GTD Model Based Cubic Spline Interpolation Method for Wide-Band Frequency- and Angular- Sweep

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Abstract **─** A hybrid frequency- and angular-sweep interpolation method is proposed for the efficient analysis of the scattering over a broad frequency and angular band. This method hybridizes the cubic spline interpolation method and the GTD model. The cubic spline interpolation method is applied to interpolate the induced currents in the angular domain. The GTD model combined with the matrix pencil method is applied to parameterize the scattering centers over a wide frequency band. In order to efficiently compute electromagnetic scattering, the flexible general minimal residual (FGMRES) iterative solver is applied to compute the coefficients of Rao-Wilton-Glisson (RWG) basis functions. Therefore, a great deal of time can be saved for the calculation of frequency- and angular- sweep. Numerical results demonstrate that this hybrid method is efficient for wide-band RCS calculation with high accuracy.

Index Terms **─** Cubic spline interpolation method, electromagnetic scattering, GTD model, matrix pencil.

I. INTRODUCTION

Electromagnetic wave scattering problems address the physical issue of detecting the diffraction pattern of the electromagnetic radiation scattered from a large and complex body when illuminated by an incident incoming wave. A good understanding of these phenomena is crucial to radar cross section (RCS) calculation, antenna design, electromagnetic compatibility, and so on. All these simulations are very demanding in terms of computer resources, and require efficient numerical methods to compute an approximate solution of Maxwell's equations. Using the equivalence principle, Maxwell's equations can be recast in the form of integral equations that relate the electric and magnetic fields to the equivalent electric and magnetic currents on the surface of the object. Amongst integral formulations, the electric field integral equation (EFIE) is widely used for electromagnetic wave scattering problems as it can handle the most general geometries. The matrix associated with the resulting linear systems is large, dense, complex, and non-Hermitian [1]. It is basically impractical to solve EFIE matrix equations using direct methods because they have a memory requirement of $0(N^2)$, where N refers to the number of unknowns. This difficulty can be circumvented by use of iterative methods, and the required matrix-vector product operation can be efficiently evaluated by multilevel fast multipole algorithm (MLFMA) [2, 3]. The use of MLFMA reduces the memory requirement to $0(NlogN)$ and the computational complexity of per-iteration to $0(NlogN)$.

To obtain the RCS over a frequency- and angular- band using MoM, one has to repeat the calculations at each frequency and angle over the band of interest. This can be computationally prohibitive despite the increased power of the present generation of computers. In order to alleviate this difficulty, many interpolation methods have been proposed and applied for acceleration.

In [4], the model-based parameter estimation (MBPE) is used to obtain the wide-band data from frequency and frequency-derivative data. In [5-7], a similar technique called asymptotic waveform evaluation (AWE) technique has been applied to frequency-domain electromagnetic analysis. Both

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MBPE and AWE interpolate the coefficients of RWG which can avoid repeated construction and solution. Other current-based methods, such as the Cauchy-method and the spline-method, are applied in fast frequency- and angular- sweep as well [8, 9]. However, coefficients of RWG are not a linear function of frequency so that many samples are required for frequency swap. There are some attempts to obtain the wide-band data by interpolating the impedance matrix [10-12]. This method saves much time for constructing the impedance matrix but can do nothing for the iterative solution repeatedly. Moreover, the impedance matrix interpolation method requires significant memory for saving impedance matrices at frequency samples. Thus, a new method is required to circumvent this difficulty.

The wide frequency band scattering can be modeled by scattering centers. Using the GTD model can easily parameterize the scattering from an arbitrary object over wide band [13, 14]. Besides, the cubic spline interpolation method is successfully applied for accelerate wide angular sweep [15]. Accordingly, frequency- and angular-sweep RCS over wide frequency- and angular-band can be efficiently computed by combining these two methods. The GTD-based cubic spline interpolation method is proposed and used to accelerate the computation of frequency- and angular- sweep scattering in this paper.

