Scattering by PEMC (Perfect Electromagnetic Conductor) Spheres using Surface Integral Equation Approach

Ari Sihvola, Pasi Ylä-Oijala, and Ismo V. Lindell

Electromagnetics Laboratory, Helsinki University of Technology,
P.O. Box 3000, FI–02015 TKK, Finland

Abstract– This article discusses bistatic scattering by totally reflecting spheres. The spheres are either perfect electric, magnetic, or electromagnetic conductors (PEMC). The PEMC medium is described by the parameter M with special cases of PMC (vanishing M) and PEC (infinite M). The scattering by a small sphere of such a material from incoming plane wave can be explained by the interplay of electric and magnetic dipoles. The special characteristics of the radiation of PEMC spherers (different from PEC and PMC) include cross-polarization which is especially marked in the backscattering direction. The radiation pattern is rotated by an angle that has a simple connection with the M parameter. Scattering patterns of PEMC spheres with size parameters up to $ka = 3$ are shown and they also display cross-polarized properties. The computations are based on a MoM software to solve the surface integral equation for the fields.

I. INTRODUCTION

The objective of the present paper is to analyze how the basic scattering characteristics and parameters of electromagnetic plane waves from a perfectly conducting (PEC) sphere are affected and changed when the material of the scattering sphere is made of a more general medium: perfect electromagnetic conductor (PEMC).

In the following, we first introduce the PEMC medium and connect it with the class of bianisotropic materials. Next we present the scattering properties of homogeneous spheres made of different materials. Then the scattering problems for spheres with different material and size parameters are solved by a surface integral equation software, developed in the Electromagnetics Laboratory of Helsinki University of Technology, and the results are given physical interpretation.

II. PEMC MEDIUM

The concept of perfect electromagnetic conductor (PEMC, [1, 2]) is a generalization of the much-used idealizations of perfect electric conductor (PEC) and perfect magnetic conductor (PMC). In PEC, the electric field and magnetic flux vanish, which can be modelled by assuming that its permittivity ϵ grows to infinity and the permeability μ decreases to zero. This behavior can be generalized: the response of PEMC media contains a scalar parameter M

$$\mathbf{D} = M\mathbf{B}, \quad \mathbf{H} = -M\mathbf{E} \tag{1}$$

in the constitutive relations linking the electric $(\mathbf{E}, \mathbf{D})$ and magnetic $(\mathbf{H}, \mathbf{B})$ field strengths and flux densities. Translating these quite strange-looking conditions (1) into field-versus-flux type relations, we can write the constitutive parameter matrix relation [3] in the following manner,

$$\begin{aligned} \begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix} &= \begin{pmatrix} \epsilon & \xi \\ \zeta & \mu \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} \\ &= q \begin{pmatrix} M & 1 \\ 1 & 1/M \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}. \\ &\text{with} \quad q \to \infty \end{aligned} \tag{2}$$

Even if the transformation between equations (1) and (2) is not immediately obvious, it can be checked by opening up the matrix: $\mathbf{D} = q(M\mathbf{E} + \mathbf{H})$ and $\mathbf{B} = q(\mathbf{E} + \mathbf{H}/M)$, which returns equation (1) as $q \to \infty$. Because $\mathbf{D}$ cannot reach infinity, the factor multiplying q has to vanish.

The admittance-like parameter M can be expressed in another convenient form by the angle ϑ,

$$M = \cot\vartheta. \tag{3}$$

It may hurt intuition that the material parameters of PEMC are infinite. In fact, this is the case for all four bi-isotropic parameters of PEMC. There is, however, the following restriction between them: $\epsilon\mu - \xi\zeta = 0$ as can be seen from equation (2). But as was noted, infinities are also encountered in PEC and PMC cases. T he PEC and PMC materials are special cases of relation (1).

For $M = \infty$ (corresponding to $\vartheta = 0$), we have $\mathbf{E} = 0$ and $\mathbf{B} = 0$ which is the requirement for PEC (electric field and magnetic flux cannot exist inside a perfect conductor). But because no restrictions are set on $\mathbf{D}$ and $\mathbf{H}$, we require $\epsilon \to \infty$ and $\mu \to 0$. Similarly, by duality [4], the case $M = 0$ (or $\vartheta = \pm\pi/2$) leaves us with PMC, as $\mathbf{H} = 0$ and $\mathbf{D} = 0$, in other words $\epsilon \to 0$ and $\mu \to \infty$.

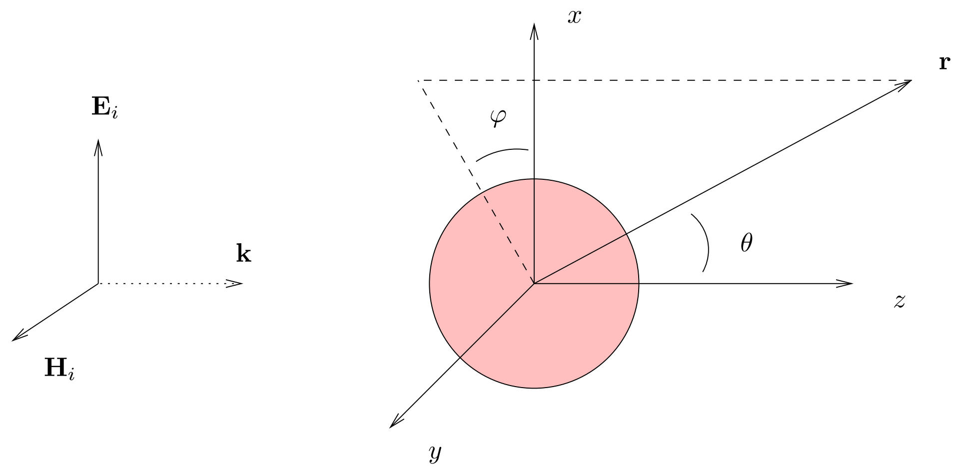

Fig. 1. Geometry of the problem: homogeneous sphere as an obstacle to the incident, linearly x-polarized electromagnetic wave which is propagating into the $+z$-direction. The scattering direction is defined by the angles θ, φ of the spherical coordinate system.

Inside PEMC media, a linear combination of the electric and magnetic fields vanishes identically: $\mathbf{H} + M\mathbf{E} = 0$. This means that on the interface between "ordinary" and PEMC materials, the boundary condition is ($\mathbf{n}$ is the unit normal)

$$\mathbf{n} \times \mathbf{H} = -M\mathbf{n} \times \mathbf{E}. \tag{4}$$

The properties of PEMC media, as the results of the present paper will show, promise quite interesting applications concerning polarization transformation, for example. A challenge which remains is how to fabricate PEMC materials in the real world. One possibility is a composite structure involving a gyrotropic (e.g., ferrite) material layer on a metal surface, with which a band-limited PEMC behavior can be simulated [5]. In today's electromagnetics research there are large-scale activities internationally concerning artificial metamaterials[1] which give hopes that new designs that behave according to more and more exotic material relations come into experimental and operative use.

In the following, we will study how an electromagnetic plane wave is scattered by a sphere which is made of PEMC material.

III. SCATTERING BY A SPHERE

The scattering of waves by a homogeneous sphere is a classical problem in electromagnetics. After the fundamental paper by Gustav Mie [7] and much preceding literature, the problem has been discussed thoroughly in textbooks; see, for example [8, 9, 10, 11].

In the following three-dimensional formulation, we shall use normalized field quantities: in terms of the SI-system electric and magnetic fields and flux densities $\mathcal{E}, \mathcal{H}, \mathcal{D}, \mathcal{B}$ with units V/m, A/m, As/m^2, Vs/m^2, respectively, we deal with fields and fluxes that are renormalized in order to have homogeneous units in each of them,

$$\mathbf{E} = \sqrt{\epsilon_0}\,\mathcal{E}, \quad \mathbf{H} = \sqrt{\mu_0}\,\mathcal{H}, \quad \mathbf{D} = \frac{\mathcal{D}}{\sqrt{\epsilon_0}}, \quad \mathbf{B} = \frac{\mathcal{B}}{\sqrt{\mu_0}} \tag{5}$$

with the free-space parameters ϵ_0, μ_0. This leaves the material parameters in equation (2) dimensionless. All four renormalized field quantities carry the dimension of square root of energy density: $\sqrt{\mathrm{VAs/m}^3} = \sqrt{\mathrm{J/m}^3}$.

