Genetic Algorithm Optimization of a Traveling Wave Array of Longitudinal Slots in a Rectangular Waveguide

(*Invited Paper*)

Anders Jensen[1] and Sembiam R. Rengarajan[2]
[1] Cushcraft Corporation, Carlsbad, CA 92010
[2] Department of Electrical and Computer Engineering, California State University, Northridge, CA 91330
e-mail: [1]anders.jensen@ieee.org, [2]srengarajan@csun.edu

Abstract — Genetic Algorithm is used to optimize the performance characteristics of a traveling wave array consisting of longitudinal slots cut in the broad wall of a rectangular waveguide. An analysis method employing a loaded transmission line to model the slot arrays is employed. External mutual coupling is considered. The self admittance of the radiating slots is computed using the method-of-moments technique applied to the pertinent integral equations. Numerical results indicate typical performance improvements possible using the genetic algorithm optimization.

I. INTRODUCTION

Longitudinal slots cut in the broadwall of a rectangular waveguide have been employed as radiating elements in linear and planar arrays for numerous radar and communication applications. The design of a traveling wave linear array of longitudinal slots was presented by Elliott circa 1977 [1]. A design procedure developed by Elliott is applicable to both traveling wave and standing wave arrays [2]. That procedure uses an iterative technique to design the array at the center frequency of the desired bandwidth. Usually the array performance is optimum at the design frequency and it degrades away from the center frequency. Optimizing an array with respect to any performance parameter is a multi-dimensional problem. Due to the presence of many extrema, local optimization techniques such as conjugate gradient and Fletcher-Powell minimization techniques may not be well-suited to this work. Genetic Algorithms (GA) has the ability to search hyperplanes extensively and they are less susceptible to get stuck at local maxima [3]. In this work we investigated the optimization of different performance parameters of a traveling wave array of longitudinal slots using GA. Many case studies are shown for the 21-element array discussed by Elliot in [1]. The purpose of this work is to show typical performance improvements possible in traveling wave arrays designed using the genetic algorithm optimization.

II. THE METHOD OF ANALYSIS

A. Self Admittance

The analysis employs a moment method solution to the pertinent integral equation for the aperture electric field of a single slot when excited by an incident TE_{10} wave. From the aperture electric field the TE_{10} mode scattered wave coefficients in the forward and backward directions are determined. Since the slot scattering is very nearly symmetric, a shunt admittance model is found to be an excellent assumption. The self admittance of the slot is determined as a function of slot offset and length from the scattered wave coefficients. The basic equations of the method of moments analysis are shown here leaving all the details since the method is very similar to that presented in [4].

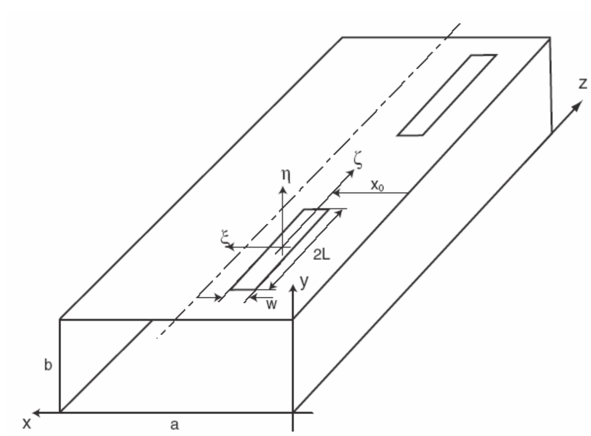

Fig. 1. Longitudinal slots cut in the waveguide broadwall.

