A Novel Space -Stepping Finite-Difference Frequency –Domain Method for Full Wave Electromagnetic Field Modeling of Passive Microwave Devices

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Abstract **–** In this paper a novel space-stepping finite-difference frequency-domain (SSFDFD) method is presented for the analysis of passive microwave devices. Unknown electromagnetic (EM) fields are solved from given EM fields at two initial planes space-step by space-step along a spatial direction. SSFDFD has the advantage over the traditional FDFD method in that all the EM field unknowns are local variables and the solution of a huge matrix equation is avoided. The stability condition for the SSFDFD method is derived as that for the finite-difference time-domain (FDTD) method. Application examples show that the stability condition is valid and the SSFDFD method is at least one magnitude faster than the traditional FDFD method with the same accuracy of analysis. SSFDFD has the potential to be a powerful and fast tool for full wave EM field modeling of passive microwave devices.

I. INTRODUCTION

Electromagnetic field analysis is very important in many scientific and technologic issues, among which full wave analysis has attracted much attention for high analysis accuracy at high frequencies. The finite -difference time-domain (FDTD) method has been one of the most widely used numerical full wave analysis methods. However, such time-domain simulations always consume huge computer time and memory space which is a big problem. The finite-difference frequency-domain (FDFD) method has been recognized as one of the most powerful numerical algorithms for full-wave solution of EM field problems because of its ability to take the frequency-dependent properties of material parameters into account [1, 2] and because FDFD simulates characteristics in the frequency domain directly. In FDFD the time variation of EM fields has the form of $e^{j\omega t}$ or e^{st}, thus the partial differential operator $\partial/\partial t$ in Maxwell's equations can be substituted with $j\omega$ or s. The remained spatial differential or integral Maxwell's equations are then approximated by central differences. The space grid mesh in FDFD is similar with the standard Yee's mesh of FDTD [3]. However, in all existing FDFD methods, the obtained difference equation is a very large matrix equation, the dimension of which is equal to the number N_v of the unknown EM field variables at all grid nodes. Although the coefficient matrix is a band diagonal matrix with bandwidth of 25, the number of multiplications needed to solve the equation by traditional Gauss elimination is still as large as N_v^{β} ($1.0<\beta<1.5$). In order to improve the efficiency of FDFD, some novel numerical methods usually used in circuit simulation, such as the CFH [4] and Pade approximation via Lanczos (PVL) [5, 6], have been implemented to accelerate the procedure for obtaining a response spectrum.

In Yee's FDTD, the EM fields can be solved from the initial condition (usually the electric fields at time t=0 and magnetic fields at time t=0.5Δt) time-step by time-step, and the EM fields at all grid nodes are local variables which are related only to the EM fields at the four surrounding nodes. Following this idea, a novel space-stepping finite-difference frequency-domain (SSFDFD) method is developed to analyze passive microwave devices. In SSFDFD, Maxwell's equations in the frequency-domain are first approximated with center difference, and a spatial coordinate axis, for example, the z-axis, is selected as the space stepping direction. Then the unknown EM fields are solved space-step by space-step along this space stepping direction from the given EM fields on two initial cross sectional planes perpendicular with the stepping direction. For passive microwave devices consist of waveguide structures, the longitude direction can be defined as the space stepping axis, and two transverse planes in the uniform guided wave structures can be chosen as the initial planes. All the EM fields to be solved at each node are local variables, which are related only to the EM fields at their neighboring nodes. Thus, the solution of a huge matrix equation as that in traditional FDFD is avoided. In SSFDFD, the number of multiplications for solution of all the unknown EM field variables is reduced to $2N_v$, and much computer memory can be saved because at each space step only the EM fields on the planes under analysis need to be saved. Application examples show that SSFDFD is at least one magnitude faster than the traditional FDFD and the finite element method (FEM). Therefore, the SSFDFD method could be useful to solve complex microwave circuits.

In FDTD the space steps (Δx, Δy, and Δz) and time step (Δt) must satisfy a stability condition to guarantee the algorithm stability [3, 7]. In this paper, it is pointed out that, as with FDTD, there is a stability problem with SSFDFD. The space step size and the angular frequency must satisfy a condition to guarantee the stability of SSFDFD. If SSFDFD steps in the z-axis direction, then the stability condition for SSFDFD is,

$$2c_{\max}\sqrt{\frac{1}{(\Delta x)^2}+\frac{1}{(\Delta y)^2}} \le \omega \le \frac{2c_{\min}}{\Delta z}$$

where $c_{\max}$ and $c_{\min}$ are the maximum and minimum possible velocity of the EM waves in the media, respectively. For a given angular frequency ω, relatively smaller space step (Δz) in the stepping direction of the algorithm and larger space steps (Δx and Δy) in the cross sectional plane are required for the stability of SSFDFD. Also, for a given reasonable set of space steps, there is a frequency band in which SSFDFD is stable.