A. *Small scatterer*

Let the geometry be fixed in such a way that the plane wave is incident in the $+z$ direction, its electric field is polarized along the x axis, and the magnetic field along the y axis, as shown in Fig. 1. The ordinary spherical angles are θ between the position vector and the z axis, and φ counted from the x axis up to the projection of the position vector onto the xy plane.

To begin with, consider the scattering by a small sphere. By a small sphere we mean a sphere the diameter of which is small compared to the wavelength of the incident field. No exact relative limit can be expressed for the object to be small; in practise, often the size around a tenth of the wavelength could be used as this limit. The results of the present paper will give more light into this question which determines when static and quasi-static considera-

[1]Such programs include the DARPA program in the USA (`www.darpa.mil/dso/thrust/matdev/metamaterials`) and the European Network of Excellence within the Union's 6th Framework programme *Metamorphose* (`www.metamorphose-eu.org`). For literature on electromagnetic metamaterials, see, for instance [6].

Fig. 2. The electric far fields radiated by an electric $(\mathbf{p}_e)$ and magnetic $(\mathbf{p}_m)$ dipole. The electric field of the electric dipole $\mathbf{E}_e$ is "longitudinal", meaning that it is in the plane spanned by the dipole and the position vector, whereas the electric field of the magnetic dipole $\mathbf{E}_m$ is "latitudinal," perpendicular to that. Note the arrow directions. The amplitude dependence of the radiated field is sine of the angle between the dipole and the position vector in both cases.

tions have to be replaced by dynamic analysis.

Let us start with the electromagnetic field radiated by dipoles. The far-field behavior of the electric field of the electric and magnetic Hertzian dipoles [12] is illustrated in Fig. 2.

Obviously, the amplitude of the radiated electric field decays with distance as r^{-1}. If the electric and magnetic dipoles vibrate in the same phase, their electric fields add up constructively. The vector characteristics (polarization) and radiation pattern in the far field can then be calculated using vector cross products,

$$\mathbf{E}_s = (\mathbf{p}_e \times \mathbf{u}_r - \mathbf{p}_m) \times \mathbf{u}_r \tag{6}$$

with $\mathbf{u}_r = \mathbf{u}_x \, \sin\theta\cos\varphi + \mathbf{u}_y \, \sin\theta\sin\varphi + \mathbf{u}_z \, \cos\theta$.

B. PEC and PMC spheres

The electric and magnetic dipole moments $\mathbf{p}_e, \mathbf{p}_m$ of a sphere which can have dielectric and magnetic properties are excited by the incident fields,

$$\mathbf{p}_e = \alpha_e \mathbf{E}_i, \quad \mathbf{p}_m = \alpha_m \mathbf{H}_i. \tag{7}$$

These dipole moments are determined by the normalized electric and magnetic polarizabilities [13]

$$\alpha_e = 3\frac{\epsilon - 1}{\epsilon + 2}, \quad \alpha_m = 3\frac{\mu - 1}{\mu + 2} \tag{8}$$

where ϵ is the relative permittivity and μ the relative permeability of the sphere.

For the PEC sphere $(\epsilon \to \infty, \mu \to 0)$, the normalized electric polarizability is 3 and the normalized magnetic polarizability is $-3/2$. Therefore we can calculate the scattered electric field of a small, perfectly conducting sphere from the vector (note that in our normalized system of electric and magnetic quantities, the electric and magnetic fields have the same unit and the free-space impedance is unity)

$$3(\mathbf{u}_x \times \mathbf{u}_r + \frac{1}{2}\mathbf{u}_y) \times \mathbf{u}_r \tag{9}$$

because the electric dipole is x-directed and the magnetic one y-directed according to Fig. 1. The resulting absolute value of the scattered field is illustrated in Fig. 3 in the two principal planes: E-plane (xz-plane) and H-plane (yz-plane). Note the greater backscattering compared to the forward scattering, and the null in the E-plane into the $\theta = 60^\circ$ direction. The threefold back-to-front ratio (ninefold in the power pattern) is a well-known fact [10, p. 157] for perfectly conducting scatterers. Of course the same behavior applies for magnetically perfectly conducting small spheres with the E- and H-plane patterns interchanged.

There is a significant qualitative difference of these patterns compared to the ordinary Rayleigh scattering diagram of a dielectric sphere (Fig. 4), with equal radiation to the front and back, and a null in the $\theta = 90^\circ$ in the E-plane. On the other hand, if the sphere has both arbitrary finite permittivity and permeability, the patterns again change. Especially, if the dielectric and magnetic material parameters are equal $(\epsilon = \mu)$, there is no reflection, but, rather, a strong forward scattering, as can also be seen in Fig. 4.

The polarization (vector direction of the electric field) of the scattering by a PEC sphere is shown in Fig. 5.

C. PEMC sphere

How does the scattering change if the sphere is made of homogeneous PEMC material where the material parameter is arbitrary $(0 < |M| < \infty)$? When the plane wave touches the sphere, dipole moments are created, just like in the cases dealt with in the previous section. The difference with the PEC and PMC cases is now that both the electric and magnetic dipole moments of the PEMC sphere are dependent on both the electric and magnetic incident fields.

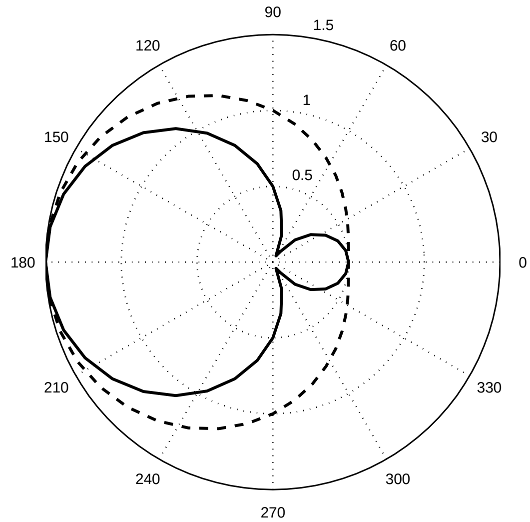

Fig. 3. The scattered far field pattern of a PEC sphere in the E-plane (solid line) and H-plane (dashed line). For the PMC sphere the patterns interchange. The incident wave is arriving from the left.

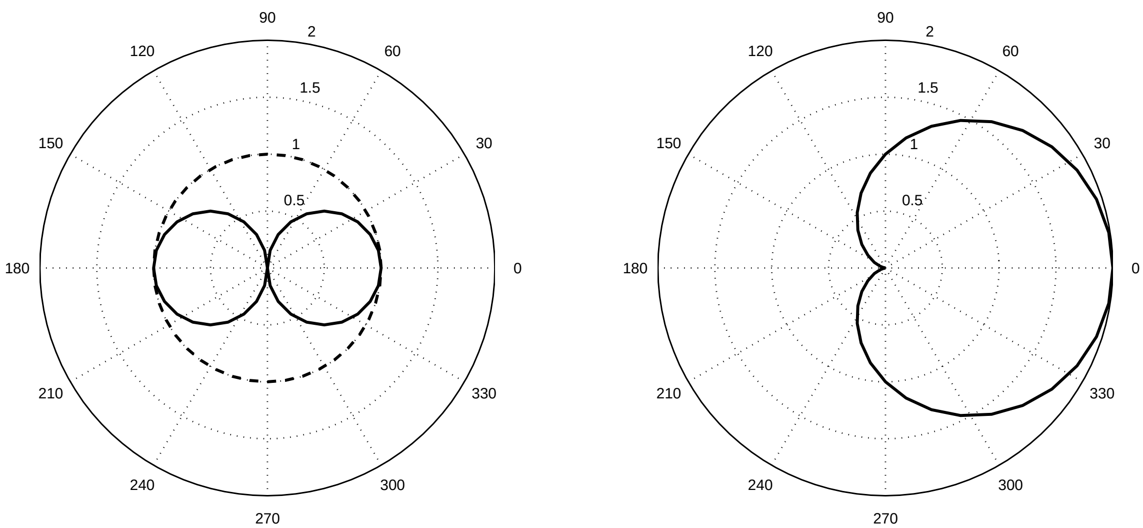

Fig. 4. Left-hand side: scattered far field pattern of a dielectric sphere in the E-plane (solid line) and H-plane (dashed line). On the right-hand side, the pattern for a reflection-matched sphere, $\epsilon = \mu$, in which case the pattern is the same in E and H planes. The incident wave is arriving from the left. Note that the amplitude of the permittivity and permeability only affects the overall magnitude of the scattering (here normalized to 1 and 2 in maximum field magnitude, respectively). However, regardless of the absolute amplitude of ϵ (which is equal to μ on the right-hand side), the radiation patterns are the same.