The continuity of the tangential component of the aperture electric field is implicitly satisfied. The boundary condition for the dominant component of the magnetic field in each aperture of the thick slot is enforced as follows,

$$H_z^{ext}(P) - H_z^{int}(P) = H_z^{inc}(P), \qquad (1)$$

$$H_z(P) = j\omega\varepsilon \int_{slot} G(P,P')\left[-\hat{n}xE_x\hat{x}\right]ds'. \qquad (2)$$

Equation (1) satisfies the continuity of the magnetic field across the slot aperture. For simplicity only one equation is shown. For a thick slot two equations are used so as to satisfy the boundary conditions at the interior as well as the exterior aperture. The Green's function inside the integral in (2) is that of the rectangular waveguide, a cavity formed by the thick slot with both openings shorted by thin conducting sheets, or the exterior half space respectively for each of the three regions. The complete expressions for all the Green's functions are found in [4]. Global sinusoidal expansion functions describe the longitudinal variation of the electric field across the slot in (3). A uniform transverse distribution is assumed in (3). The weighting functions have similar longitudinal variation but exhibit a delta function in the transverse direction as shown in (4),

$$E_x(P') = \sum_{q=1}^{N} E_q \sin\left[\frac{q\pi}{2L}(\varsigma' + L)\right], \qquad (3)$$

$$w_p(P) = \sin\left[\frac{p\pi}{2L}(\varsigma + L)\right]\delta(\xi). \qquad (4)$$

The primed coordinates (ζ',ξ') along and across the slot are used for the source region and unprimed coordinates (ζ,ξ) in Fig. 2 denote the field region. The integral equations are reduced to matrix equations and their solution yields the coefficients of expansion of the aperture electric field. It is then possible to obtain the back scattering coefficient, Γ and the normalized self admittance as shown below [4],

$$\frac{Y}{Y_0} = \frac{-2\Gamma}{1+\Gamma}. \qquad (5)$$

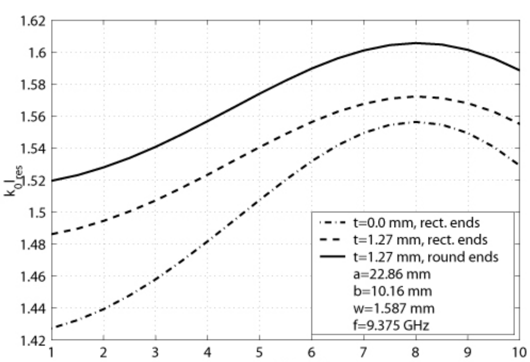

Fig. 2. Normalized resonant length versus slot offset.

The normalized self admittance computed in (5) is an important parameter in the design optimization. First, the normalized resonant length $k_0 l_{res}$ versus slot offset from the centerline of the broadwall is found as shown in Fig. 2 for a range of values of slot offsets. Fig. 2 shows that the resonant length is dependent on the slot thickness and hence the data need to be computed for the specific wall thickness of interest. The value of slot length, $2l_{res}$, that makes the phase of the back scattered TE_{10} mode wave equal to 180^o with respect to the incident wave electric field, both referenced to a plane passing through the center of the slot, is said to be the resonant length [4]. The real and imaginary parts of the self admittance as a function of the slot offset and the normalized length of the slot with respect to free space wavelength are then obtained. These data are shown in Figs. 3 and 4, respectively, over a range of values of slot offsets and lengths normalized to resonant lengths. The computed MoM data for the slot admittance as a function of the slot offset and normalized length shown in Figs. 2 through 4 are cast in the form of Stegen normalization [5]. The data are then curve-fitted as fourth order polynomials using the least mean square error criterion. The polynomial representations are easy to work with in the GA optimizations.

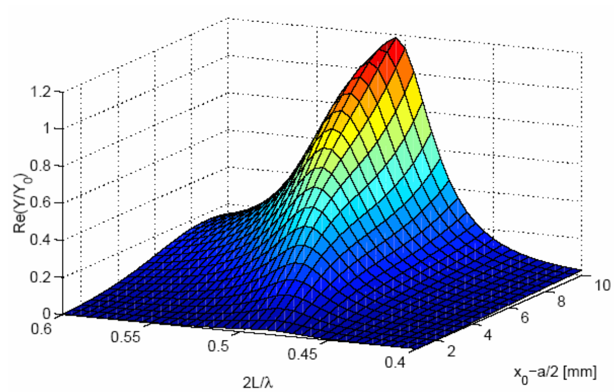

Fig. 3. Equivalent slot conductance versus slot offset and length.