The validity, efficiency, and stability conditions of the proposed SSFDFD are tested and verified by simulating the EM field response of the TE_{10} mode in a rectangular waveguide containing a dielectric segment. Two cases where the permittivity of the dielectric segment is first constant and then frequency -dependent are considered.

II. SSFDFD ALGORITHM

In order to clearly illustrate the SSFDFD algorithm, the components of Maxwell's equations in the frequency-domain are written in a modified order as

$$\frac{\partial E_x}{\partial z} = \frac{\partial E_z}{\partial x} - j\omega\mu(\omega)H_y \tag{1}$$

$$\frac{\partial E_y}{\partial z} = \frac{\partial E_z}{\partial y} + j\omega\mu(\omega)H_x, \tag{2}$$

$$E_z = \frac{1}{j\omega\varepsilon(\omega)+\sigma(\omega)}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right), \tag{3}$$

$$\frac{\partial H_x}{\partial z} = \frac{\partial H_z}{\partial x} + j\omega\varepsilon(\omega)E_y + \sigma(\omega)E_y, \tag{4}$$

$$\frac{\partial H_y}{\partial z} = \frac{\partial H_z}{\partial y} - j\omega\varepsilon(\omega)E_x - \sigma(\omega)E_x, \tag{5}$$

$$H_z = \frac{1}{j\omega\mu(\omega)}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) \tag{6}$$

where $\varepsilon(\omega)$, $\mu(\omega)$, and $\sigma(\omega)$ are the medium permittivity, permeability and conductivity, respectively, which can be frequency-dependent. Using the grid mesh in Fig. 1, equations (1) to (6) can be approximated by central difference as,

$$E_x(i,j,k+1) = E_x(i,j,k) + \frac{\Delta z}{\Delta x}\big[E_z(i+1,j,k) - E_z(i,j,k)\big] - j\omega\mu(\omega)\Delta z H_y(i,j,k) \tag{7}$$

$$E_y(i,j,k+1) = E_y(i,j,k) + \frac{\Delta z}{\Delta y}\big[E_z(i,j+1,k) - E_z(i,j,k)\big] + j\omega\mu(\omega)\Delta z H_x(i,j,k)\ , \tag{8}$$

$$E_z(i,j,k+1) = \frac{1}{\big(j\omega\varepsilon(\omega)+\sigma(\omega)\big)\Delta x}\big[H_y(i,j,k+1) - H_y(i-1,j,k+1)\big] - \frac{1}{\big(j\omega\varepsilon(\omega)+\sigma(\omega)\big)\Delta y} \cdot\big[H_x(i,j,k+1) - H_x(i,j-1,k+1)\big], \tag{9}$$

$$H_x(i,j,k+1) = H_x(i,j,k) + \frac{\Delta z}{\Delta x}\big[H_z(i,j,k+1) - H_z(i-1,j,k+1)\big] + \Delta z\,\big[j\omega\varepsilon(\omega)E_y(i,j,k+1) + \sigma(\omega)E_y(i,j,k+1)\big], \tag{10}$$

$$H_y(i,j,k+1) = H_y(i,j,k) + \frac{\Delta z}{\Delta y}\big[H_z(i,j,k+1) - H_z(i,j-1,k+1)\big] - \Delta z\,\big[j\omega\varepsilon(\omega)E_x(i,j,k+1) + \sigma(\omega)E_x(i,j,k+1)\big], \tag{11}$$

$$H_z(i,j,k+1) = \frac{1}{j\omega\mu(\omega)\Delta y}\big[E_x(i,j+1,k+1) - E_x(i,j,k+1)\big] - \frac{1}{j\omega\mu(\omega)\Delta x}\big[E_y(i+1,j,k+1) + E_y(i,j,k+1)\big]\ , \tag{12}$$

in which Δx, Δy , and Δz are grid sizes in the x, y, and z directions of the Cartesian coordinates, respectively. Next a spatial coordinate axis, for example, the z-axis, is selected as the stepping direction of the algorithm to be developed for SSFDFD. For a given value of index k, suppose the EM fields $\varphi(i,j,k)$ for all i and j have already been obtained, then the fields $\varphi\ (i,j,k+1)$ for all i and j can be computed by the following procedure:

1. Compute $E_x(i,j,k+1)$ and $E_y(i,j,k+1)$ from equations (7) and (8).
2. Compute $H_z(i,j,k+1)$ from equation (12).
3. Compute $H_x(i,j,k+1)$ and $H_y(i,j,k+1)$ from equations (10) and (11).
4. Compute $E_z(i,j,k+1)$ from equation (9).