The remainder of this paper is organized as follows. Section II demonstrates the basic theory of the 3-D scattering center parameterized by GTD model. The GTD-based cubic spline interpolation method is proposed and discussed in Section III. Numerical experiments of three geometries are presented to demonstrate the efficiency of this proposed method in Section IV. Conclusions are provided in Section V.

II. BASIC THEORY OF SCATTERING CENTER

At sufficiently high frequencies, the scattering response of an object can be well approximated as a sum of responses from individual caterers or scattering centers [13]. Using a parametric scattering model based on the geometrical theory of diffraction to parameter characterizing the geometry of each scattering center is proposed in [16]. The GTD-based model is more closely related to the physics of the electromagnetic scattering than other models, such as the Prony model.

Let the incident electric field be a plane wave propagating in the $\hat{\mathbf{k}}$ direction. The electric field at location $\mathbf{r}$ with wave number k can be described by

$$\mathbf{E}^{inc}\left(\mathbf{r}\right)=\mathbf{E}_0 e^{-jk\hat{\mathbf{k}}\cdot\mathbf{r}} , \tag{1}$$

where $\mathbf{E}_0$ is the amplitude of the incident electric field and the time-harmonic factor $e^{-j\omega t}$ is suppressed. Using GTD model, the scattering field $\mathbf{E}^{sca}$ is described by

$$\mathbf{E}^{sca}\left(\mathbf{r}\right)=\sum_{m=1}^{N} A_m\left(j\frac{k}{k_c}\right)^{\alpha_m} e^{-j2kr_m} . \tag{2}$$

As shown in (2), kc is defined as the wavenumber corresponding to the starting frequency point. The model parameters $\{A_m, \alpha_m, r_m\}$ characterize the N individual scattering centers. Each r_m gives the range of a scattering center with respect to a zero-phase reference, α_m characterizes the geometry or the type of the m^{th} scattering center which is described in Tab.1, and A_m is a complex scalar providing the magnitude and phase for a scattering center.

Table 1: Type parameters for canonical scattering geomeries

Value of α_m	Example scattering geometries
− 1	Corner diffraction
− 0.5	Edge diffraction
0	Point scatterer, doubly curved surface reflection, straight edge specular
0.5	Singly curved surface reflection
1	Flat plate at broadside, dihedral

Using the GTD model, we can successfully extrapolate the scattering over a wide frequency band. Obviously, how to obtain the parameters of this model is the most important thing. For simplicity, a more simple formulation can be used for frequency extrapolation, which is shown as follows

$$\mathbf{E}^{s}\left(k\right)=\sum_{i=1}^{N} A_i e^{-jkr_i} . \tag{3}$$

Only the amplitude and the phase of the scattering center are essential for extrapolation. First of all, the scattering field at several frequency points can be obtained by MoM-MLFMA. Then apply some super-resolution algorithms, such as matrix pencil (MP) method [17], to the scattering field output at these frequency points to extract the parameters. In order to perform the matrix pencil algorithm, two matrices $\mathbf{Y}_1$ and $\mathbf{Y}_2$ are defined as

$$\mathbf{Y}_1 = \begin{bmatrix} \mathbf{E}^s(k_0) & \mathbf{E}^s(k_1) & \cdots & \mathbf{E}^s(k_{L-1}) \\ \mathbf{E}^s(k_1) & \mathbf{E}^s(k_2) & \cdots & \mathbf{E}^s(k_L) \\ \vdots & \vdots & & \vdots \\ \mathbf{E}^s(k_{N-L-1}) & \mathbf{E}^s(k_{N-L}) & \cdots & \mathbf{E}^s(k_{N-2}) \end{bmatrix}_{(N-L)\times L} \quad (4)$$

$$\mathbf{Y}_2 = \begin{bmatrix} \mathbf{E}^s(k_1) & \mathbf{E}^s(k_2) & \cdots & \mathbf{E}^s(k_L) \\ \mathbf{E}^s(k_2) & \mathbf{E}^s(k_3) & \cdots & \mathbf{E}^s(k_{L+1}) \\ \vdots & \vdots & & \vdots \\ \mathbf{E}^s(k_{N-L}) & \mathbf{E}^s(k_{N-L+1}) & \cdots & \mathbf{E}^s(k_{N-1}) \end{bmatrix}_{(N-L)\times L} \quad (5)$$

