An Efficient Broadband Analysis of an Antenna via 3D FEM and Pade Approximation in the Frequency Domain

Brahim Essakhi[1], Lionel Pichon[2]
LGEP, UMRS 8507 CNRS, SUPELEC,
UNIVERSITE PARIS SUD, UNIVERSITE PIERRE ET MARIE CURIE
Plateau du Moulon, 91192 Gif-sur-Yvette, France
[1] essakhi@lgep.supelec.fr [2] pichon@lgep.supelec.fr

Abstract **–** The paper presents a finite element model for the calculation of the impedance of an antenna over a wide frequency band. The antenna has been designed to analyze a rectenna (rectifying antenna) in the context of wireless microwave energy transfer. The modelling approach combines a 3D edge element method (FEM) with a Padé approximation procedure. It allows to obtain an explicit expression of the impedance over a large frequency band. The comparison of the proposed technique with a standard finite element method shows that the computational cost is significantly reduced.

I. Introduction

Microwave rectennas (rectifying antennas) are devoted to power transmission and detection. Applications include long distance power beaming, signal detection and wireless control systems. Power transmitting and receiving systems must be designed so that the power transmitted from the transmitting antenna is transmitted efficiently to the rectenna and is converted into DC power by rectifiers. Efficient field-circuit simulations are required in the design and characterization of such rectennas since non linear lumped elements are included. The knowledge of the input impedance of the antenna is of particular importance in evaluation the conversion efficiency.

In [1] a rectenna structure involving a loop antenna was studied. This rectenna is devoted to low-power applications. The targeted applications include microwave power reception from various sources in a large frequency range. The operating frequency may belong to the industrial, scientific and medical band (central frequency of 2.45 GHz). Also investigations include radio-frequency identification (RFID) applications where wideband signals may be used. For these reasons a reliable circuit model of the rectenna allowing a global simulation over the band [0, 20] GHz was required. The circuit model takes into account both distributed electromagnetic portions of the antenna and the rectifier circuit. From the 3D electromagnetic modelling of the structure the input impedance was obtained as a function of the frequency. This technique provides an adequate way to incorporate the impedance into a non-linear circuit simulation. With such an approach the impedance has to be calculated over a wide frequency band with a 3D modeling tool. This can be achieved as in [1] with a standard frequency domain method (boundary element method or finite element method for example).

A finite element model provides an efficient way for solving electromagnetic problems. In a frequency domain analysis the electromagnetic fields are discretized over a meshed volume. The unknowns are the solution of a linear system whose matrix depends on frequency. In a wide frequency band analysis the linear system has to be solved for each frequency of interest. This often leads to a huge computational cost. An alternative approach is to search for a power series expansion of the solution about a center frequency. The approach requires only one single matrix inversion. The radius of convergence is limited but it is possible to extend the interval using a corresponding Padé approximant. This technique is known as an asymptotic wave form analysis (AWE).

The AWE approach has been combined with integral equations in 3D [2]-[4] to solve scattering problems involving perfectly metallic obstacles. For solving general electromagnetic problems including inhomogeneous media and complex geometries, the finite element method (FEM) provides a powerful tool. The AWE approach used in connection with FEM was shown to deal with electromagnetic problems within bounded domains in [5]-[7] where passive microwave devices such as waveguides and cavities were studied. In these almost closed structures the boundaries of the studied domain consist of perfectly conducting walls or access planes. Then the efficiency of AWE relies on the fact that dominant poles and zeros of the network transfer function can be used to build rational approximations of the solution. Indeed in such a case the resonant modes, can be computed in a first step from a generalized eigenvalue problem and can be used in a second step to give an expansion of the solution. On the other hand, for electromagnetic problems in unbounded domains like radiation of an antenna in free space or scattering problems the fields cannot be expressed with resonant modes of the structure and efficient

Padé approximation are difficult to obtain. An extension of AWE combined with finite elements for radiation problems has been proposed in [8] for the 2D case. In a 2D analysis the structure is infinite along one direction and the electromagnetic problem reduces to a scalar wave problem (TM case or TE case). In such a configuration one of the two fields (electric or magnetic) has the same direction that the infinitely long structure. The ability of this method was demonstrated in the scattering of canonical obstacles having simple shapes.