So, if we set the EM fields $\varphi(i,j,0)$ at k=0 (actually, E_x, E_y and H_z at plane z=0 and H_x, H_y and E_z at plane $z=0.5\Delta z$ in Fig. 1) as the given "initial" condition, then all the EM fields at $k > 0$ can be obtained space-step by space-step along the z-direction. The characteristic electrical parameters (such as the S-parameters) for the structure under analysis can be extracted from the obtained EM fields using the approach given in [2].

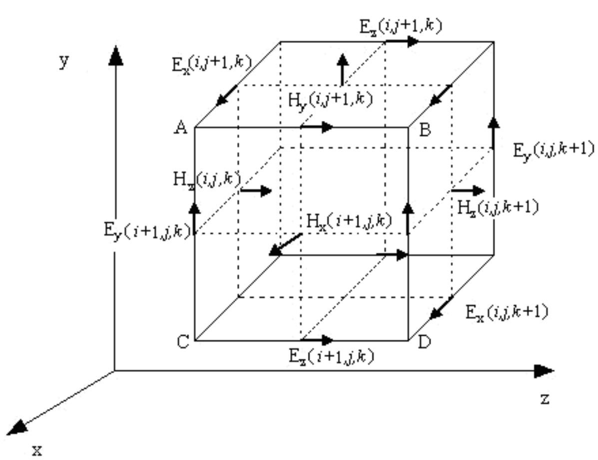

Fig. 1. The grid mesh for SSFDFD.

From the above SSFDFD algorithm, we can see that the unknown EM fields to be solved at each grid node are all local variables, which are only related to the EM fields at their four neighboring nodes. The solution of a huge matrix equation as required in the traditional FDFD method is avoided. The total number of multiplication operations of complex numbers to solve for all the N_v unknown EM field components with SSFDFD is $2N_v$, compared with the number N_v^{β} ($1.0<\beta<1.5$) in traditional FDFD. Furthermore, significant computer memory can be saved with SSFDFD, because at each space step only the EM fields on the planes under analysis need to be saved.

The SSFDFD method can be used not only for waveguide structures, but also for open problems of EM field analysis. In an open space problem, absorbing boundary conditions (ABCs) can be used at the artificial computation boundaries in a similar way as in FDTD. But unlike in FDTD, the absorbing boundary condition is not necessary in SSFDFD for the two artificial computation boundaries perpendicular with the z-axis.

III. SSFDFD STABILITY CONDITION

It is found that, as with FDTD, there is the stability problem with SSFDFD. The space step size and the angular frequency must satisfy a condition to guarantee the stability of SSFDFD. The derivation of the stability condition for SSFDFD is similar to that for the FDTD stability condition [7]. For convenience, a normalized region of space with μ=1, ε=1, and $\sigma=0$ is considered. Maxwell's equations can be written in the normalized space as,

$$j\nabla\times\vec{v}=\frac{\partial\vec{v}}{\partial t} \tag{13}$$

in which $\vec{v}=\vec{H}+j\vec{E}$. The frequency-domain form of equation (13) is,

$$\nabla\times\vec{V}=\omega\vec{V} \tag{14}$$

where $\vec{V}$ is the Fourier transform of $\vec{v}$. Suppose the SSFDFD algorithm steps in the z-axis direction, equation (14) is then rewritten as,

$$\nabla_t\times\vec{V}-\omega\vec{V}=-\frac{\partial\left(V_x\vec{a}_y-V_y\vec{a}_x\right)}{\partial z} \tag{15}$$

in which $\vec{a}_x$ and $\vec{a}_y$ are unit vectors in the x and y directions, respectively, and ∇_t is the lateral part of the operator ∇. The stability of a particular numerical representation of equation (15) can be examined simply by considering the following pair of eigenvalue problems,

$$-\frac{\partial\left(V_x\vec{a}_y-V_y\vec{a}_x\right)}{\partial z}=\lambda\left(V_x\vec{a}_y-V_y\vec{a}_x\right) \tag{16}$$

$$\nabla_t\times\vec{V}-\omega\vec{V}=\lambda\left(V_x\vec{a}_y-V_y\vec{a}_x\right). \tag{17}$$

SSFDFD employs central difference to approximate the derivative in Maxwell's equations. Using central difference for the derivative with respect to the z coordinate and letting $\vec{V}_n=\vec{V}(z=n\Delta z)$, equation (16) yields,

$$-\frac{\left(V_x\vec{a}_y - V_y\vec{a}_x\right)_{n+\frac{1}{2}} - \left(V_x\vec{a}_y - V_y\vec{a}_x\right)_{n-\frac{1}{2}}}{\Delta z} = \lambda\left(V_x\vec{a}_y - V_y\vec{a}_x\right)_n . \tag{18}$$

