BEM-FEM coupling in electromechanics: A 2-d watch stepping motor driven by a thin wire coil

Stefan Kurz, Joachim Fetzer, Thomas Kube, Günther Lehner and Wolfgang M. Rucker
Institut für Theorie der Elektrotechnik, Universität Stuttgart, Pfaffenwaldring 47, 70550 Stuttgart, Germany

***Abstract*—This paper deals with the transient numerical analysis of 2-d electromechanical devices driven by thin wire coils. Their dynamical behaviour is described by the Maxwell equations, by the constitutive relations, and by the equations of motion. For the solution of the electromagnetic problem the BEM-FEM coupling method is used. A two-dimensional watch stepping motor is presented as an example.**

I. Theory

A. Electromagnetic field equations for 2-d problems

Starting from the Maxwell equations, neglecting displacement currents, taking into account the constitutive relations [1]

$$\vec{B} = \mu_0 \vec{H} + \vec{J}, \quad (1)$$

$$\vec{g} = \kappa(\vec{E} + \vec{v} \times \vec{B}) \quad (2)$$

and the Coulomb gauge

$$\mathrm{div}\vec{A} = 0, \quad (3)$$

as well as the magnetic vector potential $\vec{A}$, the equation

$$-\frac{1}{\mu_0}\Delta A_z + \kappa_1 \frac{\mathrm{d}A_z}{\mathrm{d}t} = g_{z\mathrm{C}} + \frac{1}{\mu_0}\mathrm{rot}\vec{J}\cdot\vec{e}_z \quad (4)$$

is derived [2]. Equation (4) describes two-dimensional eddy current problems. $\vec{g}$ is the current density, $g_{z\mathrm{C}}$ the current density in the voltage driven thin wire coil, $\vec{J}$ the magnetization, κ the conductivitiy, κ_1 the conductivitiy of massive conducting media and $\vec{v}$ the velocity. The operator d/dt in (4) denotes the total time derivative. The following continuity conditions are valid on eventual interior boundaries

$$A_z = \text{continuous}, \quad (5)$$

$$\frac{1}{\mu_0}(\mathrm{rot}\vec{A} - \vec{J}) \times \vec{n} = \text{continuous}. \quad (6)$$

A possible magnetization $\vec{J}$ has been taken into account in (4) by the equivalent magnetization current density rot $\vec{J}$. In general, this formulation requires an iterative solution of the problem [3].

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S. Kurz, e-mail stefan.kurz@ite.uni-stuttgart.de;
J. Fetzer, e-mail joachim.fetzer@ite.uni-stuttgart.de.

B. Thin wire coil

If the radius of the cross-section of a single filament is smaller than the skin depth, the current density in the turn may be assumed to be constant. In most cases the section is the same for all turns and therefore the current density may be assumed to be constant in the whole coil. If there are spaces between conductors due to insulation, this can be taken into account by considering an average current density and an equivalent conductivity [4], [5]. With these assumptions the coil is considered as a homogeneous conductor with a constant current density. Considering for the sake of simplicity resting coils only ($\vec{v} = 0$), the current density of a threedimensional thin wire coil reads

$$\vec{g}_\mathrm{C} = -\kappa_2(\mathrm{grad}\varphi + \frac{\partial \vec{A}}{\partial t} - \vec{E}_\mathrm{S}). \quad (7)$$

φ is the scalar electrical potential, κ_2 is the equivalent conductivity of the coil and $\vec{E}_\mathrm{S}$ the impressed electric field in the coil. Multiplying (7) with $\vec{\tau}$, the unit vector with the direction of positive current through a filament, yields

$$g_\mathrm{C} = -\kappa_2(\mathrm{grad}\varphi + \frac{\partial \vec{A}}{\partial t} - \vec{E}_\mathrm{S}) \cdot \vec{\tau}. \quad (8)$$