The polarizabilities and the dipole moments

$$\begin{aligned} \begin{pmatrix} \mathbf{p}_e \\ \mathbf{p}_m \end{pmatrix} &= \begin{pmatrix} \alpha_{ee} & \alpha_{em} \\ \alpha_{me} & \alpha_{mm} \end{pmatrix} \begin{pmatrix} \mathbf{E}_i \\ \mathbf{H}_i \end{pmatrix} \\ &= \mathsf{A} \begin{pmatrix} \mathbf{E}_i \\ \mathbf{H}_i \end{pmatrix} \end{aligned} \tag{10}$$

can be calculated from the material parameter matrix

$$\begin{aligned} \mathsf{C} &= \begin{pmatrix} \epsilon & \xi \\ \zeta & \mu \end{pmatrix} \\ &= \lim_{q\to\infty} q \begin{pmatrix} M & 1 \\ 1 & 1/M \end{pmatrix} \end{aligned} \tag{11}$$

with the infinite parameters for the PEMC material. The polarizability matrix can be calculated from the material

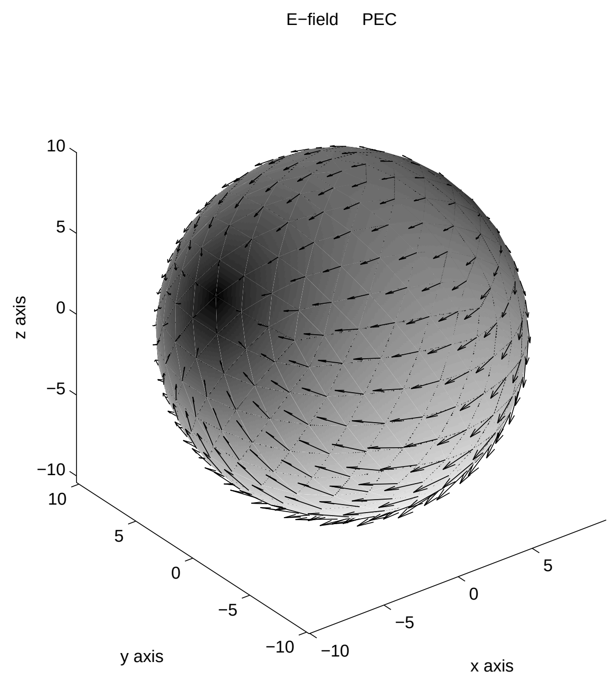

Fig. 5. The scattered far field of a small PEC sphere. Grayscale gives the amplitude and the arrows display the polarization of the radiated electric field. Note the zero in the radiation pattern in the E-plane (xz plane, cf. Fig. 3). Incident electric field is parallel to the x axis. (The figure has been calculated numerically with a dynamic solution for a sphere with the radius $a = 1\,\mathrm{m}$, and the (small) size parameter $ka = 0.1$. The radiated field has been calculated on a spherical surface with radius 10 m).

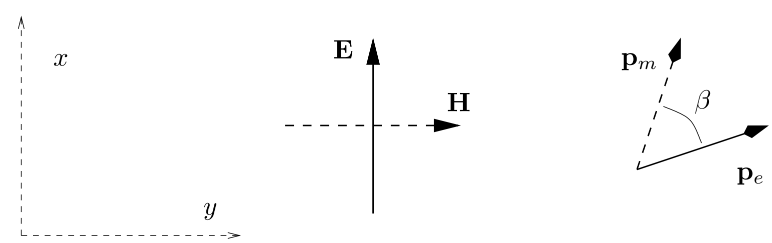

Fig. 6. The directions of the induced electric and magnetic dipole moments in the PEMC sphere are not in general parallel to the incident field vectors. The angle β between the moments depends on M. This case corresponds to $M = 1$.

parameter matrix [13] and turns out to be

$$
\begin{aligned}
\mathsf{A} &= 3(\mathsf{C}-\mathsf{I})\cdot(\mathsf{C}+2\mathsf{I})^{-1} \qquad (12)\\
&= \frac{3/2}{1+M^2}\begin{pmatrix} 2M^2-1 & 3M \\ 3M & 2-M^2 \end{pmatrix}\\
&= \frac{3}{4}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\\
&\quad -\frac{9}{4}\begin{pmatrix} \cos 2\vartheta & -\sin 2\vartheta \\ \sin 2\vartheta & \cos 2\vartheta \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}
\end{aligned}
$$

where I is the 2×2 unit matrix. Note that despite the infinities on (11) of the four material parameters of PEMC, all polarizability components of (12) behave “reasonably.” They remain finite for real values of M.

Now, both induced dipole moments contain components in x and y direction, due to the magnetoelectric coupling. The dipoles are rotated, depending on M; see Fig. 6. Furthermore, they are orthogonal only in the PEC or PMC case. The angle β between the dipoles obeys (cf. definition (3))

$$
\begin{aligned}
\cos\beta &= \frac{3M}{\sqrt{(1+4M^2)(4+M^2)}}\\
&= \frac{3\sin 2\vartheta}{\sqrt{(5-3\cos 2\vartheta)(5+3\cos 2\vartheta)}}. \qquad (13)
\end{aligned}
$$

The dependence of the angle β on the PEMC parameter

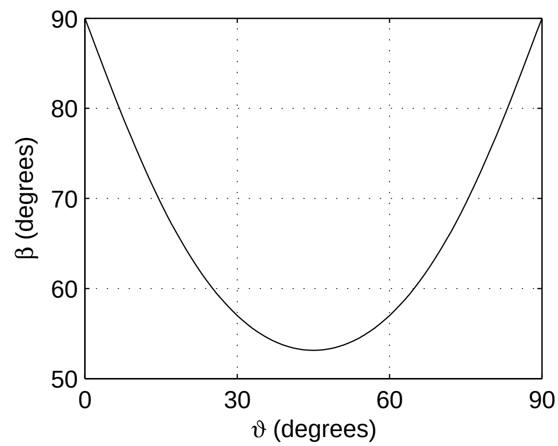

Fig. 7. The angle β as a function of the PEMC parameter ϑ. Note the orthogonality of dipoles in the PEC ($\vartheta = 0°$) and PMC ($\vartheta = 90°$) limits.

ϑ is illustrated in Fig. 7. In the PEC and PMC limit cases, the angle is obviously $90°$. The minimum angle $\beta = \arccos(3/5) \approx 53°$ comes in the case $M = \pm 1$ (corresponding to $\vartheta = \pm 45°$)

Now the scattered field can be calculated from equation (6). Figure 8 shows the radiated field for different values of the M parameter. We can observe a continuous transition from PMC to PEC as the radiation pattern rotates along with varying M. Also the polarization of the electric field changes gradually from the "longitudinal" field of the PEC case to the "latitudinal" PMC radiation field. For a general PEMC sphere, the far-field polarization pattern is helicoidal with handedness depending on the sign of M.

The rotation of the radiation pattern can be seen in Fig. 9, where the scattered field amplitude is shown in the xy-plane. Clearly the radiation of the PEMC sphere is somewhere between those of PEC and PMC.

D. Size-dependent effects

When the radius of the sphere is no longer small compared to the wavelength, the scattering characteristics of the object become decorated with more details. Of course, then the quasistatic calculations involving the electric and magnetic dipole moments with which the low-frequency results were calculated do not apply. We have used a full-wave numerical code to calculate the results.