where L is an integer number which is very important for matrix pencil. Then the matrices $\mathbf{Y}_1$ and $\mathbf{Y}_2$ could be rewritten as

$$\mathbf{Y}_1 = \mathbf{Z}_1\mathbf{R}\mathbf{Z}_2. \quad (6)$$

$$\mathbf{Y}_2 = \mathbf{Z}_1\mathbf{R}\mathbf{Z}_0\mathbf{Z}_2. \quad (7)$$

Assume $z_i = e^{-jkr_i}$, then

$$\mathbf{Z}_1 = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ z_1 & z_2 & \cdots & z_M \\ \vdots & \vdots & & \vdots \\ z_1^{N-L-1} & z_2^{N-L-1} & \cdots & z_M^{N-L-1} \end{bmatrix}_{(N-L)\times M}, \quad (8)$$

$$\mathbf{Z}_2 = \begin{bmatrix} 1 & z_1 & \cdots & z_1^{L-1} \\ 1 & z_2 & \cdots & z_2^{L-1} \\ \vdots & \vdots & & \vdots \\ 1 & z_M & \cdots & z_M^{L-1} \end{bmatrix}_{M\times L}, \quad (9)$$

$$\mathbf{Z}_0 = diag\left[z_1, z_2, \cdots, z_M\right], \quad (10)$$

$$\mathbf{R} = diag\left[A_1, A_2, \cdots, A_M\right]. \quad (11)$$

Obviously, the parameter z_i and A_i may be found as the generalized eigenvalues of the matrix pair $\{\mathbf{Y}_2, \mathbf{Y}_1\}$. Equivalently, the problem can be cast as an ordinary eigenvalue problem. Once the parameters in (3) are obtained, the frequency-dependant model of the scattering field can be constructed. Accordingly, the scattering over a wide frequency band can be calculated from this model.

III. GTD-BASED CUBIC SPLINE INTERPOLATION METHOD

In order to efficiently compute frequency- and angular- sweep RCS, a frequency- and angular-sweep interpolation scheme is required. In this section, we propose a GTD-based cubic spline interpolation method for analysis of frequency- and angular sweep scattering. This method combines the GTD modal and the cubic spline interpolation method. Using the GTD modal for frequency sweep is discussed in Section II. Accordingly, this section focuses on the fast angular sweep and how to hybridize these two methods.

First of all, we explain the basic idea of the cubic spline interpolation method. Cubic spline interpolates the induced current instead of the scattering field. Assume that the sampling angles are $\varphi_0, \varphi_1, \ldots, \varphi_n$ for a 1-D curve, which divide the whole angular band into several intervals. Within each interval such as $[\varphi_{i-1}, \varphi_i]$, the induced current $\mathbf{I}(\varphi)$ can be expanded as a third-order polynomial on φ. According to the Hermite interpolation model [18], the interpolation formula of $\mathbf{I}(\varphi)$ can be described as

$$\begin{aligned}\mathbf{I}(\varphi) = & \frac{(\varphi - \varphi_i)^2\left[h_i + 2(\varphi - \varphi_{i-1})\right]}{h_i^3}\mathbf{I}(\varphi_{i-1}) \\ & + \frac{(\varphi - \varphi_{i-1})^2\left[h_i + 2(\varphi_i - \varphi)\right]}{h_i^3}\mathbf{I}(\varphi_i) \ , \\ & + \frac{(\varphi - \varphi_i)^2(\varphi - \varphi_{i-1})}{h_i^2}\mathbf{I}'(\varphi_{i-1}) \\ & + \frac{(\varphi - \varphi_{i-1})^2(\varphi - \varphi_i)}{h_i^2}\mathbf{I}'(\varphi_i)\end{aligned} \quad (12)$$

where φ_i is the sampling point and $h_i = \varphi_i - \varphi_{i-1}$. Obviously, $\mathbf{I}(\varphi_i)$ and $\mathbf{I}'(\varphi_i)$ are needed in order to estimate the value of $\mathbf{I}(\varphi)$ in the angle range $[\varphi_{i-1}, \varphi_i]$. Suppose the number of sampling points to be n, the times of MoM solution is $2n$ [18]. It is thus a waste of time to compute the first derivative of induced current vector of each sampling node.

