Computational Electromagnetics and Model-Based Inversion: A Modern Paradigm for Eddy-Current Nondestructive Evaluation

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(Invited Paper)

Abstract ─ This is the first of a planned series of papers in which we demonstrate the application of computational electromagnetics, especially the volume-integral method, to problems in eddy-current nondestructive evaluation (NDE). In particular, we will apply the volume-integral code, VIC-3D, to solve forward and inverse problems in NDE. The range of problems that will be considered spans industries from nuclear power to aerospace to materials characterization. In this paper we will introduce the notion of model-based inversion, emphasizing the role of `estimation-theoretic metrics' to the practical application of inverse theory.

Index Terms ─ volume-integral equations, electromagnetic nondestructive evaluation, model-based inversion, model-based standards, estimation-theoretic metrics.

I. INTRODUCTION

Nondestructive evaluation (NDE) is to materials and structures what CAT scanning is to the human body–an attempt to look inside without opening up the body. As in CAT scanning, modern NDE requires sophisticated mathematical software to perform its function. This is especially true with regard to quantitative NDE, wherein we attempt to quantify defects, that is, determine their size, location, even shape, rather than just to detect their presence. Low-frequency electromagnetic methods using eddy-currents are a traditional mode of doing NDE (approximately 35% of NDE uses eddy-currents, depending upon the specific application), but the technology still suffers from a lack of algorithms and software to allow its full potential to be realized.

In its essence, electromagnetic (eddy-current) nondestructive evaluation (NDE) is a scattering problem in which the anomaly (the flaw) in Figure 1 produces a current whose associated magnetic field is coupled into the probe coil. The change in driving-point impedance seen at the terminals of the coil is the measurable that indicates the presence of the anomaly. The 'anomalous current' associated with the flaw, then, is the principle electromagnetic quantity that is to be computed in order to determine the change in impedance, and to this end we have introduced VIC-3D© [1], a volume-integral code [2] [3].

The anomalous current is defined to be

$$\begin{aligned}\mathbf{J}_a(\mathbf{r}) &= (\sigma_f(\mathbf{r}) - \sigma_h)\mathbf{E}(\mathbf{r}) \\ &= \sigma_a(\mathbf{r})\mathbf{E}(\mathbf{r})\end{aligned} \tag{1}$$

where σ is the conductivity of the flaw region, σ_h is the (uniform) conductivity of the host, and $\mathbf{E}(\mathbf{r})$ is the total electric field, which is the sum of the incident field due to the probe coil and the secondary field due to $\mathbf{J}_a(\mathbf{r})$. Clearly, because the anomalous current is identically zero away from the flaw (or anomalous region), only this region needs to be gridded in order to transform the volume-integral equation into its discrete form via the Galerkin variant of the method of moments. Furthermore, if the grid is uniform in all three directions, the resulting discretized equations have matrix elements that are either Töplitz or Hankel. The *ij*th element of a Töplitz matrix is a function of $(i - j)$ and is a function of $(i + j)$ for a Hankel matrix. This allows one to compute matrix-vector products very quickly using the FFT when solving large problems with an iterative scheme, such as the conjugate-gradient method. Indeed, we solve problems with 100,000 unknowns quite routinely in a matter of minutes on personal computers using the volume-integral method. See [2] and [3] for the technical details.

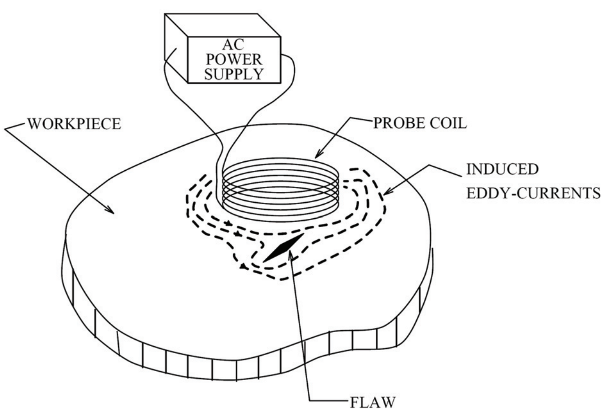

Fig. 1. Illustrating eddy-current nondestructive evaluation as a scattering problem.