The code is based on the surface integral equation method. The electric field integral equation, EFIE, is first written for both the electric surface currect density $\mathbf{J}_e = \mathbf{n} \times \mathbf{H}$ and magnetic $\mathbf{J}_m = -\mathbf{n} \times \mathbf{E}$ surface current density as

$$\left(\frac{-1}{i\omega\varepsilon_0} \left(\nabla\nabla \cdot + k_0^2 \right) \mathcal{S}(\mathbf{J}_e) - \nabla \times \mathcal{S}(\mathbf{J}_m) \right)_{\tan} = -\mathbf{E}^{\rm inc}_{\tan}. \tag{14}$$

Here

$$\mathcal{S}(\mathbf{F}) = \int_S G(\mathbf{r}, \mathbf{r}') \, \mathbf{F}(\mathbf{r}') \, dS' \tag{15}$$

and G is the free space Green's function. For $M \neq 0$, boundary condition (4) gives the following relation between $\mathbf{J}_e$ and $\mathbf{J}_m$ on the surface of a PEMC object

$$\mathbf{J}_m = \frac{1}{M} \mathbf{J}_e. \tag{16}$$

Using equation (16) $\mathbf{J}_m$ can be removed from equation (14) and the integral equation for $\mathbf{J}_e$ reads

$$\left(\frac{-1}{i\omega\varepsilon_0} \left(\nabla\nabla \cdot + k_0^2 \right) \mathcal{S}(\mathbf{J}_e) - M \nabla \times \mathcal{S}(\mathbf{J}_e) \right)_{\tan} = -\mathbf{E}^{\rm inc}_{\tan}. \tag{17}$$

In the PEC case, i.e., $M = \infty$, $\mathbf{J}_m$ vanishes and equation (17) reduces to the usual EFIE for the electric surface current. In the PMC case, $M = 0$, or for small values of M, instead of equation (16) we write

$$\mathbf{J}_e = M \mathbf{J}_m \tag{18}$$

and equation (14) is written for the magnetic current only. In both cases the resulting integral equations are solved numerically with the method of moments [14] using triangular rooftop basis and testing functions [15]. Details of the numerical implementations are similar as those presented in [16] and [17] in the case of PEC and dielectric objects. It should be noticed that, similarly as in the cases of PEC and PMC objects, also in the case of a PEMC

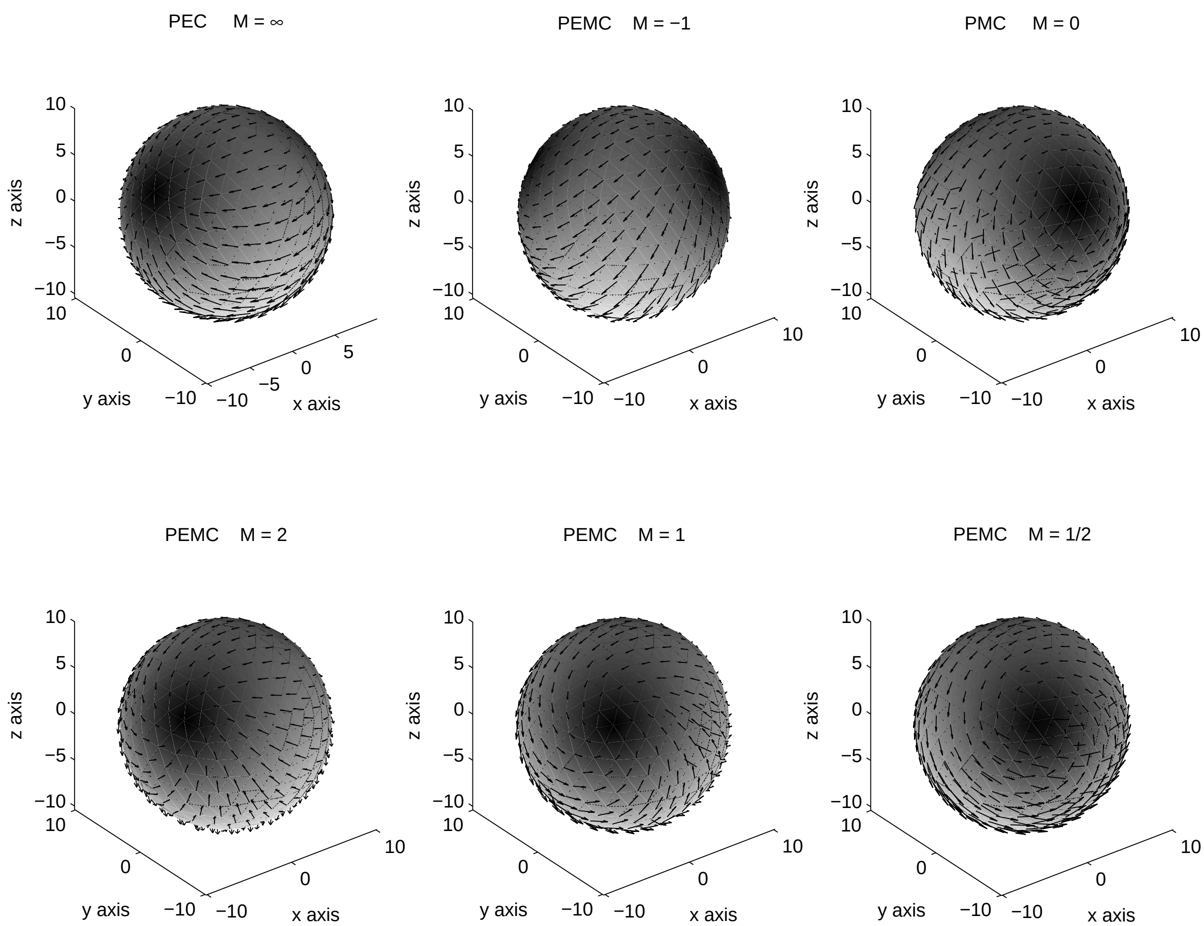

Fig. 8. The scattered far field of a small PEMC sphere with varying M parameters. $M = 0$ gives PMC and $M = \infty$ corresponds to PEC. Grayscale shows the amplitude and the arrows display the polarization of the radiated electric field. Incident electric field is parallel to the x axis.

object, a combined field integral equation formulation is needed to avoid problems with internal resonances.

The combined effects of the electrical size of the sphere and the PEMC parameter are shown in the set of Figs. 10 to 13, where the size of the sphere increases from $ka = 0.1$ to 3. These correspond to sphere diameters from $2a = 0.032\lambda$ to 0.95λ.

IV. DISCUSSION

We can draw several conclusions from the calculated results.

A. Co- and cross-polarization

The main property in the scattering behavior and characteristics of a PEMC sphere is the appearance of cross-polarization. Cross-polarization is strongest when the parameter $|M| = 1$ and also in the backscattering direction.

Quantitatively, the effect of the parameter M on backscattering is the following. The co- and cross-polarized components of the backscattered electric field as functions of the x and y components of the incident field read as

$$\begin{aligned} \begin{pmatrix} \mathbf{E}_x \\ \mathbf{E}_y \end{pmatrix}_s &= \begin{pmatrix} \frac{1-M^2}{1+M^2} & \frac{2M}{1+M^2} \\ \frac{-2M}{1+M^2} & \frac{1-M^2}{1+M^2} \end{pmatrix} \begin{pmatrix} \mathbf{E}_x \\ \mathbf{E}_y \end{pmatrix}_i \qquad (19) \\ &= -\begin{pmatrix} \cos 2\vartheta & -\sin 2\vartheta \\ \sin 2\vartheta & \cos 2\vartheta \end{pmatrix} \begin{pmatrix} \mathbf{E}_x \\ \mathbf{E}_y \end{pmatrix}_i . \end{aligned}$$

The appearance of a cross-polarized component in reflection is a sign of non-reciprocity. From equation (19) we can observe that the reflection matrix is a rotation matrix defined by the angle $\pi - 2\vartheta$. For example, in the case $M = \pm 1$ (meaning that the angle of equation (3) is $\vartheta = \pm 45^\circ$) the reflection is fully cross-polarized, in other words the rotation is $180^\circ - 2\vartheta = \pm 90^\circ$. In the other limit, the cross-polarization vanishes for the PEC (180° rotation) and PMC (0° rotation) cases.