The cubic-spline interpolation method applies another way to obtain the first derivative of each sampling node instead of solving the linear equations (2) repeatedly [18]. This method just needs to compute the first derivative of φ_0 and φ_n. However, it is unnecessary to do this since we can put them into zero under the natural boundary condition [18], that is to say, $\mathrm{I}'(\varphi_0) = 0$ and $\mathrm{I}'(\varphi_n) = 0$. Also, we can calculate the precise derivative of these two samples with a little time cost. The first derivative of other sampling points are then given by

$$\begin{bmatrix} 2 & \lambda_1 & & & \\ \mu_2 & 2 & \lambda_2 & & \\ & \ddots & \ddots & \ddots & \\ & & \mu_{n-1} & 2 & \lambda_{n-1} \\ & & & \mu_{n-1} & 2 \end{bmatrix} \begin{bmatrix} \mathbf{I}'(\varphi_1) \\ \mathbf{I}'(\varphi_2) \\ \vdots \\ \mathbf{I}'(\varphi_{n-2}) \\ \mathbf{I}'(\varphi_{n-1}) \end{bmatrix} = \begin{bmatrix} g_1 - \lambda_1 \mathbf{I}'(\varphi_0) \\ g_2 \\ \vdots \\ g_{n-2} \\ g_{n-1} - \mu_{n-1}\mathbf{I}'(\varphi_n) \end{bmatrix}, \quad (13)$$

where the parameter h_i is the interval between φ_i and φ_{i+1}.

$$\lambda_i = \frac{h_{i+1}}{h_i + h_{i+1}}, \quad \mu_i = \frac{h_i}{h_i + h_{i+1}}, \quad (14)$$

and

$$g_i = 3\left[\mu_i \frac{\mathbf{I}(\varphi_{i+1}) - \mathbf{I}(\varphi_i)}{h_{i+1}} + \lambda_i \frac{\mathbf{I}(\varphi_i) - \mathbf{I}(\varphi_{i-1})}{h_i}\right]. \quad (15)$$

Due to the large time saved in calculating the derivative, the cubic-spline interpolation approach is able to reduce a great deal of cost.

For frequency- and angular- sweep RCS surface, GTD based cubic spline interpolation method includes three steps. The first step is to compute the induced current $\mathbf{I}(f_s, \varphi_s)$ at each sampling frequency and each sampling angle. If the number of frequency samples is p and the number of angular samples is q, the times of MoM procedure is $p \times q$ and the memory cost for samples is $p \times q \times N$, where N is the number of unknowns. Then the cubic spline interpolation method is used to evaluate the scattering field at each sampling frequency. We can obtain p curves described by $\mathbf{E}^{sca}(f_s, \varphi)$, where f_s is the sampling frequency and φ is the angle. Finally, using matrix pencil to evaluate the parameters of scattering center and calculate the whole frequency- and angular- sweep RCS surface RCS(f, φ).

GTD Cubic-Spline interpolation algorithm:

Assume the whole surface can be described by function (f, φ), where f and φ are unknown variables. (f_0, φ_0), (f_0, φ_1), … , (f_i, φ_j), … , (f_m, φ_n) are the control points on the surface.

Step 1: Calculate the induced current at sampling points $\mathbf{I}(f_s, \varphi_s)$.

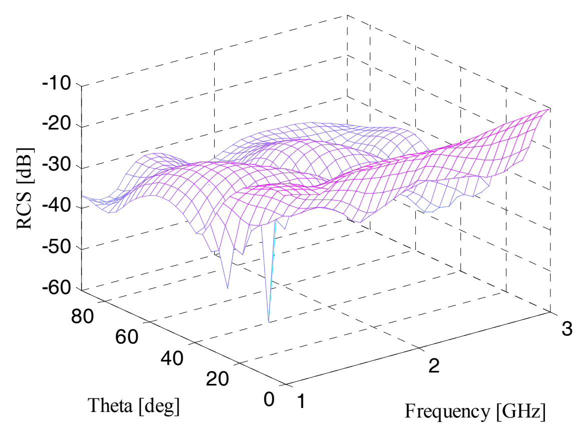
Step 2: For φ direction, assume f is a const. applying the cubic spline interpolation method to approximate line $\mathbf{E}^{sca}(f_s, \varphi)$ (s = 1, 2, … , m).