B. Array Analysis Procedure

Elliott's design equations were rearranged by Hamadallah [6] to facilitate the analysis of a slot array. The direct method of analysis discussed in [6] is used in the GA optimization process. The basic analysis equations alone are reproduced here for ready reference and the details are omitted. The complete equations and the definitions of all symbols are found in [2, 6]. The slot voltages found in the column matrix on the left side of (6) are computed directly from the solution of the matrix equation (6).

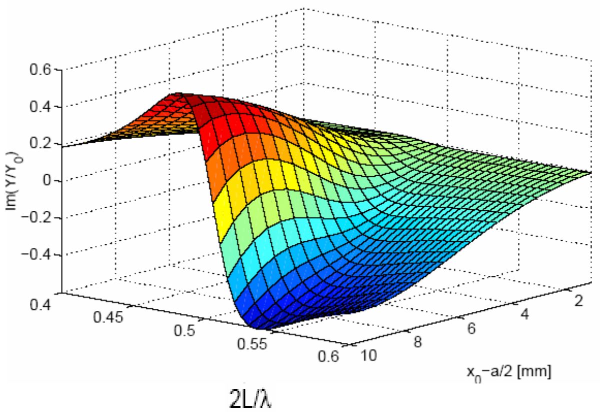

Fig. 4. Equivalent slot susceptance versus slot offset and length.

C. Array Analysis Procedure

Elliott's design equations were rearranged by Hamadallah [6] to facilitate the analysis of a slot array. The direct method of analysis discussed in [6] is used in the GA optimization process. The basic analysis equations alone are reproduced here for ready reference and the details are omitted. The complete equations and the definitions of all symbols are found in [2, 6]. The slot voltages found in the column matrix on the left side of (6) are computed directly from the solution of the matrix equation (6),

$$\begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1N} \\ g'_{21} & g_{22} & & g_{2N} \\ \vdots & & \ddots & \\ g'_{N1} & g'_{N2} & & g_{NN} \end{bmatrix} \begin{bmatrix} V_1^s \\ V_2^s \\ \vdots \\ V_N^s \end{bmatrix} = \left(\frac{k_2}{k_1}\right) V_1 \begin{bmatrix} f_1 \cdot E_1 \\ f_2 \cdot E_2 \\ \vdots \\ f_N \cdot E_N \end{bmatrix} \tag{6}$$

where

$$\begin{aligned} g'_{mn} &= g_{mn} \quad if \quad m \ge n, \\ g'_{mn} &= g_{mn} - jk_2 f_m f_n \sin(n-m)\phi \quad if \quad m < n. \end{aligned} \tag{7}$$

The mutual coupling terms g_{mn} are given by

$$\begin{aligned} g_{mn} = \int_{-L_m}^{L_m} \int_{-L_n}^{L_n} \cos\left(\frac{\pi \zeta_m}{2L_m}\right) \cos\left(\frac{\pi \zeta'_n}{2L_n}\right) \frac{e^{-jkR}}{R^5} \cdot \\ \left[k^2 R^4 + \left(2 + 2jkR - k^2 R^2\right) \cdot \right. \\ \left. \left(z_c + \zeta'_n - \zeta_m\right)^2 - \left(1 + jkR\right) x_c^2 \right] d\zeta'_n \, d\zeta_m . \end{aligned} \tag{8}$$

The analysis procedure discussed in [6] provides the expression for the input admittance, not shown here, from which the input VSWR is also determined.

D. Genetic Algorithms

GA is one of the most popular global optimization techniques. GA optimizes the trade-off between exploring new evaluations and exploiting information computed previously. It has an implicit parallelism, wherein extensive search of hyperplanes is carried out without directly testing all hyperplanes. Its ability to maintain multiple solutions concurrently makes it less susceptible to problems of local maxima.