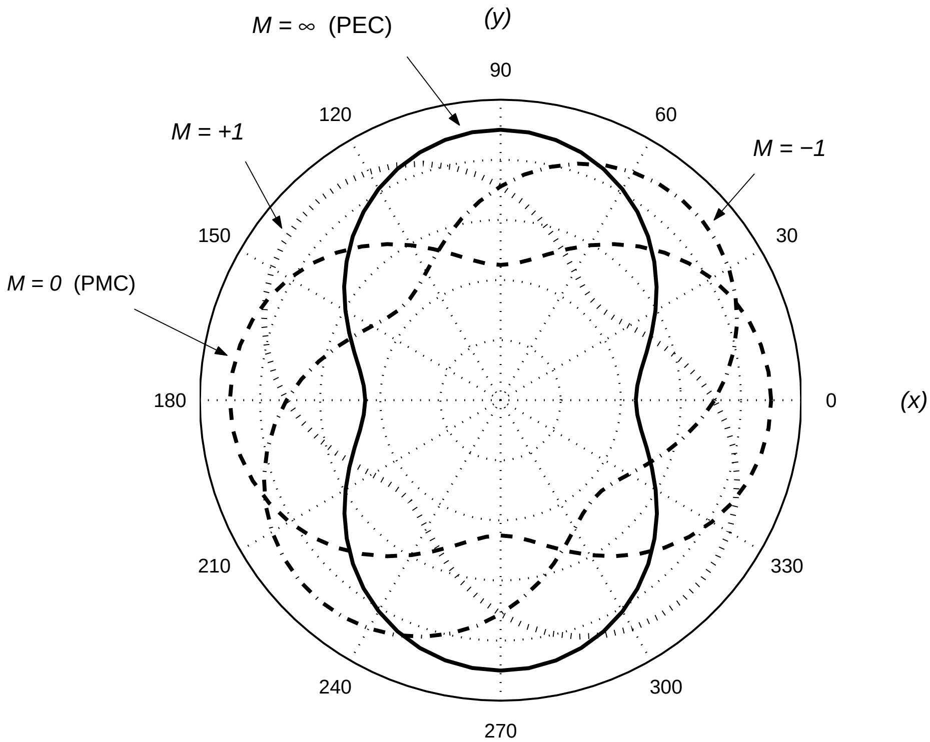

Fig. 9. The xy-plane far-field scattering plot of various spheres: PEC, PMC, and PEMC with $M = \pm 1$.

On the other hand, the forward-scattered field of a PEMC sphere with $|M| = 1$ is fully co-polarized. This is valid not only for electrically small spheres but also can be seen to hold when the sphere increases in the cases of $ka = 1, 2, 3$ (Figs. 11 to 13). When the M values starts to differ from unity, co-polarization appears in the backscattered radiation and cross-polarization in the forward direction. Of course, in the limits of both PEC and PMC, the scattering is fully co-polarized in both E and H planes.

This phenomenon of cross-polarization in reflection is important to notice. It is a sign of non-reciprocity (for this concept, see, for example [12, 18]). Whereas in transmission problems a rotation of polarization is perfectly consistent with reciprocity, the situation for the reflection problem is different. An example of reciprocal type of polarization rotation is the optical activity in chiral materials: a linearly polarized electromagnetic wave undergoes a uniform twist in the plane of the polarization the angle of which is proportional to the chirality parameter, frequency, and the distance propagated. The handedness of the material determines the sense (clockwise or counterclockwise) into which the rotation is space happens. (It is important not to confuse this polarization rotation with the Faraday rotation that can take place with wave propagation in magnetoplasma or ferrites. But there the external magnetic field makes the phenomenon anisotropic and non-reciprocal whereas the optical activity of chiral materials is isotropic and reciprocal [19].)

B. *Radiation pattern and polarization*

The phenomenon of cross-polarization is connected with the fact that the radiation pattern is rotated when the M parameter changes in the PEMC sphere, as can be seen for example in Figs. 8 and 9. The minimum radiation on the E-plane of a small PEC sphere moves continuously into the H-plane in the PMC case. The direction of the rotation depends on the sign of M.

Furthermore, the polarization of the radiated electric field changes continuously from the longitudinal electric-dipole type radiation (PEC), through a helicoidal pattern into the latitudinal magnetic-dipole type pattern in the PMC limit. This evolution is clear from Fig. 8.

C. *Size dependence*

When the size of the sphere increases so that it is no longer electrically small, the radiation pattern becomes more complex. This is a well-known fact from the theory of Mie scattering of “ordinary” spheres. For PEMC spheres, the effect of varying M parameter can be seen in Figs. 10 to 13. The expected increase into the forward

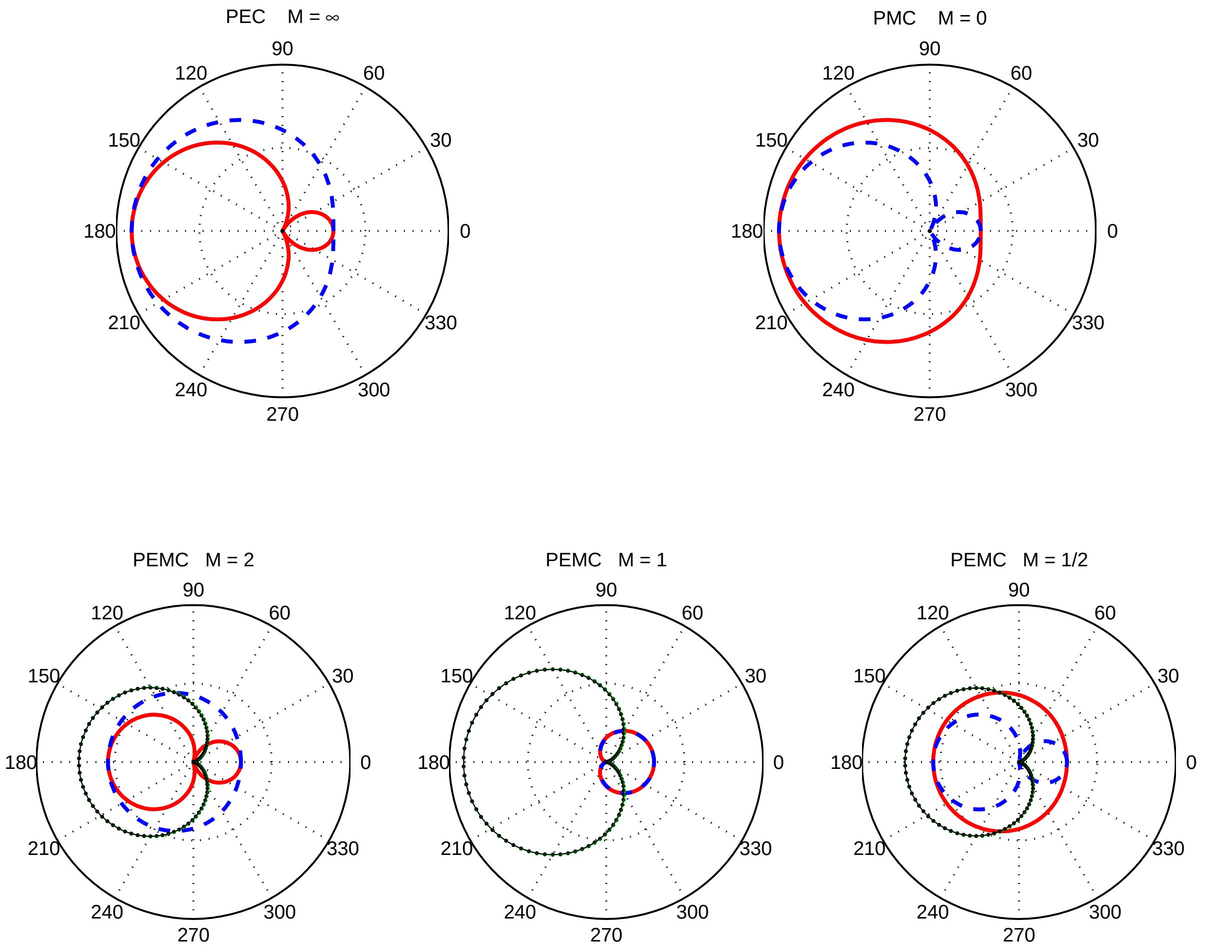

Fig. 10. The scattered far field power pattern of a small reflecting sphere $(ka = 0.1)$ with various values of the M parameter. Co-polarized in E-plane (solid line) and H-plane (dashed line). The dotted line shows the cross-polarized scattering (the same in both planes). The incident field is coming from the $180°$ direction.

scattering takes place regardless of the magnitude of M, as the size of the sphere increases from $ka = 0.1$ to 3. ($ka = 3$ means that the diameter of the sphere is around 95% of the wavelength.) But the results show clearly the cross-polarization in the backscattering direction, which is at its maximum for $|M| = 1$.