Integrating (8) along one turn $c(\vec{r})$ of the coil results in

$$g_\mathrm{C} w(\vec{r}) = -\kappa_2 \oint_{c(\vec{r})} \frac{\partial \vec{A}}{\partial t} \cdot \vec{\tau}\, \mathrm{d}l + \kappa_2 E_\mathrm{S} w(\vec{r}), \quad (9)$$

$$w(\vec{r}) = \oint_{c(\vec{r})} \mathrm{d}l = \text{length of one turn.}$$

Afterwards an integration of (9) over the cross-section of the coil is performed, yielding

$$g_\mathrm{C} = -\frac{\kappa_2}{V} \int_V \left(\frac{\partial \vec{A}}{\partial t} \cdot \vec{\tau}\right) \mathrm{d}V + \kappa_2 E_\mathrm{S}, \quad (10)$$

$$V = \text{volume of the whole coil.}$$

Considering only two-dimensional problems, (10) can be written as

$$g_{z\mathrm{C}} = -\frac{\kappa_2}{V}\left(\int_{V_1}(\frac{\partial \vec{A}}{\partial t}\cdot\vec{\tau}_1)\mathrm{d}V + \int_{V_2}(\frac{\partial \vec{A}}{\partial t}\cdot\vec{\tau}_2)\mathrm{d}V\right) + \kappa_2 E_{\mathrm{S}z}, \quad (11)$$

$V = V_1 + V_2$,
$V_1,\ V_2$ = volumes of both halves of the coil,
$\vec{\tau}_1 = \vec{e}_z$,
$\vec{\tau}_2 = -\vec{e}_z$.

Taking into account that the integrals in (11) do not depend on z we get

$$g_{z\mathrm{C}} = -\frac{\kappa_2}{A}\left(\int_{A_1}\frac{\partial A_z}{\partial t}\mathrm{d}a - \int_{A_2}\frac{\partial A_z}{\partial t}\mathrm{d}a\right) + \kappa_2 E_{\mathrm{S}z} \quad (12)$$

A_1, A_2 = sections of both halves of the coil,
$A = A_1 + A_2 = 2NS$,
N = number of turns of the coil,
S = cross-section of one turn.

C. Equations of motion

Only rigid body motions are considered. The most general motion of a rigid body can be broken up into a translation of the center of gravity C and a rotation with respect to C. The translational part is governed by the momentum law

$$\dot{\vec{p}} = \vec{F}, \quad \dot{\vec{r}} = \frac{\vec{p}}{m}, \quad (13)$$

where m is the mass, $\vec{r}, \vec{p}$ are location and momentum of C, respectively and $\vec{F}$ is the total force.

The orientation of the body is specified by the Euler angles Φ, Θ, Ψ [6]. The rotational part of the motion is governed by the angular momentum law

$$\dot{\vec{L}} = \vec{M}, \quad (14)$$

where $\vec{L}$ is the angular momentum and $\vec{M}$ the total torque with respect to C. The time rate of change of the Euler angles can be expressed as a function of the angular momentum, the principal moments of inertia and the Euler angles themselves. In the case of rotation around a fixed axis, which is assumed to be the z-axis ($\Phi = \Theta = 0$, $\Psi \neq 0$), the general relation reduces to

$$\dot{\Psi} = \frac{L_z}{J_z}, \quad (15)$$

where J_z is the moment of inertia with respect to an axis through C parallel to the z-axis. Rotation around a fixed axis is the most common case when dealing with electric machines.

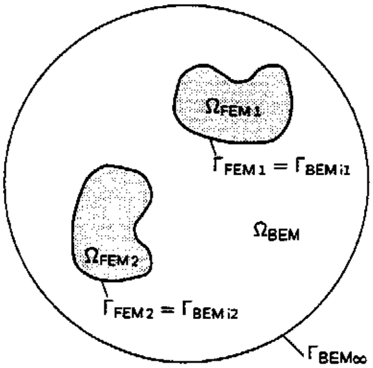

Fig. 1. Structure of the considered domain
$\Omega_{\mathrm{FEM}} = \Omega_{\mathrm{FEM1}} \cup \Omega_{\mathrm{FEM2}}, \quad \Omega = \Omega_{\mathrm{FEM}} \cup \Omega_{\mathrm{BEM}}$
$\Gamma_{\mathrm{FEM}} = \Gamma_{\mathrm{FEM1}} \cup \Gamma_{\mathrm{FEM2}}, \quad \Gamma_{\mathrm{BEMi}} = \Gamma_{\mathrm{BEMi1}} \cup \Gamma_{\mathrm{BEMi2}}$

II. The BEM-FEM Coupling Method

The coupling of the boundary element method (BEM) and the finite element method (FEM) is used to solve (4). The domain Ω of the boundary value problem is decomposed into (possibly) several FEM subdomains $\Omega_{\mathrm{FEM}\nu}$ and one multiply connected BEM subdomain Ω_{BEM}, Fig. 1. The decomposition has to be such that conducting and/or permeable bodies are described by the FEM. The surrounding unbounded air space is treated by the BEM. The boundary $\Gamma_{\mathrm{BEM}\infty}$ does not give any contribution and has therefore not to be taken into account.