II. MODEL-BASED INVERSION

In solving problems in eddy-current NDE, one often models the anomaly as a region that can be defined in terms of a few parameters. For example, we can model corrosion pitting in aerospace structures or heat-exchanger tubes in nuclear power plants by truncated right-circular cylinders–'pillboxes'–for which the parameters would be height and diameter (and perhaps the coordinates of the center of the pillbox). The inverse-scattering problem in which these parameters are to be determined from measurements of the driving-point impedance of the probe coil is what we call 'model-based inversion.'

Figure 2 illustrates a system representation for three important problems: (a) a direct problem, in which the input and system are known, and the output is to be determined; (b) a signal-detection (communication) problem, in which the system (a communication channel) and output are known, and the problem is to determine the input signal; and (c) the inverse problem, in which the input and outputs are known, and we must determine the system.

For the most part, the problems solved in [2]-[6] are direct problems; we assume knowledge of the probe and flaw, and determine the response of the probe, namely the driving-point or transfer impedances. The second problem of Figure 2 is dealt with in communication and information theory texts, and has a close relation to inverse problems.

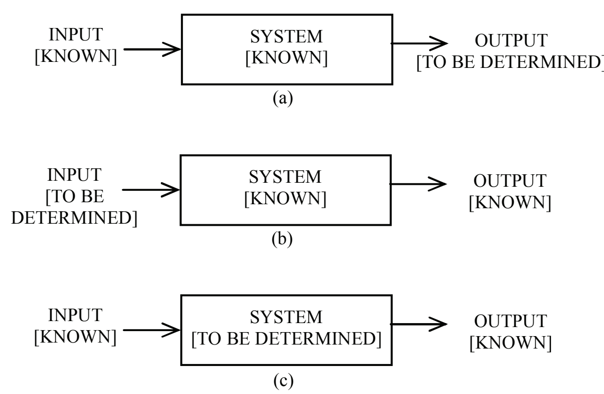

Fig. 2. System representation of direct and inverse problems: (a) The direct problem; (b) The signal-detection (communication) problem; (c) The inverse problem.

Nonlinear Least-Squares Parameter Estimation

Let

$$Z = g(p_1, \ldots, p_N, f), \tag{2}$$

where $p_1,\ldots, p_N$ are the N parameters of interest, and f is a control parameter at which the impedance, Z is measured. The parameter f can be frequency, scan-position, lift-off, etc. It is, of course, known; it is not one of the parameters to be determined. To be explicit during our initial discussion of the theory, we will call f 'frequency.'

In order to determine $p_1,\ldots,p_N$, we measure Z at M frequencies, $f_1,\ldots,f_M$, where $M > N$:

$$\begin{aligned} Z_1 &= g(p_1,\ldots,p_N,f_1) \\ &\vdots \\ Z_M &= g(p_1,\ldots,p_N,f_M). \end{aligned} \tag{3}$$

The right-hand side of (3) is computed by applying the volume-integral code to a model of the problem, usually at a discrete number of values of the vector, **p**, forming a multidimensional interpolation grid.

Because the problem is nonlinear, we use a Gauss-Newton iteration scheme to perform the inversion. First, we decompose (3) into its real and imaginary parts, thereby doubling the number of equations (we assume the $p_1,\ldots,p_N$ are real). Then we use the linear approximation to the resistance, R_i, and reactance, X_i, at the ith frequency:

$$\begin{bmatrix} R_1 \\ X_1 \\ \vdots \\ R_M \\ X_M \end{bmatrix} \approx \begin{bmatrix} R_1(p_1^{(q)},\ldots,p_N^{(q)}) \\ X_1(p_1^{(q)},\ldots,p_N^{(q)}) \\ \vdots \\ R_M(p_1^{(q)},\ldots,p_N^{(q)}) \\ X_M(p_1^{(q)},\ldots,p_N^{(q)}) \end{bmatrix} + \begin{bmatrix} \frac{\partial R_1}{\partial p_1}\cdots\frac{\partial R_1}{\partial p_N} \\ \frac{\partial X_1}{\partial p_1}\cdots\frac{\partial X_1}{\partial p_N} \\ \vdots \\ \frac{\partial R_M}{\partial p_1}\cdots\frac{\partial R_M}{\partial p_N} \\ \frac{\partial X_M}{\partial p_1}\cdots\frac{\partial X_M}{\partial p_N} \end{bmatrix}_{p_1^{(q)},\ldots,p_N^{(q)}} \begin{bmatrix} p_1 - p_1^{(q)} \\ \vdots \\ p_N - p_N^{(q)} \end{bmatrix}, \tag{4}$$

where the superscript (q) denotes the qth iteration, and the partial derivatives are computed numerically by the software. The left side of (4) is taken to be the measured values of resistance and reactance. We rewrite (4) as

$$0 \approx r + Jp, \tag{5}$$

where r is the $2M$-vector of residuals, J is the $2M \times N$ Jacobian matrix of derivatives, and p is the N-dimensional correction vector. Equation (5) is solved in a least-squares manner starting with an initial value, $(x_1^{(0)},\ldots,x_N^{(0)})$, for the vector of unknowns, and then continuing by replacing the initial vector with the updated vector $(x_1^{(q)},\ldots,x_N^{(q)})$ that is obtained from (4), until convergence occurs [7].