In this work the AWE technique used in conjunction with finite elements is successfully extended in three dimensions (3D) for solving radiation problems in free space. The approach combines the vectorial finite element method and a Padé approximation. The numerical method is shown to provide a fast computation of the impedance of an antenna over a wide frequency band. The antenna is the loop antenna of the rectenna considered in [1] for which only a three dimensional analysis allows to obtain the distribution of the electromagnetic fields. The method is based on first order edge finite elements. A Silver-Müller boundary condition is used for the truncation of the domain. Once the finite element matrix has been built for one frequency an explicit expression (power series) of the fields and the impedance are available over a frequency band. From the power series a Padé approximation (rational function) can be derived. It is shown that a very good approximation is obtained even if several sharp resonance peaks are included in the studied range. The comparisons between the presented technique and a standard finite element analysis clearly underline the advantages of the proposed model.

II. Electromagnetic Problem

We consider the 3D problem of an antenna radiating in its surrounding medium. The dimensions of the studied device are shown in Fig. 1. The loop is assumed to be infinitely thin and perfectly conducting. However for the sake of generality the induced current density J is included in the equations since the presented analysis remains also valid in this case. The loop is excited by an impressed current J_{imp} between the ends of the two arms.

For a full wave analysis we deal with the Maxwell equations in the frequency domain:

$$\nabla \times \mathbf{E} = -j\omega \mathbf{B} \quad (1)$$

$$\nabla \times \mathbf{H} = \mathbf{J}_{imp} + \mathbf{J} + j\omega \mathbf{D}, \quad (2)$$

where E and H are the electric and magnetic field respectively, μ is the permeability, ε is the permittivity.

The constitutive relations are given by:

$$\mathbf{B} = \mu \mathbf{H} \quad (3)$$

$$\mathbf{D} = \varepsilon \mathbf{E}. \quad (4)$$

The Ohm law gives:

$$\mathbf{J} = \sigma \mathbf{E}. \quad (5)$$

From equations (1-4), we establish the vector wave equation in terms of the electric field E:

$$\nabla \times \frac{1}{\mu} \nabla \times \mathbf{E} + j\omega(\sigma + j\varepsilon\omega)\mathbf{E} = -j\omega \mathbf{J}_{imp}. \quad (6)$$

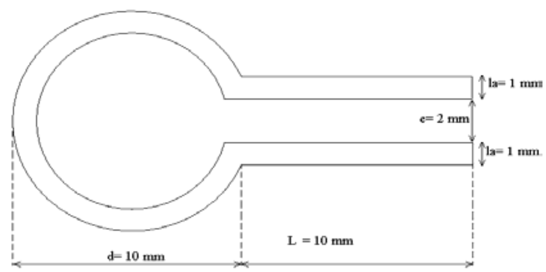

Fig. 1. The loop antenna and its dimensions.

III. Finite Element Formulation

A computation using the finite element method is performed in a finite region which includes the antenna and some of its surrounding medium. In order to truncate the volume of the computational domain the Silver-Müller condition is applied as an absorbing boundary condition [9]. It is given by:

$$n \times \nabla \times \mathbf{E} = jk\mathbf{E}_{\tan} \quad (7)$$

where k is the wave number in free space and $\mathrm{E}_{\tan}$ is the tangential electric field on the outer boundary surface.

This ABC preserves the sparsity and symmetric features of the final matrix. It is exact for normal incidence. Let denote Ω the computational domain and Γ the outer boundary. As usual with FEM, we define the space of the work:

$$V = \left\{ u \in (L^2(\Omega))^3, \nabla \times u \in (L^2(\Omega))^3 \right\}. \quad (8)$$

A weak formulation of the problem is obtained after multiplying the vector wave equation by a test function F in V:

$$\langle j\omega(\sigma + j\varepsilon\,\omega)\mathbf{E},\mathbf{F}\rangle_\Omega + \langle \frac{1}{\mu}\nabla\times\mathbf{E},\nabla\times\mathbf{F}\rangle_\Omega +$$

$$\langle j\omega\sqrt{\frac{\varepsilon}{\mu}}\mathbf{E},\mathbf{F}\rangle_\Gamma = -j\omega\langle \mathbf{J}_{imp},\mathbf{F}\rangle_\Omega \qquad (9)$$

where $\langle,\rangle_\Omega$ denotes the scalar product in *V*.