Step 3: For f direction, assume φ is a const. applying the matrix pencil method to approximate final result RCS(f, φ).

As described above, the computational complexity is $O(p \times q)$, where p is the number of frequency samples and q is the number of angular samples. The memory cost for this algorithm is $O(p \times q \times N)$, where N is the number of unknowns. That is, most of the memory consumption is to save the induced current. Neither computational complexity nor memory consumption is large, since the number of samples p and q are small. Therefore, GTD based cubic spline interpolation method can greatly improve the efficiency of analysis of frequency- and angular- sweep RCS with low memory cost.

IV. NUMERICAL RESULTS

In this section, a number of numerical results are presented to demonstrate the accuracy and efficiency of the hybrid interpolation method for the fast calculation of RCS over wide band. The flexible general minimal residual (FGMRES) [19, 20] algorithm is applied to solve linear systems. The dimension size of Krylov subspace is set to be 30 for outer iteration and the dimension is set to be 10 for inner iteration. The tolerance of inner iteration is 0.1 in this paper. All experiments are conducted on an Intel Core II 8300 with 1.96 GB local memory and run at 2.66 GHz in single precision. The iteration process is terminated when the 2-norm residual error is reduced by 10^{-3}, and the limit of the maximum number of iterations is set as 1000.



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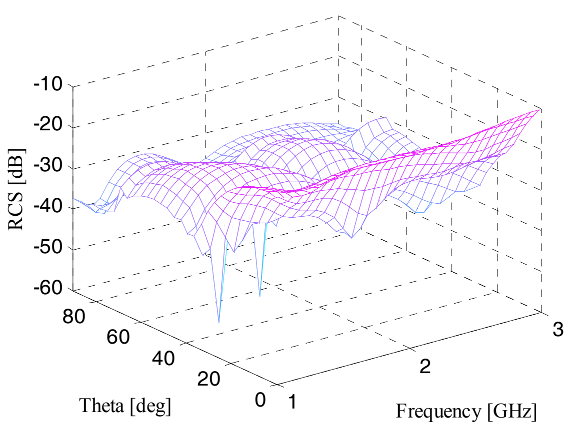

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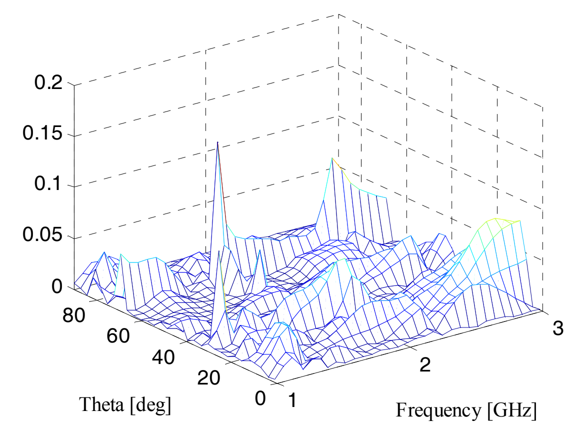
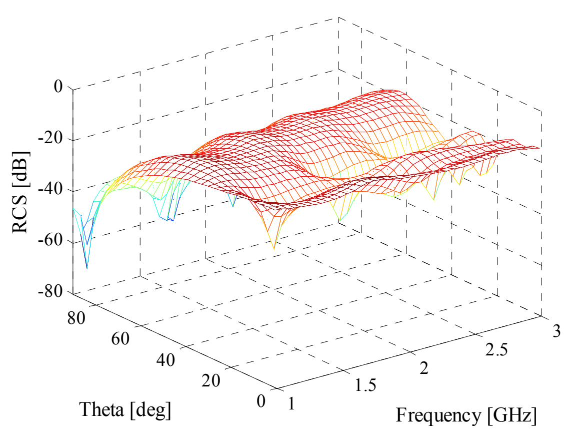


(c)

Fig. 1. Numerical results of NASA almond: (a) without interpolation; (b) hybrid interpolation; (c) numerical error.