Defining a growth factor,

$$q = \left|\left(V_x\vec{a}_y - V_y\vec{a}_x\right)_{n+\frac{1}{2}}\right| / \left|\left(V_x\vec{a}_y - V_y\vec{a}_x\right)_n\right|$$

and substituting it into equation (18), we get,

$$q = -\frac{\lambda\Delta z}{2} \pm \left(1 + \left(\frac{\lambda\Delta z}{2}\right)^2\right)^{1/2} . \tag{19}$$

Algorithm stability requires $q \le 1$ for all possible spatial modes in the lattice. For this to occur,

$$\mathrm{Re}\,\lambda = 0,\ \left|\mathrm{Im}\,\lambda\right| \le \frac{2}{\Delta z} . \tag{20}$$

We now let,

$$\vec{V}(l,m) = \vec{V}_0 e^{-j(k_x l\Delta x + k_y m\Delta y)} \tag{21}$$

represent an arbitrary spatial mode. Using the central difference for the derivative with respect to x and y, we have,

$$\begin{aligned}\nabla_t \times \vec{V} &= \frac{\partial V_z}{\partial y}\vec{a}_x - \frac{\partial V_z}{\partial x}\vec{a}_y + \frac{\partial V_y}{\partial x}\vec{a}_z - \frac{\partial V_x}{\partial y}\vec{a}_z \\ &= -js_y V_z\vec{a}_x + js_x V_z\vec{a}_y - js_x V_y\vec{a}_z + js_y V_x\vec{a}_z \end{aligned} \tag{22}$$

where

$$s_x = 2\sin\left(\frac{k_x\Delta x}{2}\right)\Big/\Delta x,\ s_y = 2\sin\left(\frac{k_y\Delta y}{2}\right)\Big/\Delta y . \tag{23}$$

Substituting equation (22) into equation (17) yields,

$$-s_y V_z + j\omega V_x = j\lambda V_y \tag{24}$$

$$s_x V_z + j\omega V_y = -j\lambda V_x , \tag{25}$$

$$-s_x V_y + s_y V_x + j\omega V_z = 0 . \tag{26}$$

Solving equations (24) to (26), we get,

$$\lambda^2 = s_x{}^2 + s_y{}^2 - \omega^2 . \tag{27}$$

Substituting equation (27) into equation (20) yields the stability condition for SSFDFD,

$$s_x{}^2 + s_y{}^2 \le \omega^2 \le \left(\frac{2}{\Delta z}\right)^2 + s_x{}^2 + s_y{}^2 . \tag{28}$$

For any spatial modes in real media, equation (28) can be rewritten as,

$$2c\sqrt{\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2}} \le \omega \le \frac{2c}{\Delta z} \tag{29}$$

where $c = 1\big/\sqrt{\mu\varepsilon}$ is the EM wave velocity in the real media. Considering that c is unknown and is not the same in different media region, equation (29) is modified as,

$$2c_{\max}\sqrt{\frac{1}{(\Delta x)^2} + \frac{1}{(\Delta y)^2}} \le \omega \le \frac{2c_{\min}}{\Delta z} \tag{30}$$

where $c_{\max}$ and $c_{\min}$ are the maximum and minimum velocity of EM waves in the media, respectively.

From the stability condition shown in equation (30) we can see that for a given angular frequency ω, a relatively smaller space step (Δz) in the stepping direction of the algorithm and larger space steps (Δx and Δy) in the cross sectional plane are required for the stability of SSFDFD, and that Δz must be smaller than Δx and Δy. The role of Δz in the stability condition of SSFDFD is similar with that of the time step Δt in the stability condition of FDTD. This is because a simulation with SSFDFD is supposed to iterate in the z-axis direction, while a simulation with FDTD iterates with increasing time t. From equation (30) we can also conclude that for a given reasonable set of space steps (Δx, Δy and Δz, $\Delta z<\Delta x$ and Δy), there is a frequency band in which SSFDFD is stable.

The stability condition in equation (30) is more difficult to satisfy for smaller ω. This is because the smaller ω is, the larger Δx and Δy are required to be to satisfy the stability condition, while in practical EM field simulations, Δx and Δy should not be too large in the interest of analysis accuracy.