Another interesting observation is the following: for PEMC spheres, the radiated electric field is elliptically polarized in the E and H planes. As is known, this does not happen for PEC and PMC spheres even if the sphere is electrically large. This phenomenon can be seen in Figs. 14 to 15 which display the real and imaginary parts of the scattered electric field for a PEMC sphere with $ka = 1$. Both in the E and H planes we can observe that the real and imaginary parts are generally nonzero and are non-parallel, which implies that the temporal evolution of the electric field follows an ellipse.

D. Amplitudes

The preceding analysis has not paid any particular attention to the absolute amplitudes of the bistatic scattering cross section. This is because the behavior is quite uninteresting, as far as the effect of M parameter is concerned. Since the PEMC material is totally reflecting, in other words it is lossless and no field can penetrate into it, just like in the case of PEC and PMC, the scattered power remains independent of M. The effect is only on the distribution of power into the spatial directions and polarization states.

For example, the well-known [8, Sec. 10.61] result for the scattering efficiency of a small PEC sphere

$$Q_{\text{sca}} = \frac{10}{3}(ka)^4 \tag{20}$$

is correct for a PEMC (and PMC) sphere with any real value of M.

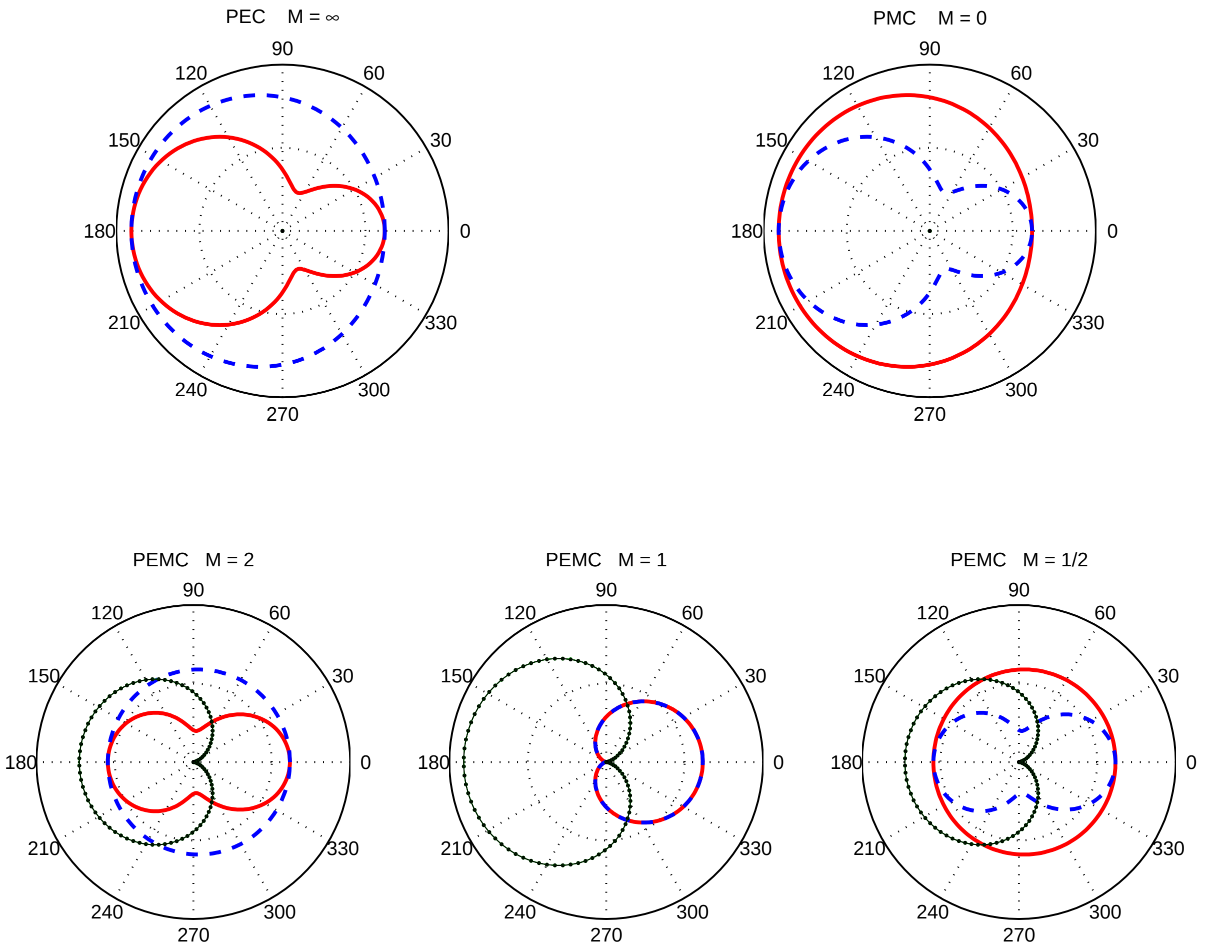

Fig. 11. The scattered far field power pattern of a small reflecting sphere $(ka = 1)$ with various values of the M parameter. Co-polarized in E-plane (solid line) and H-plane (dashed line). The dotted line shows the cross-polarized scattering (the same in both planes). The incident field is coming from the $180°$ direction.

V. CONCLUSIONS

The present paper has demonstrated that the scattering problem of particles made of rather complex materials can be solved. Using a computational surface integral equation software, where the required boundary condition is modified to account for the perfect electromagnetic conductor effect, the fields can be calculated. The results showed clearly the effect of the PEMC parameter on the scattering patterns: compared with a corresponding PEC scatterer, the polarization of the fields is rotated in reflection. The angle of rotation is determined by the PEMC amplitude between the limit cases of PEC and PMC materials. In the scattering patterns, this effect obviously manifests itself as cross-polarization which increases when the scattering direction deviates from the incident wave (forward) direction.

Because the polarization-rotating effect is present for all sphere sizes and the angle of rotation was seen to be independent of the size of the sphere relative to the wavelength, the results were only calculated for sphere sizes $ka = 0 \ldots 3$. If the size parameter becomes higher, the scattering pattern will display more and more sidelobes, as is well known. However, the effect of the M parameter of the scatterer on the patterns remains the same as for the lower frequency cases studied in this paper.