The length and offset of each slot are quantized into 64 values or 6 genes each. Thus for the 21-element slot array each set of input values of slot lengths and offsets may be represented by a chromosome of 252 genes. The population or family size used for these GA optimizations was 50, the cross over probability was 0.5 and the mutation probability was 0.02. A Fortran computer program developed by Carroll was used with the array analysis program [7]. During search a figure of merit is assigned to each array design (organism) according to the performance function (*pf*) as shown below,

$$pf = D^{\alpha}\left(1 - \frac{P_{load}}{P_{in}}\right)^{\beta} \frac{1}{VSWR^{\gamma}} \frac{1}{SLL^{\tau}} \tag{9}$$

where D is the directivity, SLL is the sidelobe level, and P_{load}/P_{in} is the ratio of power dissipated in the load to the input power. The directivity is calculated easily from the power radiated which is P_{in} - P_{ref} - P_{load} and the power density at the maximum far field direction. The population size was kept at 50. The values of α, β, γ and τ were varied depending on the parameters that were optimized. When the bandwidth was optimized the performance function was computed over five frequencies within the band.

III. NUMERICAL RESULTS AND DISCUSSION

The results presented here are for the 21-element traveling wave array investigated by Elliott [1]. The slots are cut in a standard X-band waveguide (a = 0.9 in, b = 0.4 in., wall thickness = 0.05 in.) using round ended slots of width 1/16 in. The design frequency is 9.375 GHz and the slot spacing is 0.685 in. for a beam peak at 45^{o}. The slots are all offset on the same side of the centerline to provide the correct phase of excitation. The results obtained in this work are typical of the improvements in performance achievable using GA optimization for a slot array. Initially we used the analysis equations [6] with MoM data for the above-mentioned traveling wave array and obtained the following results:

VSWR=1.05, P_{load}/P_{in}= 16.7%, SLL = -20.1 dB. These results compare well with the experimental results shown in Elliott's paper, VSWR=1.05 and SLL = -22 dB. The slight discrepancy in the SLL may be attributed to the edge diffraction effects of the finite ground plane, especially in the E plane. The value of P_{load}/P_{in} could not be measured but it was computed to be 12.3 % [1]. We believe that our computed results are more accurate than Elliott's since our results are closer to the experimental ones. In subsequent optimizations we nominally kept the slot data close to the original design data [1] and varied the values of slot lengths and offsets to optimize certain performance functions.

Figure 5 shows the GA optimized design where the VSWR and P_{load}/P_{in} are minimized with values of 1.011 and 2.1%, respectively. The SLL improved to -22.1 dB. Subsequently we optimized the sidelobe level to a low value of -25.2 dB. VSWR became 1.016 and P_{load}/P_{in} was 4.6%. The resulting radiation pattern is shown in Fig. 6. When the design was optimized for 5% bandwidth, the SLL is better than -21.9 dB and P_{load}/P_{in} is lower than 6.2% within the band. The radiation pattern at 9.375 GHz is shown in Fig. 7 and the computed input VSWR as a function of frequency is shown in Fig. 8. In the bandwidth optimization exercises performance parameters were computed at five frequency points within the bandwidth and minimized.

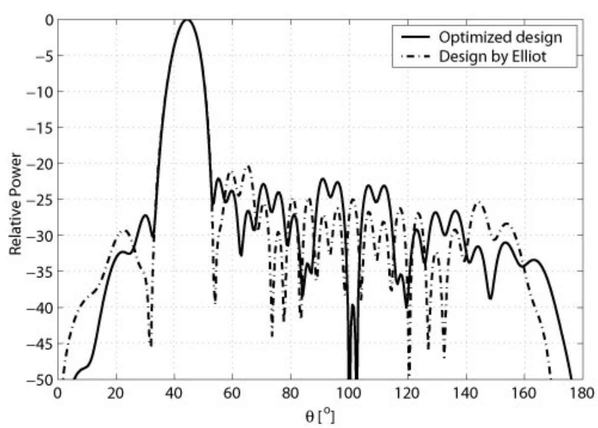

Fig. 5. Radiation pattern at 9.375 GHz (design optimized for VSWR and P_{load}).