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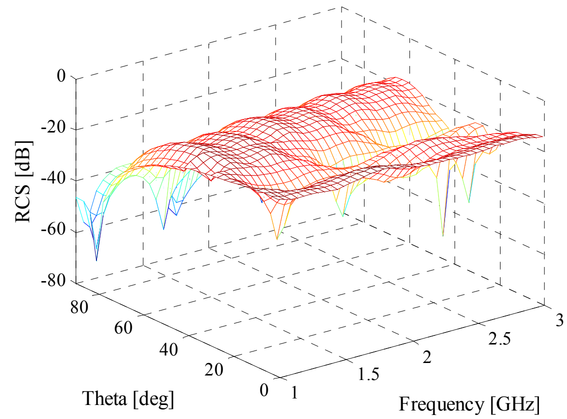

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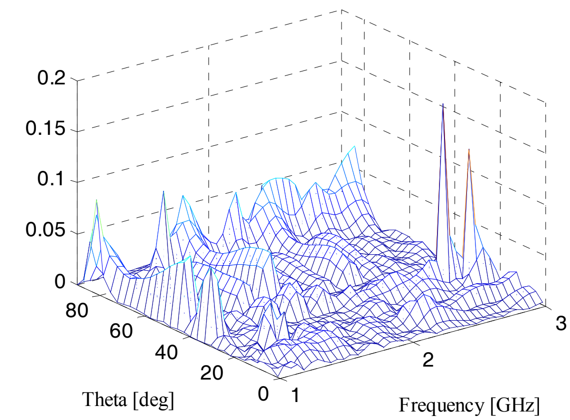
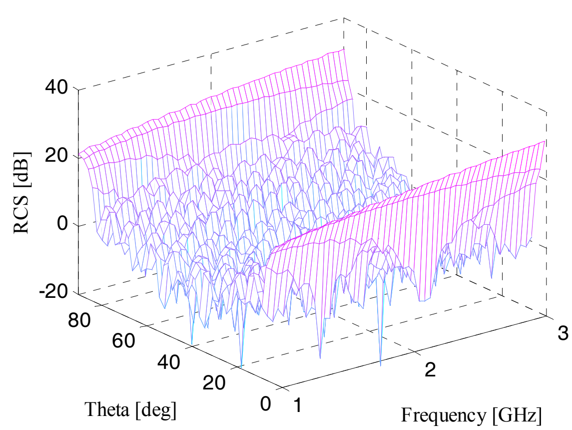


(c)

Fig. 2. Numerical results of PEC Ogive: (a) without interpolation; (b) hybrid interpolation; (c) numerical error.



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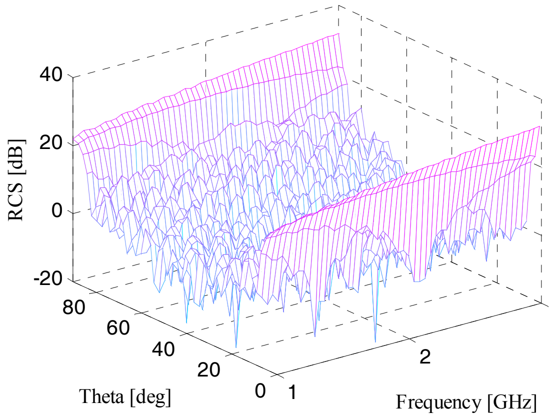

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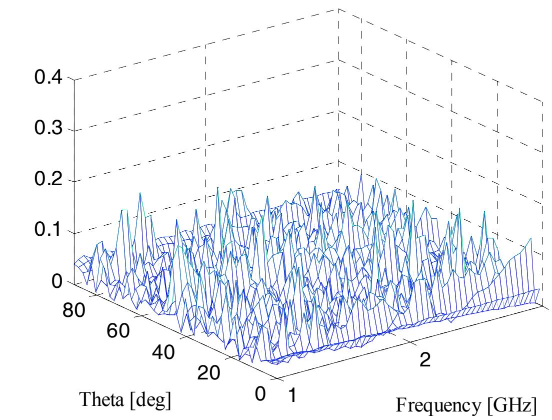

(c)

Fig. 3. Numerical results of metallic cube: (a) without interpolation; (b) hybrid interpolation; (c) numerical error.