IV. DISCUSSION OF SSFDFD

There are several points to be noted for the practical use of SSFDFD. The first is that, in equations (7) to (12) describing the proposed SSFDFD method, $E(i,j,k+1)$ and $H(i,j,k+1)$ are formally independent of $E(i,j,k+2)$ and $H(i,j,k+2)$ at the neighboring nodes. This is because SSFDFD simulates the frequency-domain steady-state EM field responses from the given EM fields at planes z=0 and z=0.5Δz, not the temporal propagation procedure of EM waves as FDTD does, although the stepping algorithm is employed in SSFDFD as in FDTD. It can be supposed that some harmonic incident EM waves propagate in the $-z$ direction, react with (and may be multiply reflected between) the objects, and transmit through the two planes at z=0 and z=0.5Δz. When steady-state is reached after a long enough time and assuming that the steady-state EM fields at planes z=0 and z=0.5Δz are given (as in SSFDFD), then SSFDFD can be used to derive the EM field distribution of the steady-state response at z>0.5Δz. For given EM fields E_x, E_y and H_z at the z=0 plane and H_x, H_y and E_z at the z=0.5Δz plane, different incident EM waves are required for different objects. In the time domain and before steady-state is reached, $E(i,j,k+1)$ and $H(i,j,k+1)$ of course depend on $E(i,j,k+2)$ and $H(i,j,k+2)$ if there is an EM wave (either a reflected wave or a source wave) transmitting from $z=(k+2)\Delta z$ to $z=(k+1)\Delta z$. But at steady-state, the EM fields at the plane $z=k\Delta z$ contain the information of that at the plane $z=(k+2)\Delta z$, so the EM fields at the plane $z=(k+1)\Delta z$ can indeed be derived from those at the plane $z=k\Delta z$ according to the frequency-domain Maxwell's equations using a difference method as in SSFDFD. The steady-state EM fields at the plane $z=(k+1)\Delta z$ can also be derived from those at the plane $z=(k+2)\Delta z$ in a similar way. Actually, equations (7) to (12) are strictly derived from Maxwell's equations with no other approximations than the second order difference. The time-domain response can be obtained only by inverse Fourier transformation of the frequency -domain responses.

The second point is that, as the "initial" condition, the fields E_x, E_y and H_z at the plane z=0 and H_x, H_y and E_z at the plane z=0.5Δz cannot be arbitrarily given. These must be the sum of EM fields of actual propagation modes at the analysis frequency. In practical use, the position of z=0 and z=0.5Δz can be put within a uniform waveguide or at two parallel planes in free space for an open space problem, so that the mode fields at z=0 and z=0.5Δz are known or are able to be solved with a 2-dimensional (2-D) method such as the 2-D FDFD [8]. The physical insight of SSFDFD is to derive the EM field distribution of the steady-state response and the incident EM fields from the given EM fields at planes z=0 and z=0.5Δz. This is to say the "initial" planes (planes at z=0 and z=0.5Δz) in SSFDFD can be understood as the output port of the structure under analysis, where there are only EM waves propagating in the $-z$ direction. As a result, the EM fields E_x, E_y and H_z at the plane z=0 and H_x, H_y and E_z at the plane z=0.5Δz can be composed of any linear combination of the existing mode fields propagating in the $-z$ direction. In this way, the problem of the unknown ratio of the incident waves and reflected waves at the incident planes is avoided.

For a given incident wave, there may be evanescent waves and non-propagating waves at planes z=0 and z=0.5Δz resulting from the structure interaction. In SSFDFD, however, only existing propagation modes (eigen modes) in uniform waveguides or in free space are considered for giving the values of the EM fields E_x, E_y and H_z at the plane z=0 and H_x, H_y and E_z at the plane z=0.5Δz. This is reasonable for two reasons. One reason is that only the steady-state response of the EM fields is simulated by SSFDFD and the evanescent waves and non-propagating waves can be neglected. Another reason is that we can assume that the incident waves are so composed that the transmitted waves are exactly the given waves at the output port.

The third point concerns the case where there is a perfect conductor with the objects. The incident fields should be perturbed behind this conductor. However, it seems that calculating the EM wave going through the conductor from the output (i.e., from z=0 and z=0.5Δz) to the input with SSFDFD will not show any perturbation of the fields unless the structure is reached. But actually, considering that the task of SSFDFD is to derive the steady-state field distribution from the given response fields at the output planes (not from the incident fields), the perturbation effects of the conductor to the EM fields will be taken into account in the simulation results of the field distribution of the incident EM waves. The situation for a cavity structure is similar. Another confusion with a perfect conductor is that, if the grid mesh in Fig. 1 is used for the conductor, then the electric fields at the surface ABCD are tangential fields and should have a value of zero, while the simulated value from SSFDFD may not be zero. This confusion can be alleviated through moving the mesh surface ABCD by 0.5Δz along the z-direction, such that the electric fields on the surface are perpendicular and can be computed with SSFDFD. On the other five surfaces of the mesh of a perfect conductor, the EM fields are still tangential and the treatment of the electric boundary condition is similar to that in FDTD.