The VSWR is better than 1.02 in the frequency range of interest in Fig. 8. When the design was optimized for 10% bandwidth, SLL was found to be better than -20.9 dB and P_{load}/P_{in} is lower than 7.1%. A typical radiation pattern is similar to that shown in Fig. 7. The computed input VSWR plot is shown in Fig. 9. Clearly the VSWR values are higher than those for the 5% bandwidth case but still the array is found to be well-matched.

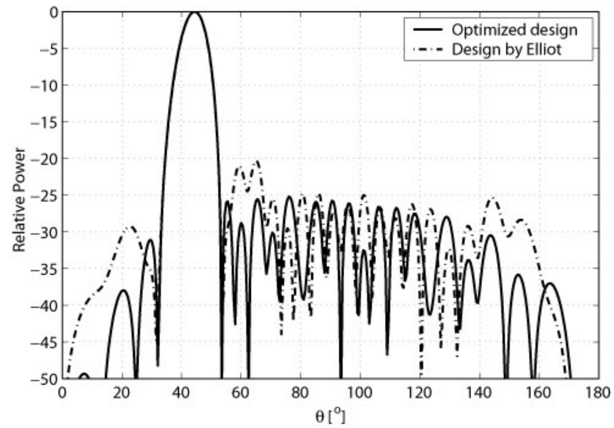

Fig. 6. Radiation pattern at 9.375 GHz (design optimized for sidelobe level).

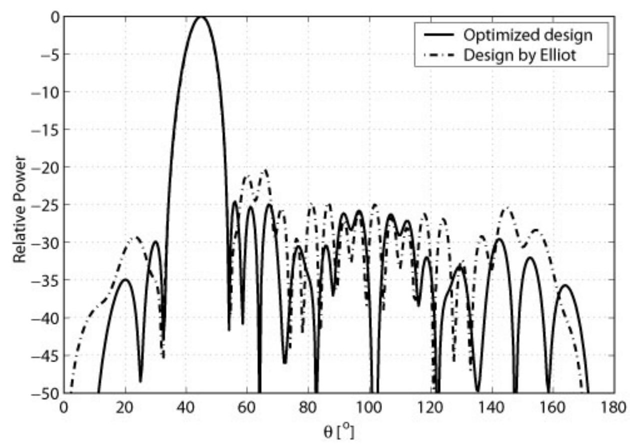

Fig. 7. Radiation pattern at 9.375 GHz (design optimized for 5% bandwidth).

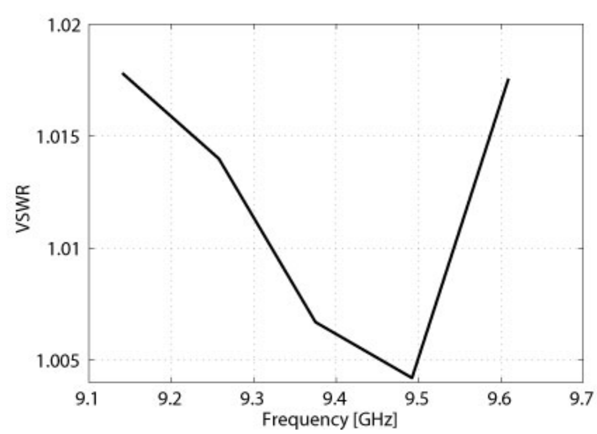

Fig. 8. VSWR versus frequency.

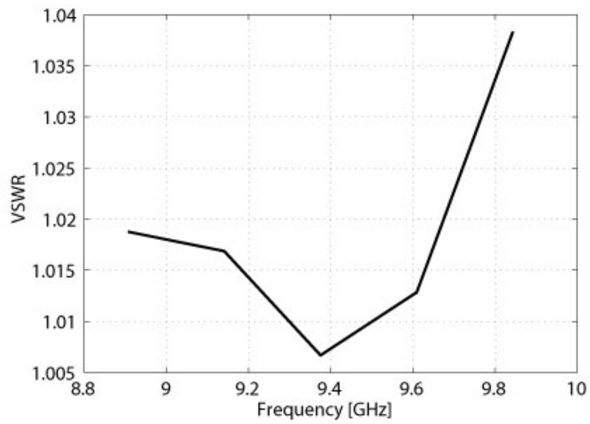

Fig. 9. VSWR versus frequency.