III. ESTIMATION-THEORETIC METRICS

We are interested in determining a bound for the sensitivity of the residual norm to changes in some linear combination of the parameters. Given an $\varepsilon > 0$ and a unit vector, v, the problem is to determine a sensitivity (upper) bound, σ, such that

$$\left\| r(x^* + \sigma v) \right\| \le (1+\varepsilon) \left\| r(x^*) \right\|. \tag{6}$$

A first-order estimate of σ is given by

$$\sigma_v = \left(\frac{\left\| r(x^*) \right\|}{\left\| J(x^*) \cdot v \right\|} \right). \tag{7}$$

Note that if $\left\| J(x^*) \cdot v \right\|$ is small compared to $\left\| r(x^*) \right\|$, then σ is large and the residual norm is insensitive to changes in the linear combination of the parameters specified by v. If $v = e_i$, the ith column of the $N \times N$ identity matrix, then (7) produces σ_i, the sensitivity bound for the ith parameter. Since σ_i will vary in size with the magnitude of x_i^*, it is better to compare the ratios σ_i / x_i^*, for $i = 1,\ldots,N$ before drawing conclusions about the fitness of a solution.

The importance of these results is that we now have metrics for the inversion process: $\Phi = \left\| r(x^*) \right\|$, the norm of the residual vector at the solution, tells us how good the fit is between the model data and measured data. The smaller this number the better, of course, but the 'smallness' depends upon the experimental setup and the accuracy of the model to fit the experiment. Heuristic judgment based on experience will help in determining the quality of the solution for a given Φ.

The sensitivity coefficient, σ, is more subtle, but just as important. It, too, should be small, but, again, the quality of the 'smallness' will be determined by heuristics based upon the problem. If σ is large in some sense, it suggests that the solution is relatively independent of that parameter, so that we cannot reasonably accept the value assigned to that parameter as being meaningful, as suggested in Figure 3, which shows a system, *S*, for which the system is sensitive to variable, x_i, at the solution point, x_i^*, and another system, *I*, for which the system is insensitive to x_i.

An example occurs when one uses a high-frequency excitation, with its attendant small skin depth, to interrogate a deep-seated flaw. The flaw will be relatively invisible to the probe at this frequency, and whatever value is given for its parameters will be highly suspect. When this occurs we will either choose a new parameter to characterize the flaw, or acquire data at a lower frequency.

These metrics are not available to us in the current inspection method, in which analog instruments acquire data that are then interpreted by humans using hardware standards. The opportunity to use these metrics is a significant advantage to the model-based inversion paradigm that we propose in this paper.

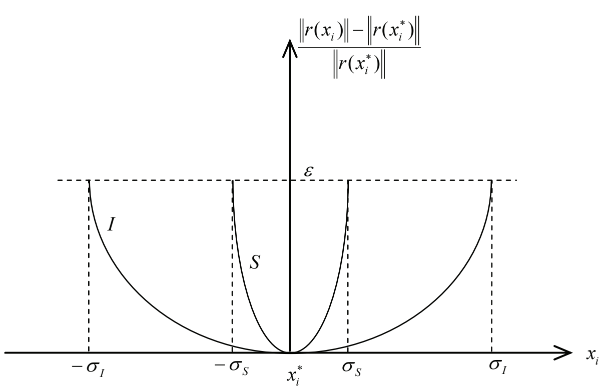

Fig. 3. Showing sensitivity parameters for two system responses to x_i. Response *S* is sensitive to x_i at $\boldsymbol{x_i^*}$, whereas response *I* is not.

IV. INVERSE METHOD QUALITY METRICS

Given the potential of inverse methods, it is important to develop a rigorous method for quantifying the performance and reliability of inversion schemes [8]. Although empirical studies provide the means for evaluating the quality of NDE techniques incorporating inverse methods, opportunities also exist with inverse methods to use the model calculations with quantitative measures to evaluate key estimation performance metrics without considerable experimental burden.