The last point concerns the stability condition. Suppose $\Delta x = \Delta y$, from equation (30) Δx and Δy must be larger than $\sqrt{2}\lambda_{\max}/\pi$ for the stability of SSFDFD, where $\lambda_{\max}$ is the maximum wavelength in the media. With such space steps, accuracy problems will be caused in an analysis of a steep distribution of EM fields and for small details of objects. Fortunately, although small Δx and Δy may make SSFDFD unstable, the phenomena of instability will occur only after a number of iterations of SSFDFD, and the analysis results along the z-direction before instability occurs can still be used. Our application practice indicates that the analysis results before instability occurs are always sufficient to reach an analysis target.

From the above discussion, although there are still some difficulties (especially the stability problem) with SSFDFD, it has great potential as a powerful and fast tool for full wave modeling of passive microwave circuits, considering that it is a breakthrough in terms of analysis efficiency and computer memory.

V. APPLICATION EXAMPLES

To verify the proposed SSFDFD method, the steady-state EM field response of the TE_{10} mode in a rectangular waveguide containing a segment of dielectric (shown in Fig. 2) is simulated. We set the relative permittivity of the dielectric to be a constant, that is, ε_r=4.0 as the initial condition. The EM fields E_x, E_y and H_z at plane z=0 and H_x, H_y and E_z at plane z=0.5Δz are given with the value of the mode fields of TE_{10} wave propagating in the z-direction, and the steady-state response for incident waves from the other side of the dielectric segment is simulated. After the simulation with SSFDFD, the total fields including both the incident and the reflected waves are obtained within the dielectric segment and at the incident side of the dielectric segment.

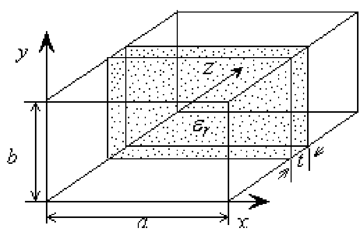

Fig. 2. Geometry of a dielectric loaded rectangular waveguide, where a=2cm, b=1cm, and the thickness of the dielectric segment is t=1.2 cm.

First the stability condition in equation (30) is tested. Because for the TE_{10} mode the EM fields are uniform in the narrow edge (y-axis) direction of the rectangular waveguide, the stability condition in equation (30) of SSFDFD becomes,

$$\frac{2c_{\max}}{\Delta x} \le \omega \le \frac{2c_{\min}}{\Delta z} \qquad (31)$$

we set $c_{\max} = c_0$ and $c_{\min} = c_0/\sqrt{\varepsilon_r}$, where c_0 is the light velocity in vacuum. Suppose the analysis frequency is 40 GHz, the exact stability condition for the TE_{10} wave is Δx≥0.2387 cm, Δz≤0.1194 cm. Practical simulations show that SSFDFD is stable when Δx=0.25 cm and Δz=0.11 cm, but is unstable when Δx=0.25 cm and Δz=0.12 cm, or when Δx=0.2 cm and Δz=0.11 cm. The distribution of the real part of the simulated complex electric field E_y along the longitudinal direction (z-axis direction) at x=0.5 cm with Δx=0.25 cm and Δz=0.11 cm is given in Fig. 3. Although the result is not accurate (compared with the result from Δx=0.25 cm and Δz=0.01 cm, which is also shown in Fig. 3) because of the large value of Δz, it is stable. The numerical results for the two unstable cases are shown in Fig. 4.

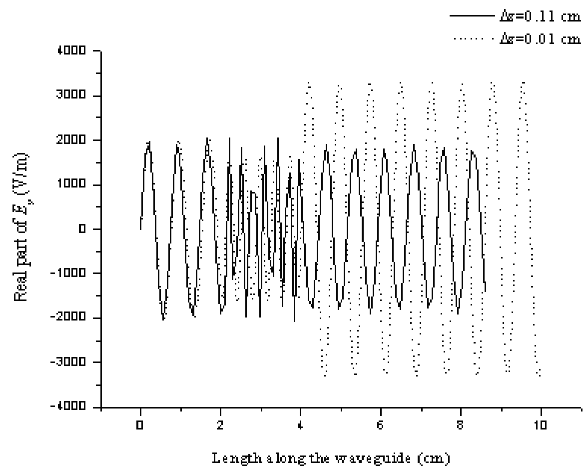

Fig. 3. Distribution of the real part of E_y along the z-axis in the dielectric loaded rectangular waveguide.