VI. ACKOWLEDGEMENT

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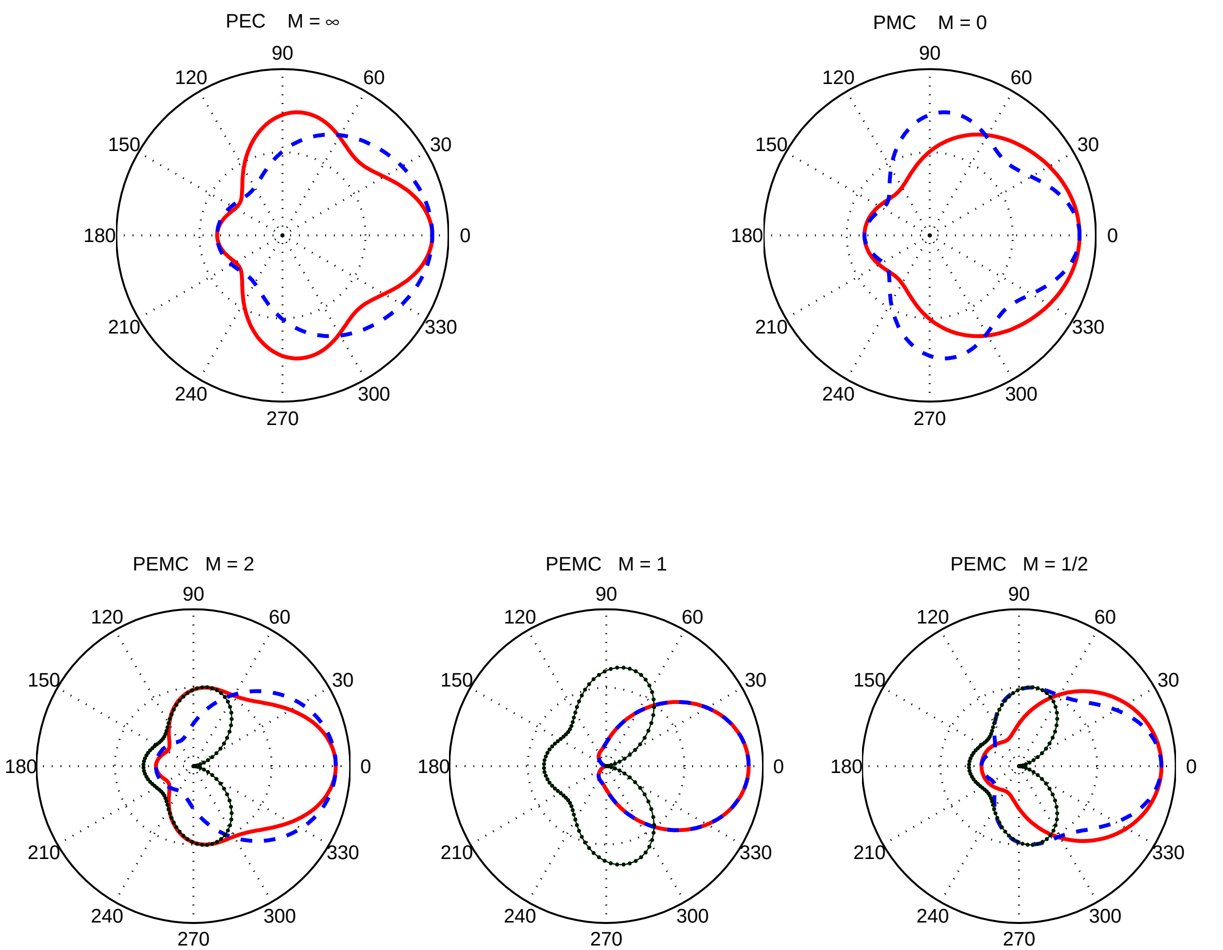

Fig. 12. The scattered far field power pattern of a small reflecting sphere $(ka = 2)$ with various values of the M parameter. Co-polarized in E-plane (solid line) and H-plane (dashed line). The dotted line shows the cross-polarized scattering (the same in both planes). The incident field is coming from the 180° direction.

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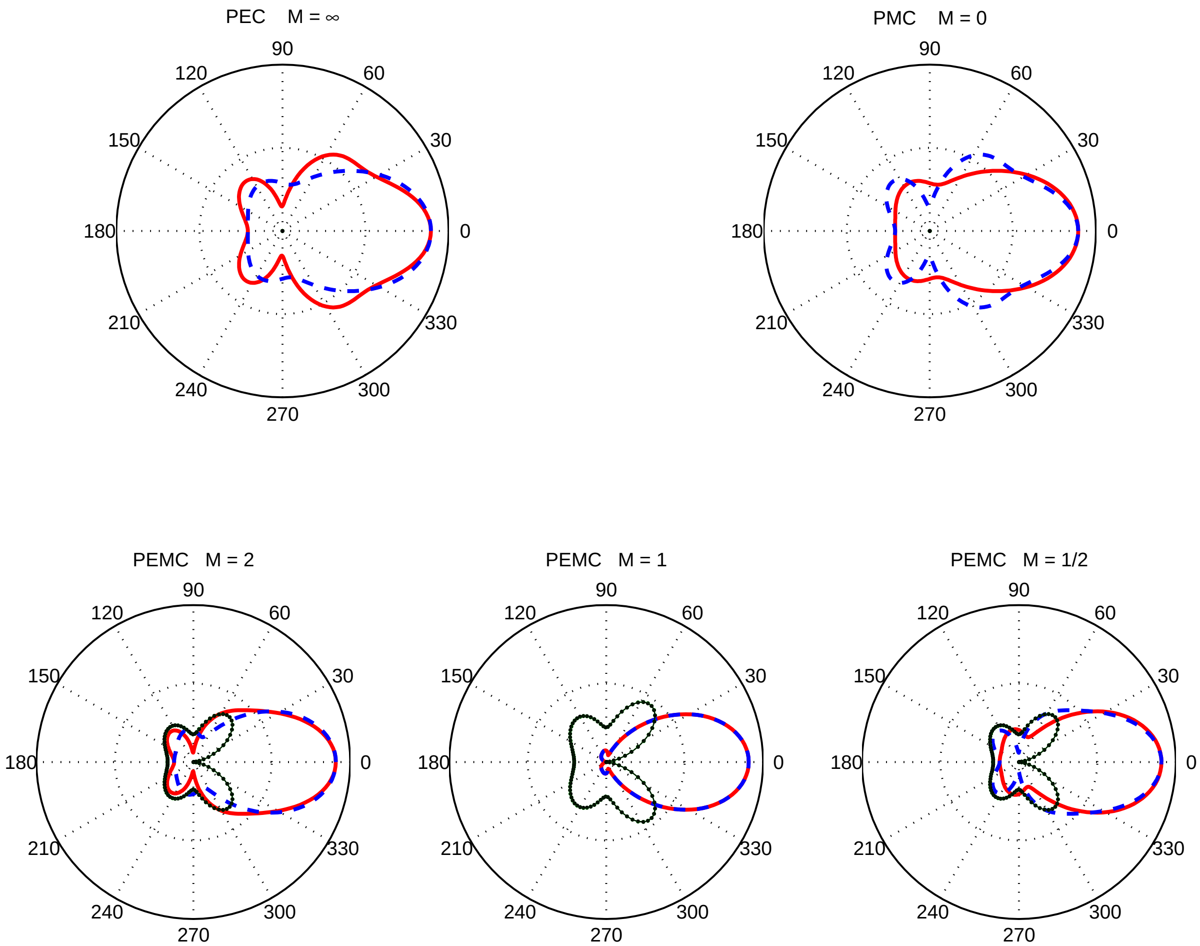

Fig. 13. The scattered far field power pattern of a small reflecting sphere $(ka = 3)$ with various values of the M parameter. Co-polarized in E-plane (solid line) and H-plane (dashed line). The dotted line shows the cross-polarized scattering (the same in both planes). The incident field is coming from the $180°$ direction.