IV. CONCLUSIONS

We have presented the results of a genetic algorithm optimization of a traveling wave array of longitudinal slots cut in a rectangular waveguide. The input parameters, lengths, and offsets of slots were varied to optimize the performance functions such as directivity, SLL, input VSWR, power dissipated in the load, and bandwidth. GA optimizations produced significant improvement in performance over the conventional design.

REFERENCES

[1] R. S. Elliott, "On the design of traveling-wave-fed longitudinal shunt slot arrays," *IEEE Transactions on Antennas and Propagation*, vol. AP-25, no. 5, pp. 717-720, Sep. 1977.
[2] R. S. Elliott, "An improved design procedure for small arrays of shunt slots," *IEEE Transactions on Antennas and Propagation*, vol. AP-31, no. 1, pp. 48-53, Jan. 1983.
[3] D. Goldberg, Genetic Algorithms in Search, Optimization and Machine Learning, Addison-Wesley, 1989.
[4] L. G. Josefsson, "Analysis of longitudinal slots in rectangular waveguides," *IEEE Transactions on Antennas and Propagation*, vol. AP-35, pp. 1351-1357, 1987.
[5] R. S. Elliott, Antenna Theory and Design, Prentice Hall, Englewood Cliffs, NJ, 1981.
[6] M. Hamadallah, "Frequency limitations on broad-band performance of shunt slot arrays," *IEEE Transactions on Antennas and Propagation*, vol. 37, no. 7, pp. 817-823, July 1989.
[7] D. Carroll, University of Illinois, GA Driver.

Anders Jensen was born in Denmark in Feb 1975. He received his M.S.E.E. in 2000 from the Technical University of Denmark. He became a Member of IEEE in 2001. He was a visiting scholar at California State University, Northridge in 1999.

He has held positions at Saab Erricson Space in Sweden and Remec Inc in California. He is currently employed by Cushcraft Corporation as a Senior Designer Engineer in Carlsbad, California.

Sembiam Rengarajan received the Ph.D. degree in Electrical Engineering from the University of New Brunswick, Canada in 1980.

Since 1980 he has been with the department of Electrical and Computer Engineering, California State University, Northridge, CA, presently serving as a Professor. His experience includes periods at Bharat Electronics Ltd., India, NASA/JPL, UCLA, Chalmers University of Technology, Sweden, Universidad de Santiago de Compostela, Spain, Rome Laboratory of the US Air Force, the University of Pretoria, South Africa, Naval Researc Lab., Washington, D.C., and the Technical University of Denmark. He has served as a consultant to JPL, Lockheed Martin, Hughes Aircraft Company, Rantec, Saab Ericsson Space, Sweden, and the United Nations Development Program in India. His research interests include analytical and numerical techniques in electromagnetics with applications to antennas, scattering, and passive microwave components.

Dr. Rengarajan has authored/co-authored about 175 journal and conference papers. He is a Fellow of IEEE (1994), a member of USNC/URSI Commission B, and the Electromagnetics Academy. He served as the chair of the LA Chapter of IEEE Antennas and Propagation Society (1983-84), Chair of the San Fernando Valley Section of IEEE (1995), and as an Associate Editor of the IEEE Transactions on Antennas and Propagation (2000-2003). He was the chair of the Education Committee of the IEEE Antennas and Propagation Society and was an Associate Editor of the IEEE Antennas and Propagation Magazine. He has received several awards for his innovative research and technical contributions to NASA. He also received the Preeminent Scholarly publication Award from the California State University, Northridge in 2005 and a Distinguished Engineering Educator of the Year Award from the Engineers' Council of California in 1995. He serves as the secretary of the Commission B of the United States National Committee of the International Union of Radio Science (2002-present). He is an Adjunct Professor at the Electromagnetics Academy at Zhejiang University in China.