Application of the Galerkin method to the weak integral form of (4) and (12) using nodal finite elements yields the matrix formulation

$$([K] + [C]\frac{\mathrm{d}}{\mathrm{d}t})\{A_z^{\mathrm{FEM}}\} - [T]\{Q^{\mathrm{FEM}}\} = \{F(E_{\mathrm{S}z}, \vec{J})\}. \quad (16)$$

$\{A_z^{\mathrm{FEM}}\}$ are the potential values at the nodes in $\bar{\Omega}_{\mathrm{FEM}}$ and $\{Q^{\mathrm{FEM}}\}$ are the values of $Q = \frac{\partial A_z}{\partial n} - (\vec{J} \times \vec{n}) \cdot \vec{e}_z$ at the nodes on Γ_{FEM}. $[K]$ is the stiffness matrix, $[C] = [C_{\mathrm{massive}}] + [C_{\mathrm{coil}}]$ the damping matrix and $[T]$ the boundary matrix.

Application of the BEM to (4) with $\kappa_{1,2} = 0$, $\vec{J} = 0$ yields another linear system,

$$[G]\{Q^{\mathrm{BEM}}\} = [H]\{A_z^{\mathrm{BEM}}\}, \quad (17)$$

where $\{A_z^{\mathrm{BEM}}\}$, $\{Q^{\mathrm{BEM}}\}$ are the values of the vector potential and its normal derivative at the nodes on Γ_{BEMi}. Eq. (16) and (17) can be coupled using the boundary conditions (5) and (6) on the common boundaries $\Gamma_{\mathrm{FEM}} = \Gamma_{\mathrm{BEMi}}$ [7] resulting in

$$\left([K] + [T][G]^{-1}[H] + [C]\frac{\mathrm{d}}{\mathrm{d}t}\right)\{A_z^{\mathrm{FEM}}\} = \{F(E_{\mathrm{S}z}, \vec{J})\}. \quad (18)$$

III. Computation of Forces

The magnetic forces are calculated using the Maxwell stress tensor [8], [9]. With this method, a normal force density $\vec{f}_n$ and a tangential force density $\vec{f}_t$ are calculated on the boundary of the considered part as

$$\vec{f}_n = \frac{1}{2}\left(\frac{B_n^2}{\mu_0} - \mu_0 \vec{H}_t^{\,2}\right)\vec{n}, \qquad (19)$$

$$\vec{f}_t = B_n \vec{H}_t . \qquad (20)$$

The net force and torque are then computed by simple integration of $\vec{f}$ and $\vec{r} \times \vec{f}$ over the boundary. Assuming that the boundary is a coupling boundary $\Gamma_{BEMi\nu} = \Gamma_{FEM\nu}$ and the integration is performed on the BEM side ($\vec{J} = 0$, $\vec{B} = \mu_0 \vec{H}$), the required field quantities B_n and $\vec{H}_t$ can be obtained from

$$B_n = \vec{n} \cdot \mathrm{rot}\,\vec{A}, \qquad (21)$$

$$\vec{H}_t = \frac{1}{\mu_0}\vec{B}_t = \vec{n} \times \left(\frac{1}{\mu_0}\mathrm{rot}\vec{A} \times \vec{n}\right). \qquad (22)$$

IV. Coupling of Electrical and Mechanical System

The discretization of a device which contains moving parts by finite elements only is not easy. Even small displacements cause a distortion of the mesh, which leads in general to a reduction of mesh quality. Large displacements require frequent remeshing of the whole device, due to the continuously changing positions of the moving parts. This disadvantage can be avoided by application of the BEM-FEM coupling. With this approach, the structure of the mesh does not change during the motion because it moves with the material.

The electrical and mechanical equations are solved step-by-step using the implicit Euler method. The weak coupling is established by the following algorithm [10]:

- Solve the electromagnetic problem (18) at the time $t = n\Delta t$.
- Compute forces and torques for this state.
- Solve the equations of motion (13), (14) and (15) to determine the displacements of the moving parts.
- Modify the location of the moving parts using the calculated displacements.
- Next time step.