In estimation theory, the Cramer-Rao Lower Bound (CRLB) provides the minimum variance that can be expected for an unbiased estimator of a set of unknown parameters. In other words, the CRLB provides a way of quantifying the inversion algorithm performance. For Gaussian noise, there is a simple inverse relationship between the CRLB and the Fisher information [9]:

$$\mathrm{var}\left(\hat{\theta}_i\right) = \left[C_{\hat{\theta}}\right]_{ii} \geq \left[I^{-1}(\theta)\right]_{ii}, \tag{8}$$

where $\boldsymbol{C}$ is the covariance matrix, the Fisher information is defined as

$$I(\theta)_{ij} = -E\left[\frac{\partial^2 \ln f(Z;\theta)}{\partial\theta_i \partial\theta_j}\right]. \tag{9}$$

θ is the parameter being estimated, and Z is the measurement vector. Fisher information represents the amount of information contained in a measurement and depends on the derivatives of the likelihood function which is based on the forward model and the noise parameters. The variance in a measurement is inversely related to the amount of information contained in the measurement, so it is not a surprise that (8) shows that the variance in the measurement is greater than or equal to the inverse of the Fisher information matrix. In eddy current NDE, the measurement is often the real and imaginary component of the impedance, $Z=[R,X]$, and the Fisher information becomes a square matrix with dimensions equal to the number of parameters being estimated.

The covariance matrix can be evaluated as a performance metric for inverse methods. First, the diagonal terms of the covariance matrix (the CRLB variances) provide a metric of sensitivity of a parameter estimated using inverse methods to measurement variation. Second, the off-diagonal terms represent the interdependence between select parameters being estimated to measurement variation. The corresponding metric is the correlation coefficient given by

$$\rho_{i,j} = \frac{C_{ij}}{\sqrt{C_{ii}C_{jj}}}. \tag{10}$$

These metrics can be used with parametric studies involving frequency or other probe parameters to optimize the NDE system design. As a general design rule for inverse methods, it is desirable to minimize the sensitivity to variation (the CRLB variances) and to have the correlation coefficient between the parameters being estimated approach zero.

Another tool used in numerical linear algebra for sensitivity analysis is singular value decomposition (SVD). SVD essentially provides a measure of sensitivity of measurements to perturbations in the unknown parameters [10]. To evaluate the sensitivity of an inverse problem for a set of measurements to changes in fit parameters, SVD can be applied to the Jacobian matrix such that

$$\mathrm{J} = U \Sigma V', \tag{11}$$

where U is an orthogonal matrix that contains the left singular vectors of J, V is an orthogonal matrix that contains the right singular vectors, and Σ is a diagonal matrix that contains the singular values of J.

The condition number (CN) of the matrix is defined as the ratio of the largest and smallest singular values resulting from SVD. For inversion, CN has been used to quantify the well-posedness of the inverse problem for select parameters [11]. The ability to estimate parameters independently increases as the condition number approaches 1. It should be noted that SVD does not incorporate noise; it depends only on the noiseless relationship between the measurement output and the parameter changes.

V. OPTIMIZING LAYER ESTIMATION USING METRICS

An inversion experiment is revisited [12] for the purpose of demonstrating estimation theory metrics. In this experiment, the thickness of an AISI-304 stainless steel plate and probe liftoff were estimated. The estimation procedure is represented in (12), which is a specialization of (4) to this problem with two unknown parameters. The left side is the measured impedance, the Jacobian is simply the derivative information from the forward model, and the thickness and liftoff parameters are updated until this equation converges,

$$\begin{bmatrix} R(f,t,l) \\ X(f,t,l) \end{bmatrix} \approx \begin{bmatrix} R(f,t_0,l_0) \\ X(f,t_0,l_0) \end{bmatrix} + \begin{bmatrix} \frac{\partial R}{\partial t} & \frac{\partial R}{\partial l} \\ \frac{\partial X}{\partial t} & \frac{\partial X}{\partial l} \end{bmatrix}_{t_0,l_0} \begin{bmatrix} t-t_0 \\ l-l_0 \end{bmatrix}. \tag{12}$$

Four scenarios in particular are investigated. Impedance values were generated for combinations of lift-off values of 0.75 and 1.5 mm and a plate thickness values of 1.0 mm and 2.0 mm with Gaussian noise of 1% of the impedance value added as shown in Figure 4(a). For each of these "measurements", the NLSE algorithm is applied to estimate the thickness and liftoff simultaneously. Figure 4(b) shows the inversion results in the parameter space. Note that for high liftoff, visual inspection indicates the variance in the estimation is much greater for liftoff and likewise for the thicker plate, the variance of the estimation of thickness is greater.

