larger cloak sizes.

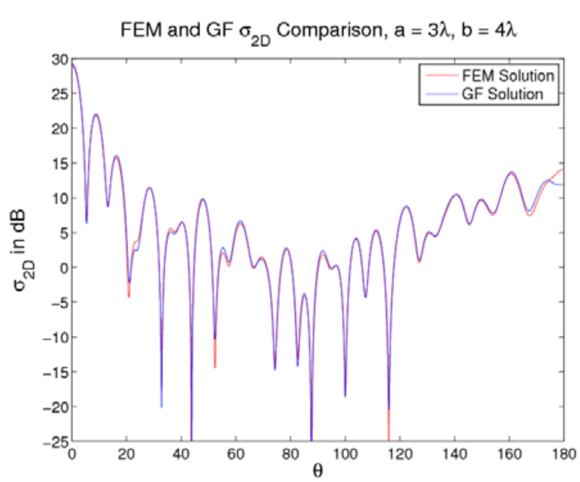

Fig. 8. σ_{2D} for larger cylinder and cloak sizes.

Note the similarities between the FEM and Green's function solutions. For the 20-layer simulation, Δ was 0.14 m^2, which shows good agreement between the two solutions. However, there is a noticeable difference in computation time. The Green's function solution took 2.28 s. The FEM solution was obtained by first creating a non-uniform mesh over the computational domain. This was necessary because the spacing between layers was only 0.05λ and 0.025λ for the 20 and 40-layer simulations respectively. A uniform mesh with MEL < 0.01λ was not possible due to memory limitations. The MEL in the concentric layers was limited to 0.007λ. This did not give us the desired number of elements in each layer for the 40-layer simulation, but this problem could not be avoided due to memory limitations. For the rest of the computation domain, an MEL = 0.05λ was used. For the 20-layer FEM simulation, the mesh had 911,004 elements and a solution time of 125 s.

The 40-layer simulation resulted in Δ = 0.04 m^2. The Green's function solution time was 2.82 s. The FEM solution used a similar non-uniform mesh with 881,892 elements and a solution time of 124 s. Obviously, the Green's function method is faster, particularly if a number of simulations are to be performed to conduct an optimization or an error analysis based on parameter or thickness variations in the layers. Additionally, if more layers are to be used, an FEM solution will require finer meshing within the layers, increasing the number of unknowns which will increase solution time.

Another benefit of using the Green's function to calculate scattering widths is when large cloak geometries are simulated. Up until this point, all previous simulations have used the cloak parameters such that $a = \lambda$ and $b = 2\lambda$. If a and b are increased, the computational domain in an FEM simulation increases. This will increase the number of unknowns if the same limits on MEL are used, ultimately resulting in a longer solution time. The MEL can be increased in order to prevent out-of-memory errors during solution at the penalty of reduced accuracy. Increasing the cloak radii in the Green's function does result in having to include more terms in the summation, but this increase in computational budget is minimal compared to the increased burden in an FEM simulation. As an example, consider the simulation geometry shown in Fig. 7. More elements are going to be needed since the computational domain is 10λ×10λ. We performed a simulation of a 20-layer cloak of homogeneous material approximating the material parameters shown in equation (13) using COMSOL and our Green's function. The results are shown in Fig. 8. The Δ between the two simulations was 1.39 m^2, still reasonable, but getting worse due to the fact MEL had to increase in some portions of the geometry.

To reduce memory requirements for the FEM solution, a non-uniform mesh was applied. The MEL for the concentric ring was 0.01λ, while for the remaining areas, MEL = 0.3λ. Note the MEL has been increased compared to the same 20-layer simulation where $a = \lambda$ and $b = 2\lambda$, meaning FEM solution accuracy will decrease. The mesh consisted of 972,698 elements and a solution time of 116 s. In addition to the solution time, the code took 168 s to simply create the mesh. The solution time for the Green's function was 3.89 s, slightly longer due to the fact $m = 31$ in the summation due to the requirement in equation (4). Further increases in cloak size result in having to significantly increase MEL in order prevent mesh size from growing beyond the computational capabilities. The Green's function formulation can handle the larger problem sizes with a minimal impact to computation time.

V. CONCLUSION

We have shown a Green's function approach for determining scattering widths from a cylindrical cloak results in a significant computational savings. This savings can be useful if an error analysis or optimization studies are to be performed on a particular cloak geometry. Additionally, the computational domain size is directly related to the cylindrical cloak's radius. A larger cloak results in a larger domain size. The increase in computational domain requires either a longer solution time due to the increased number of elements or a reduction in mesh density which impacts solution accuracy. The Green's function implementation is much faster than an FEM solution and is more adept at handling larger problem geometries.

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