To solve equation (9) numerically, the domain is discretized with tetrahedral elements. The electric field can be written in terms of basis functions associated with the edges of these elements [10, 11]. From equation (9) and by using test functions F the same as interpolation functions (Galerkin method) we get:

$$\left(A_0 + \omega A_1 + \omega^2 A_2\right) v = b \qquad (10)$$

where *v* is the unknowns' vector, *b* is the excitation currents vector and A_0, A_1, and A_2 are matrices which only depend on the mesh and on the medium.

IV. An Efficient Computational Scheme for Broadband Analysis

Consider an arbitrary ω_0 such that A_0 is non-singular, the Taylor series expansion of the matrix polynomial in equation (10), about the frequency ω_0 can be written as:

$$A(\omega) = \overline{A}_0 + (\omega-\omega_0)\overline{A}_1 + (\omega-\omega_0)^2\overline{A}_2 \quad (11)$$

where the matrices A_i, $(i = 0,1,2)$ can be obtained from equation (10) and equation (11):

$$\overline{A}_0 = A_0 + \omega_0 A_1 + \omega_0^2 A_2, \qquad (12a)$$

$$\overline{A}_1 = A_1 + 2\omega_0 A_2, \qquad 12b)$$

$$\overline{A}_2 = A_2. \qquad (12c)$$

The solution vector $v(\omega)$ has power series representation about ω_0, given by:

$$v(\omega) = \sum_{i=0}^{\infty} v_i\,(\omega-\omega_0)^i . \qquad (13)$$

The power series representation of the excitation vector $b(\omega)$ is written as:

$$b(\omega) = \sum_{i=0}^{\infty} b_i\,(\omega-\omega_0)^i . \qquad (14)$$

We can evaluate the coefficients of the power series of $v(\omega)$ by the following procedure:

$$\left(\overline{A}_0 + \left(\omega-\omega_0\right)\overline{A}_1 + \left(\omega-\omega_0\right)^2\overline{A}_2\right)(v_0 + \ldots$$

$$v_1\,(\omega-\omega_0) + \ldots) = b_0 + b_1\,(\omega-\omega_0) + \ldots \qquad (15)$$

If we equate both sides of equation (15) term by term, we obtain the following iterative expression:

$$v_i = \overline{A}_0^{-1} b_i - \sum_{j=1,\, j\le i}^{2} \overline{A}_0^{-1}\overline{A}_j v_{i-j} \quad , \qquad i = 0,1,\ldots. \qquad (16)$$

It is very important to note that only a single inverse $\overline{A}_0^{-1}$ is needed in the iteration procedure.

V. Padé Approximation

A Padé approximation is derived by expanding a function as a ratio of two power series and determining both the numerator and denominator coefficients. Padé approximations are usually superior to Taylor expansion when functions contain poles, because the use of rational functions allows a good representation, and it provides an extension beyond the interval of convergence of the series [12]. The solution is expressed as a power series of the form:

$$v(\omega) = \sum_{i=0}^{\infty} v_i\,(\omega-\omega_0)^i , \qquad (17)$$

where the coefficients v_i, $i = 0,1,2,\ldots.$ can be computed iteratively using (16). The expansion is convergent within the region $\left|\omega-\omega_0\right| < R$, where *R* is the radius of convergence of this power series.

Since $v(\omega)$ is complex, and since the Padé approximants are rational functions, we concentrate on a single component of $v(\omega)$, say $v^j(\omega)$, and we write its power series representation in the form:

$$v^j(\omega) = \sum_{i=0}^{\infty} v_i^j(\omega-\omega_0)^i , \qquad (18)$$

where the coefficient v_i^j is scalar. A Padé approximant of the power series equation (18) is a rational function of the form:

$$[N/M](\omega) = \frac{Q_N(\omega)}{P_M(\omega)}, \qquad (19)$$

where

$$Q_N(\omega) = \sum_{i=0}^{N} q_i\,(\omega-\omega_0)^i \text{ et}$$

$$P_M(\omega) = \sum_{i=0}^{M} p_i\,(\omega - \omega_0)^i \,. \tag{20}$$

We take $p_0 = 1$, the $M+N+1$ unknowns can be obtained by the condition that the equation

$$v^j(\omega) \approx [N/M](\omega)\,, \tag{21}$$

holds up to terms $O(v^{N+M+1})$. This equation implies that

$$Q_N(\omega) = P_M(\omega)\sum_{i=0}^{\infty} v_i^j(\omega - \omega_0)^i \,. \tag{22}$$