The calculations required for the CRLB involve taking numerical derivatives of the impedance changes with respect to the parameter changes from the forward model. These calculations thus require far less computational expense with respect to Monte-Carlo simulation. Following (9), the Fisher information for this particular case is given by:

$$I = \begin{bmatrix} J_{11}^2 + J_{21}^2 & J_{12}J_{11} + J_{22}J_{21} \\ J_{11}J_{12} + J_{21}J_{22} & J_{12}^2 + J_{22}^2 \end{bmatrix}. \tag{13}$$

The covariance matrix is then calculated from the Fisher information by (8):

$$C = \sigma^2 I^{-1}. \tag{14}$$

The Jacobian is also decomposed into its singular values and singular vectors in the form of the right hand side of (11). The ratio of the smallest to largest singular values provides the condition number.

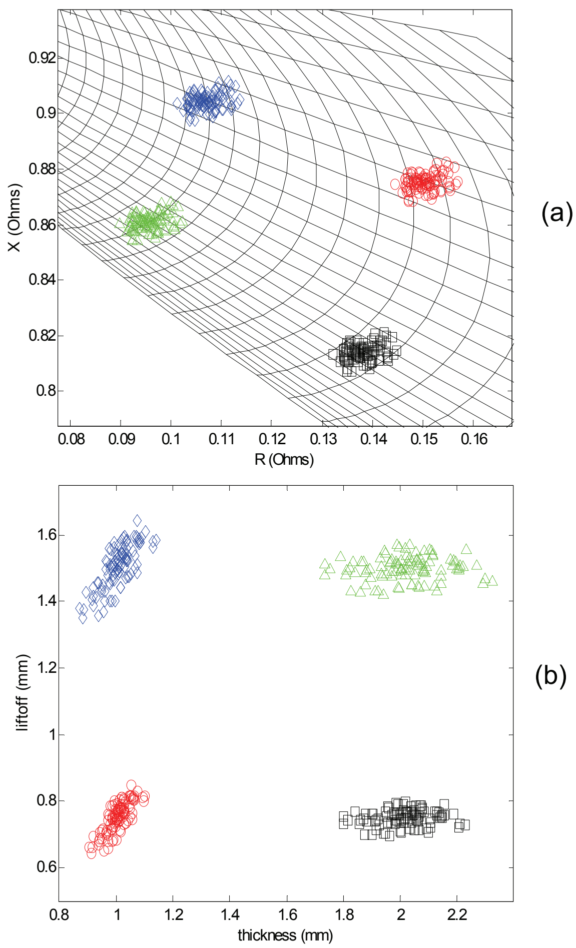

Fig. 4. (a) Distribution of source data in impedance plane and (b) corresponding estimated values in liftoff-thickness parameter space.

Figure 5 shows the CRLB of the estimation of the thickness and liftoff of a 1 mm thick plate and 1 mm lift-off for multiple frequencies. The agreement between the CRLB and the Monte-Carlo approach is quite good. This analysis demonstrates that there is an optimal frequency to achieve highest accuracy in the estimation of thickness. Estimating conductivity and thickness simultaneously is typically more ill-conditioned than estimating thickness and liftoff simultaneously. The CRLB for conductivity and thickness estimation along with the condition number and correlation number as a function of frequency are all displayed in Figure 6. The behavior of the CRLB as a function of frequency for estimating conductivity and thickness simultaneously follows a similar trend and this is expected since the impedance changes due to conductivity and thickness are similar. The condition number reaches a maximum around 95 kHz which implies that selectivity is good and the correlation is zero at this frequency which further confirms that point.