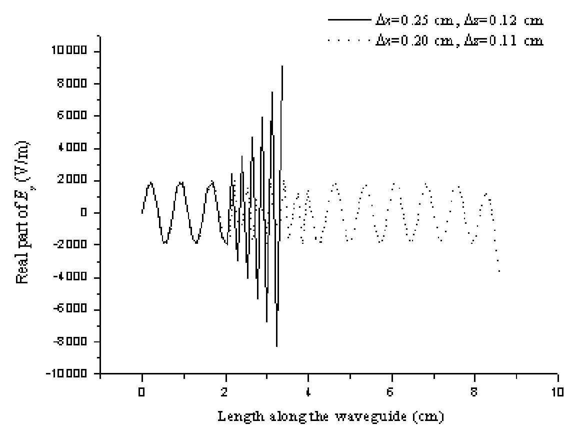

Fig. 4. Distribution of the real part of E_y along the z- axis in the two unstable cases.

Next the validity and efficiency of the proposed SSFDFD is tested. Let Δx=0.4 cm and Δz=0.05 cm in the simulation, then the frequency band where SSFDFD is stable is between 23.87 GHz and 95.49 GHz. If a frequency outside of the stable frequency band is simulated, the result will diverge with the stepping procedure. Figure 5 shows the unstable result at 10 GHz. From Fig. 5 we can see that, although the computation diverges, a slice of the result along the z-direction is correct and useful. The length of slice with correct results depends on the dielectric. If the length of the structure under analysis is shorter than the length in the z-direction where SSFDFD doesn't diverge, SSFDFD can still be used for the frequencies outside of the stable band. In Fig. 5 SSFDFD doesn't diverge until z=7 cm, then if the distance between the input port and output port is less than 7 cm, correct S-parameter results are still available. Actually we set the ports separation to 6 cm, between which the dielectric segment is included. The obtained S-parameters are plotted in Fig. 6 for comparison the results from traditional FDFD and FEM are given as well, showing that excellent agreement is achieved. The computation times for the different methods are listed in Table 1.

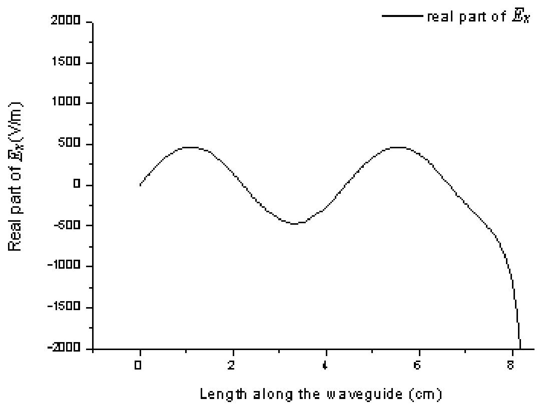

Fig. 5. Distribution of the real part of Ex along the z-axis at f=10.0 GHz.

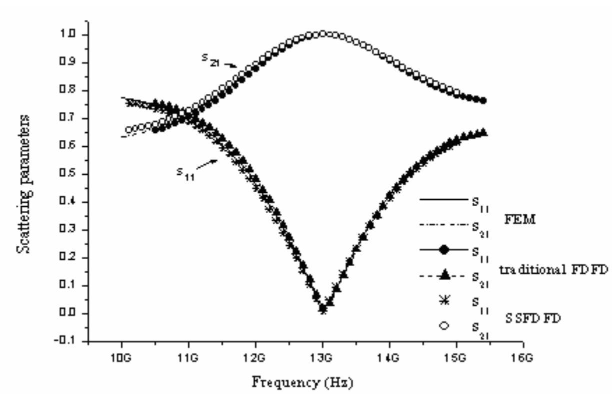

Fig. 6. Magnitude of the S-parameters from three different methods.

Table 1. Efficiency comparison among different approaches: SSFDFD, traditional FDFD, FEM with discrete frequency sweep and FEM Adaptive Lanzcos-Pada Sweep (ALPS). The CPU time is that for getting the curves in Fig. 6.

	SSFDFD	FDFD	FEM (discrete)	FEM (ALPS)
Mesh grids	3000 cubes	3000 cubes	3051 tetrahedra	3051 tetrahedra
CPU time	3 sec.	3 min. 20 sec.	4 min. 18 sec.	1 min. 19 sec.

These results are obtained with an Intel Pentium-IV 1.6 GHz computer, and using MATLAB software and ANSOFT HFSS software, respectively. From Table 1, we can see that the SSFDFD method is at least one order of magnitude faster than traditional FDFD, FEM and FEM's Adaptive Lanzcos-Pada Sweep (ALPS) which is a fast sweep approach. An algorithm for solving sparse matrix equations was used in the traditional FDFD. If compared with the FDTD method, suppose 1000 time steps are needed for FDTD to converge to a result and 100 sample frequencies are needed for SSFDFD to get a response spectrum, then, theoretically, SSFDFD should be 10 times faster than FDTD. Figure 7 gives the simulated S-parameters with SSFDFD for a wider range of frequencies.