Then we have

$$\sum_{i=0}^{N} q_i\,(\omega - \omega_0)^i = \sum_{i=0}^{\infty} l_i(\omega - \omega_0)^i \tag{23}$$

where

$$l_i = \sum_{k=0}^{i} v_{i-k}^j p_k \,. \tag{24}$$

Hence q_i and p_i can be determined from the following system

$$p_0 = 1 \tag{25a}$$

$$q_i = \sum_{k=0}^{i} v_{i-k}^j q_k \,, \quad \text{if } 1 \le i \le N \tag{25b}$$

$$p_i = 0 \qquad , \text{ if } i > N \tag{25c}$$

$$\sum_{k=1}^{i} v_{i-k}^j p_k = -p_0 v_i^j \,, \text{ if } N < i < N+M \tag{25d}$$

$$p_i = 0 \,, \text{ if } i > M \,. \tag{25e}$$

Hence the unknown coefficients of the Padé approximant can be determined from linear system. We use the diagonal Padé approximation $(N = M)$ which is more accurate; in this case we have $2N+1$ unknown coefficients.

VI. Numerical Results

Figure 2 shows a typical mesh used in the computation for the loop and the surrounding air. The electromagnetic analysis was performed over a broad band I = [0 GHz, 20 GHz]. In a first step, the studied frequency band is divided in L = 4 intervals $I_i, i=1,..,L$ such that I_1=[0 GHz, 5 GHz], I_2=[5 GHz, 10 GHz], I_3=[10 GHz, 15 GHz] and I_4=[15 GHz, 20 GHz]. In each band the centre frequency has been chosen in the middle.

We denote $[N/M]_i(\omega), i = 1,..,L$ the Padé approximation of the impedance in each interval I_i. In this case $N=M=2$. The impedance can be written as:

$$Z(\omega) = \sum_{j=1}^{L} Z^j(\omega)\,, \tag{26}$$

where

$$Z^j(\omega) = \begin{cases} [2/2]_j(\omega) & \text{if} \quad \omega \in I_j \\ 0 & \text{otherwise} \end{cases} .$$

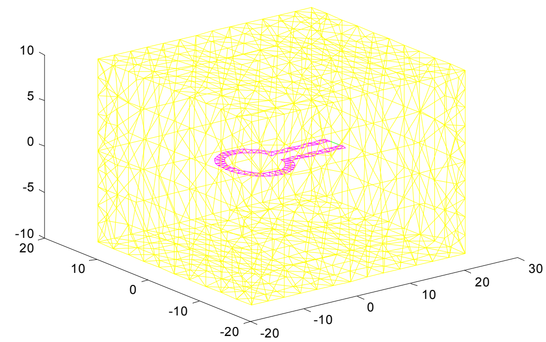

Fig. 2. The volume of the computation.

The comparison between a standard finite element model and the Padé approximation is shown on Fig. 3 and Fig. 4. The two curves are in an excellent agreement over the whole wide frequency band. In the standard approach the finite element problem has been solved for a number of 200 frequencies to obtain the behaviour of the curve. With the Padé approximation only 4 frequencies are needed. The computational cost is then significantly reduced since the amount of time required to find the Padé coefficients is negligible.

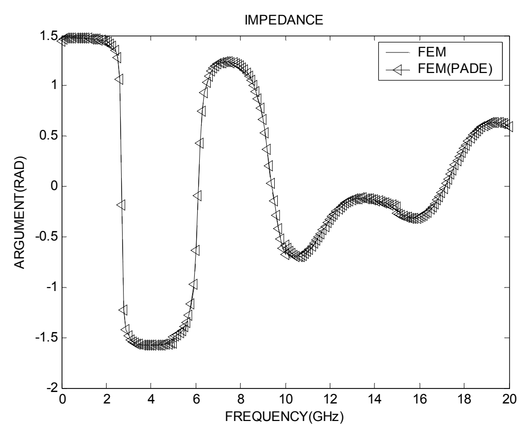

Fig. 3. Argument of the impedance : The comparison between 3D model and the Pade approximation [2/2] over four bands.