V. Numerical Example

A. Considered device

As a numerical example a watch stepping motor used in alarm clocks is considered (Fig. 2, Fig. 3). The rotor consists of a permanent magnetized ferrite with a remanent magnetic induction of $B_r = 0.25$T. The yoke of the stator is made of a nonconducting ferromagnetic sheet with a constant relative permeability $\mu_r = 5000$. The driving coil of the stepping motor consists of 6800 turns with a resistance of 860Ω. Effects due to friction have not been taken into account. The watch stepping motor is considered as a two-dimensional problem (Fig. 3).

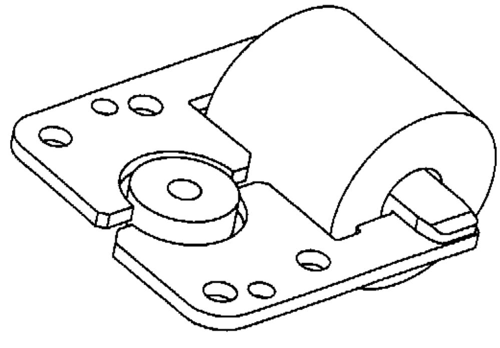

Fig. 2. Watch stepping motor used in alarm clocks

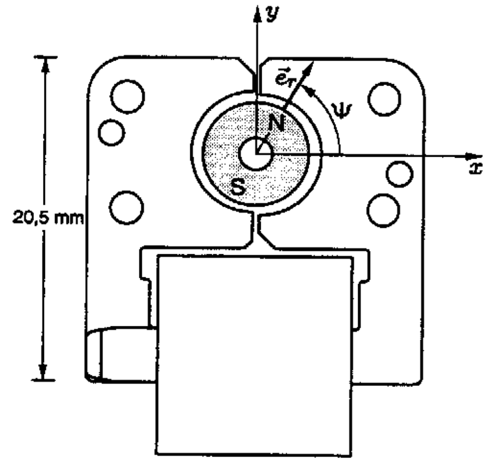

Fig. 3. Top view of the watch stepping motor

B. Numerical results

Fig. 4 shows the voltage per length applied to the driving coil. The magnetic flux density in the motor and the position of the rotor for $t = 0_+$, $t = 0.04$s, $t = 0.1$s and $t = 0.2$s are depicted in Fig. 5 - Fig. 8. Fig. 9 shows the dynamic behaviour and the two resting positions of the rotor, Fig. 10 the torque per length acting on the rotor. In Fig. 11 the current in the driving coil is depicted. In Table I the mesh and computation data for the stepping motor can be seen. Comparing the numerically calculated results (Fig. 9 - Fig. 11) with the measured data of the real watch stepping motor shows a very good correspondence of the typical features (current drop, resting positions).