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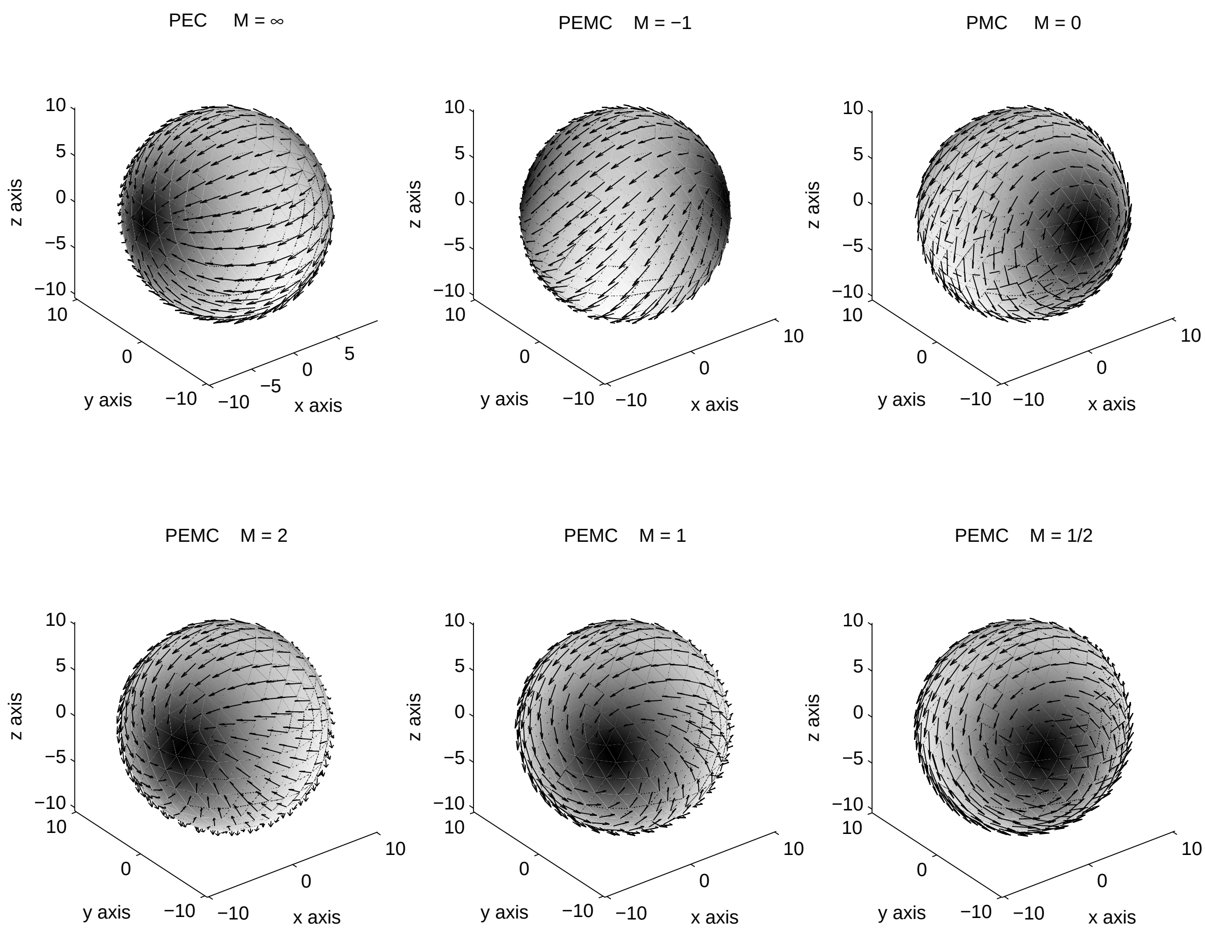


Fig. 14. The scattered far field of the PEMC sphere with varying M parameters: real part of the electric field. Grayscale shows the amplitude and the arrows display the polarization of the radiated electric field. Incident electric field is parallel to the x axis. The electrical size of the sphere is $ka = 1$.

Ari Sihvola is professor of electromagnetics at the Helsinki University of Technology (TKK), Espoo, Finland, and Academy Professor of the Academy of Finland. He received the degree of Doctor of Technology from TKK in 1987, and has been a visiting scientist and professor at the Massachusetts Institute of Technology (1985–1986), the Pennsylvania State University (1990-1991), Lund University (1996), and the Swiss Federal Institute of Technology, Lausanne (2000–2001). Ari Sihvola is the chairman of the Finnish National Committee of URSI (the International Union of Radio Science).

Pasi Ylä-Oijala received the M.Sc. degree in 1992 and the Ph.D. degree in 1999, both in applied mathematics in the University of Helsinki, Finland. Currently he is working as Academy Research Fellow in Electromagnetics Laboratory, Helsinki University of Technology, Finland. His field of interest includes numerical techniques in computational electromagnetics based on the integral equation methods.

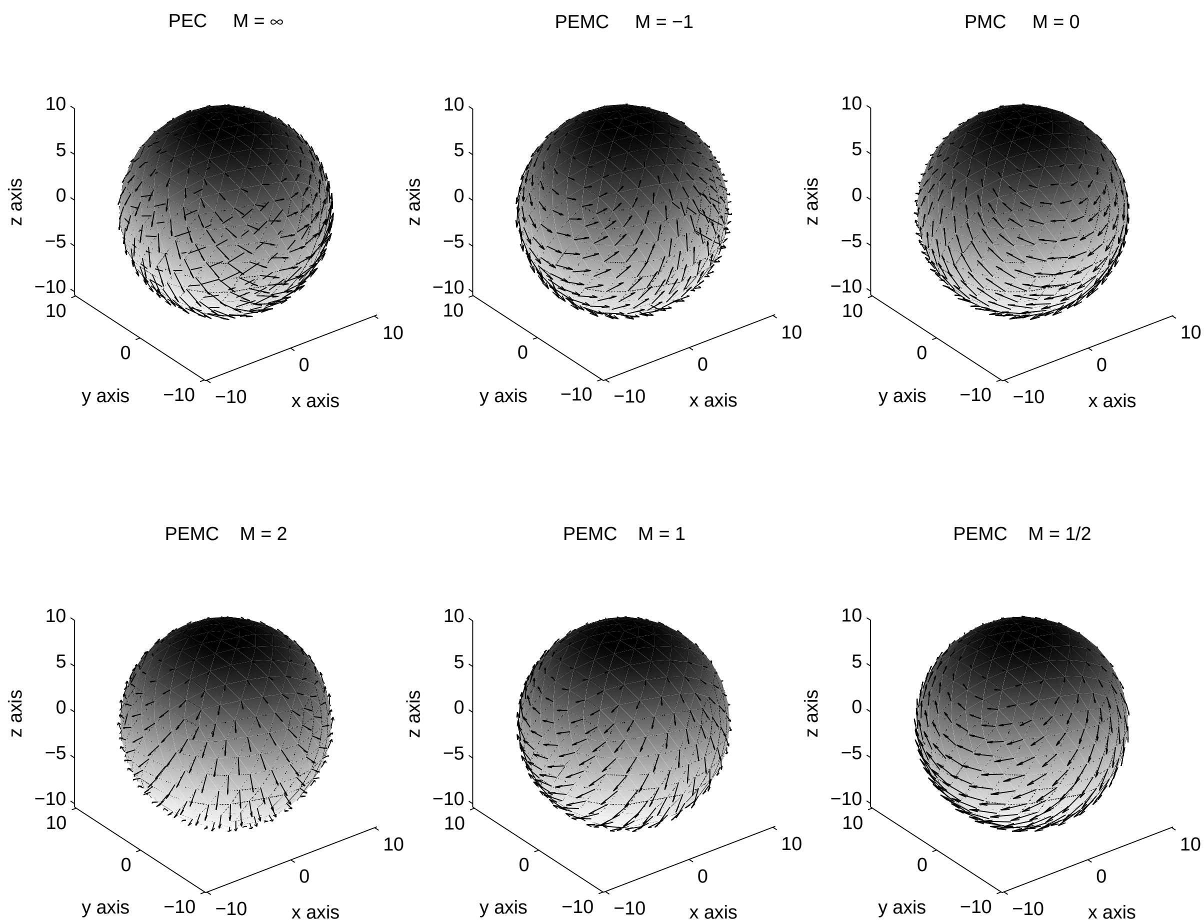
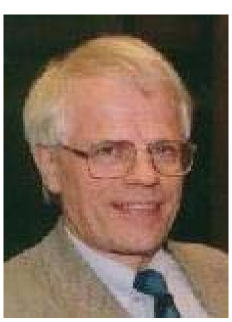


Fig. 15. The scattered far field of the PEMC sphere with varying M parameters: imaginary part of the electric field. Grayscale shows the amplitude and the arrows display the polarization of the radiated electric field. Incident electric field is parallel to the x axis. The electrical size of the sphere is $ka = 1$.



Ismo V. Lindell is professor emeritus at the Helsinki University of Technology, Espoo, Finland. He received the degree of Doctor of Technology in 1971. Besides having worked in TKK, he has served as visiting scientist and professor in University of Illinois and Massachusetts Institute of Technology. He served as the Academy Professor of the Academy of Finland during 1996–2001. Ismo Lindell is Life Fellow of the IEEE and the recipient of the IEEE S.A. Schelkunoff Prize (1987), the IEE Maxwell Premium (both 1997 and 1998), and the URSI Balthasar van der Pol Gold Medal (2005).