Three geometries are applied to illustrate the performance of our proposed hybrid method. They consist of a NASA almond with 1815 unknowns [21], a PEC ogive with 2571 unknowns [21] and a metallic cube (0.8m × 0.6m × 0.6m) with 51174 unknowns. As shown in Table 1, The direction of the incident plane wave is $\theta = 0 \sim 90°$, $\varphi = 0°$ for all examples. The frequency bands are 2 ~ 5 GHz, 1 ~ 3 GHz and 1 ~ 3 GHz for these three examples, respectively.

In our simulations, when using the interpolation method proposed in this paper, we decide the number of sample points by doing several times of trials. Eleven frequency samples and 6 angular samples are required for the first example. Nine frequency samples and 6 angular samples are required for the second example. Twenty-one frequency samples and 19 angular samples are required for the third example. Figure 1(a), Fig. 2(a), and Fig. 3(a) show the frequency- and angular- sweep RCS of the three geometries calculated without the interpolation method. When compared with the results shown by Fig. 1(b), Fig. 2(b), and Fig. 3(b), it can be observed that there is little difference between the frequency- and angular- sweep RCS obtained with and without our hybrid method. In order to demonstrate the accuracy of the novel method, the numerical error of all examples is shown in Fig. 1(c), Fig. 2(c), and Fig. 3(c). The relative error of scattering field is defined by

$$Error = \frac{\left|\mathbf{E}^{inter} - \mathbf{E}^{sca}\right|}{\left|\mathbf{E}^{sca}\right|}, \tag{6}$$

where E^{inter} is the electric field computed by the interpolation method while E^{sca} is computed by the direct solution of integral equations repeatedly at each frequency and angular points. It can be concluded that almost the same results can be obtained whether the GTD-based cubic spline interpolation method is applied or not.

Table 2: Total solution time for fast frequency- and angular- sweep

Object	Unknown	Frequency	Frequency samples	Angle	Angular samples	Without interpolation	Interpolation
Almond	1815	2~5 GHz	11	0° ~90°	6	5.4 h	33.3 min
Ogive	2571	1~3 GHz	9	0° ~90°	6	6.9 h	47.6 min
Cube	51174	1~3 GHz	21	0° ~90°	19	24.9 h	80.6 min

As shown in Tab. 2, the total solution time for computing frequency- and angular- sweep RCS are compared between the traditional method and the interpolation method for these three examples. It can be found that the computational cost of the interpolation method is much less. The main cost of the GTD-based cubic spline interpolation method is the time for frequency- and angular-samples. This table also lists the number of unknowns, frequency band, and angular band for all these three examples. It can be also found by comparison that the large calculation time can be saved when the hybrid interpolation technique is used.

V. CONCLUSIONS

In this paper, the hybrid interpolation technique is proposed for the efficient analysis of the scattering from electrically large objects over a wide frequency- and angular- band. The MLFMA and FGMRES iterative solver are used to accelerate the convergence. This hybrid interpolation technique is combined with both the GTD scattering center model and the cubic spline interpolation method. The GTD model is used for frequency sweep and cubic spline interpolation method is used for angular sweep. Numerical experiments demonstrate that our proposed hybrid interpolation method is more efficient when compared with the method solving linear systems at each frequency and angle repeatedly for electromagnetic scattering from the electrically large objects.

From the numerical results, the relative error of the RCS results computed by the proposed method rates up to 0.25 (25%). Although the large error occurs at the point with low RCS value (less than -30dB), this method is suitable for pre-designer not suitable for critical solution.

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