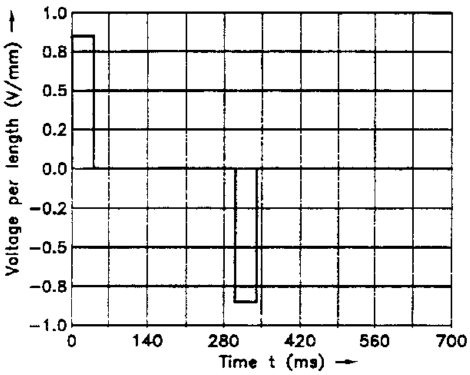

Fig. 4. Voltage per length applied to the driving coil

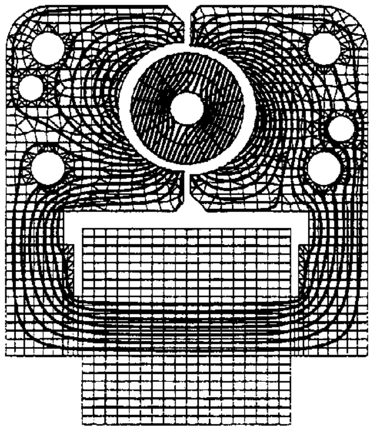

Fig. 5. Magnetic flux density in the 2-d stepping motor for $t = 0_+$

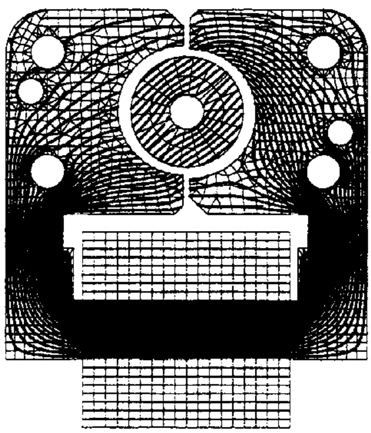

Fig. 6. Magnetic flux density in the 2-d stepping motor for $t = 0.04$s

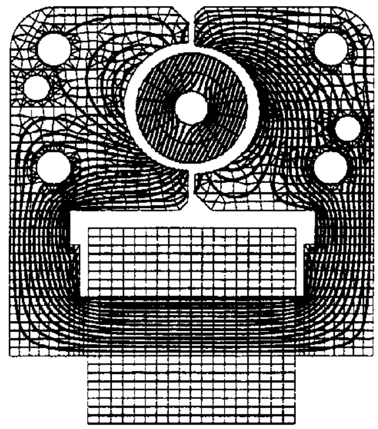

Fig. 7. Magnetic flux density in the 2-d stepping motor for $t = 0.1$s

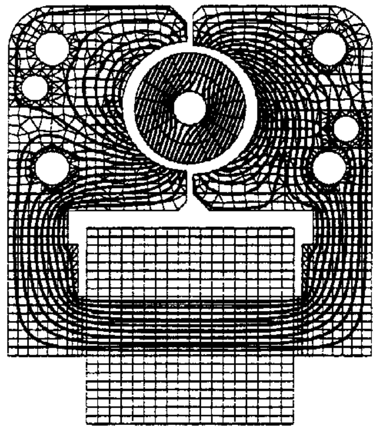

Fig. 8. Magnetic flux density in the 2-d stepping motor for $t = 0.2$s

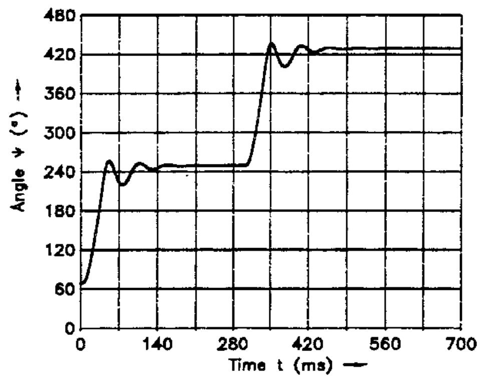

Fig. 9. Angular position of the rotor

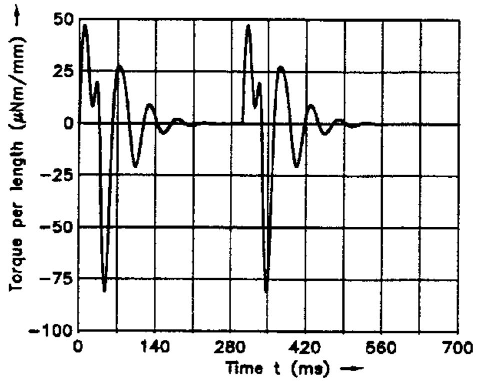

Fig. 10. Torque per length versus time acting on the rotor

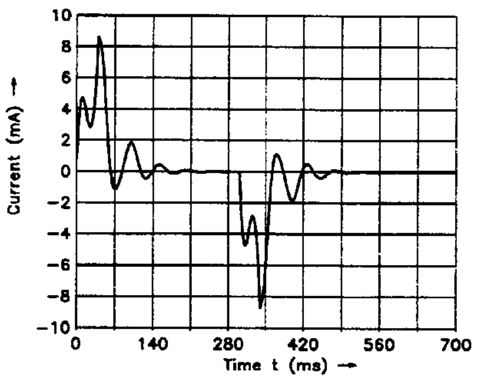

Fig. 11. Current in the voltage driven coil

TABLE I
MESH AND COMPUTATION DATA FOR STEPPING MOTOR

Number of nodes	4931
Number of FEM elements	1654
Number of BEM elements	456
Number of time steps	7000
Time step Δt	0.1 ms
Used memory	66.8 MB
CPU time (Cray C94)	1528 s

VI. CONCLUSION

For the correct numerical modelling of electromechanic systems driven by thin wire coils, it is not sufficient to treat the system as a sequence of static problems. For complete description of the transient behaviour of such devices, it is necessary to take into account, that the velocity of the moving parts (rotor) affects the electric and magnetic field. Furthermore the forces acting on the moving parts are determined by the fields. This interaction has to be taken into account to describe the transient behaviour of an electromechanic device.

This interaction results in a torque vs. time and in a current vs. time in the coil respecively as depicted in Fig. 10 and in Fig. 11.

VII. ACKNOWLEDGEMENT

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