For three bands it was shown that the Padé approximant [3/3] gives very good results. Thus, for two bands a [3/3] approximation gives a bad approximation at the junction between the two bands Fig. 5. In this case an increase of the order of the Padé approximant is required: we show the corresponding results for a [5/5] approximation in Fig. 6.

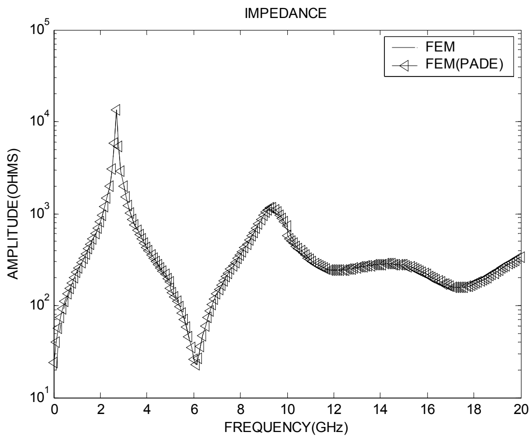

Fig. 4. The Amplitude : The comparison between 3D model and the Pade approximation [2/2] over four bands.

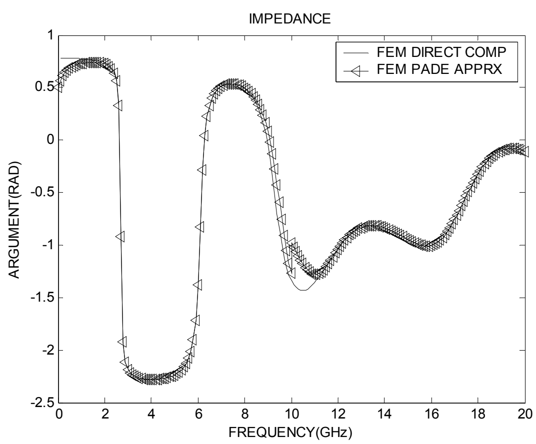

Fig. 5. Argument of the impedance: The comparison between 3D model and the pade approximation [3/3] over two bands.

VII. Conclusion

This paper describes an accurate and very efficient FEM approach to compute the impedance of an antenna over a wide frequency band. The electromagnetic problem addressed in this work is a fully three-dimensional one. In a standard finite element approach the linear system has to be solved for each frequency of interest. The power series expansion method presented in this work resolves this problem very efficiently. The advantage of this approach is that only a single resolution of the linear system is required to evaluate the series expansion using a Padé approximation. Then the approach allows one to cover a whole given frequency band with a minimum number of resolution of the linear system. In particular the resonance peaks are very well recovered. It is worth to be noted that satisfactory results are obtained when using a Silver-Müller radiation condition: the impedance is computed from the near-field and numerical values near the antenna are not very sensitive to the boundary condition. Work is in progress to combine a Padé approximation with PML (perfectly matched layer) in order to address more general radiation problems in three dimensions for which global quantities have to be obtained over a wide band.

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Brahim Essakhi was born in Rabat, in 1974. He obtained a Master's in Applied Mathematics from Ecole Normale Supérieure (ENS) in Cachan (France) in 2001. In 2002 he joined the Laboratoire de Génie Electrique de Paris (LGEP) associated to CNRS, SUPELEC, University of Paris-Sud and University of Pierre et Marie Curie where he earned a PhD in 2005. His main scientific interests are numerical methods, wave propagation modeling, and electromagnetic compatibility.

Lionel Pichon was born in Romorantin, France, in 1961. He obtained the Dip. Eng. from Ecole Supérieure d'Ingénieurs en Electronique et Electrotechnique (ESIEE) in 1984. In 1985 he joined the Laboratoire de Génie Electrique de Paris LGEP associated to CNRS, SUPELEC, University of Paris-Sud, University of Pierre et Marie Curie where he earned a PhD in 1989. He is now Chargé de Recherche at the CNRS. His research interests include computational electromagnetics in microwaves, and electromagnetic compatibility.