As a second case of the above example, the dielectric segment is next supposed to be Lorentz material [9] with a frequency-dependent complex permittivity defined by,

$$\begin{aligned} \vec{D}(\omega) &= \varepsilon_0(\varepsilon_\infty + \chi(\omega))\vec{E}(\omega) \\ \chi(\omega) &= \frac{(\varepsilon_s - \varepsilon_\infty)\omega_0^2}{\omega_0^2 + j2\omega\delta - \omega^2} \end{aligned} \tag{32}$$

where $\vec{E}(\omega)$ is the electric field vector, $\vec{D}(\omega)$ is the electric flux density vector, $\chi(\omega)$ is the electric susceptibility function, ε_0 is the permittivity in vacuum, ε_∞ is the limiting permittivity at infinite frequency, ε_s is the permittivity at zero frequency, ω_0 is the resonant frequency, and δ is the damping coefficient. For the Lorentz material in this example, $\varepsilon_\infty = 4.3$, $\varepsilon_s = 6.0$, $\omega_0 = 50\pi \times 10^9$, $\delta = 0.001\omega_0$, and $\sigma = 0$. We calculate the S-parameters for a frequency range from 10 GHz to 16 GHz by SSFDFD and compare the results in Fig. 8 with those from the FEM method. The results agree well as shown in Fig. 8, indicating that SSFDFD is able to simulate a frequency-dependent dielectric as well. The simulation efficiency is the same as that of the first case with constant dielectric permittivity.

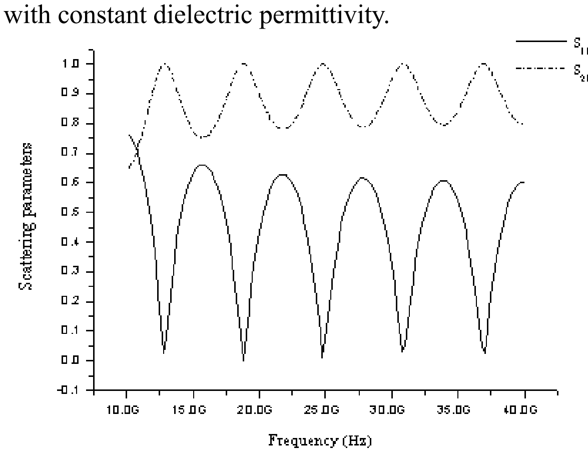

Fig. 7. Magnitude of the S-parameters from SSFDFD for a wider range of frequency.

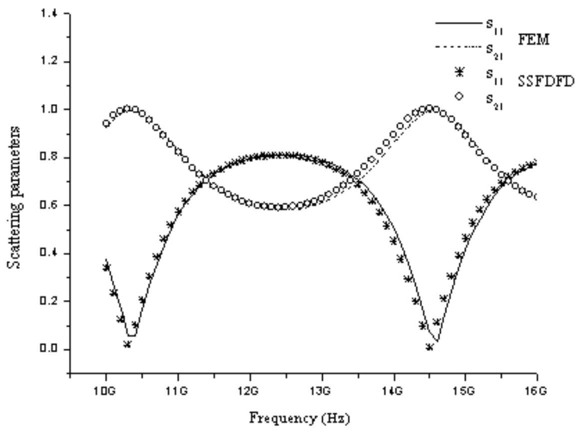

Fig. 8. Magnitude of the S-parameters from SSFDFD and FEM for the case of frequency-dependent complex dielectric permittivity.

VI. CONCLUSION

A novel SSFDFD method is presented in this paper for full wave EM field modeling of passive microwave devices. In SSFDFD all EM fields are local variables, which can be solved from the given EM fields at the initial planes space-step by space-step along a coordinate direction. The solution of a huge matrix equation as in the traditional FDFD is avoided, making the proposed SSFDFD at least one magnitude faster than the traditional FDFD, FEM and FDTD methods under the precondition of keeping similar simulation accuracy. Also, significant computer memory can be saved. The stability condition for SSFDFD is derived and analyzed. SSFDFD has great potential as a powerful and fast tool for full wave EM field analysis of guided wave structures, although much work is still required to make it practically applicable to some complex problems of microwave circuits.

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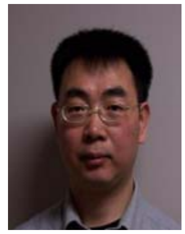

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