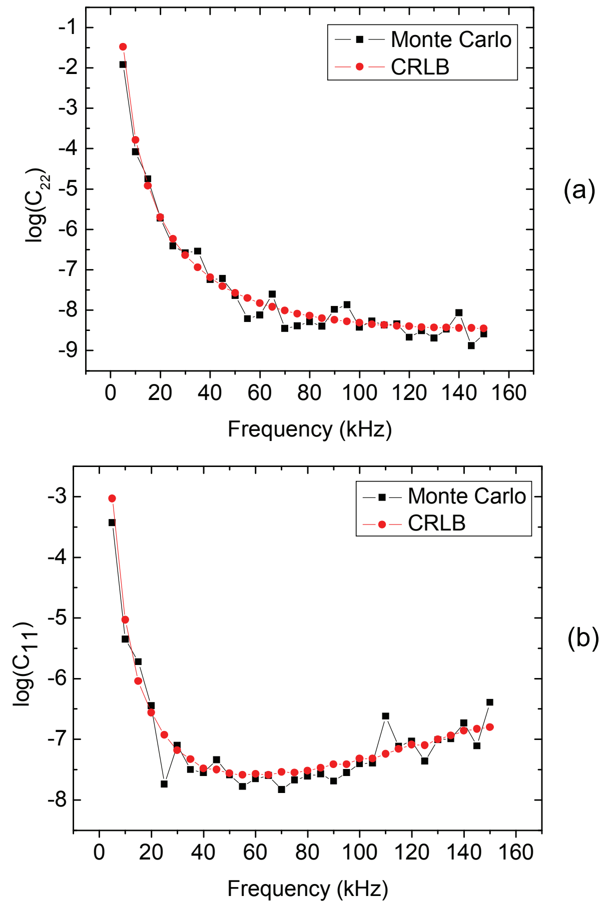

Fig. 5. Comparison of variance for varying frequency using CRLB and Monte Carlo methods for estimating (a) liftoff and (b) thickness respectively.

VI. CONCLUSION

The electromagnetics volume-integral code, VIC-3D(c), was developed to address the forward problem in eddy current NDE and provide the foundation for flaw characterization using inverse methods. The numerical method addresses a range of problems spanning industries from nuclear power to aerospace to materials characterization. The notion of model-based inversion was introduced, emphasizing the role of estimation-theoretic metrics to the practical application of inversion theory. Several metrics from estimation theory were proposed to evaluate the quality of inversion schemes. These metrics can be used in the design and validation of NDE inspection systems. Here, the CRLB has been evaluated for two parameter estimation problems. The CRLB was found to converge to the variance from Monte-Carlo simulations for Gaussian noise. The condition number derived from SVD and the correlation terms were also presented for two parameter estimation often with similar trends corresponding to parameter selectivity. It is interesting to note that if the CRLB and the condition number are used to determine the optimal frequency for inversion, they may not be in agreement. Further work will be conducted to understand the proper way to compromise between these estimation metrics. Furthermore, studies

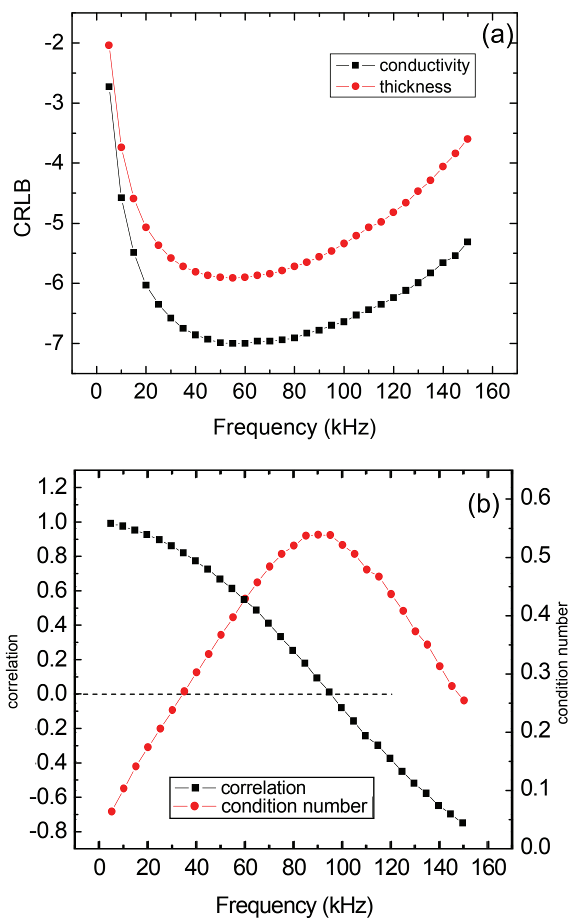

Fig. 6. Comparison of inversion metrics for varying frequency: (a) CRLB variance for thickness and conductivity estimation and (b) correlation and condition number.

addressing more challenging estimation problems including more than two parameters and multiple frequencies will be pursued. Lastly, estimation metrics will be extended to non-Gaussian noise cases.

ACKNOWLEDGEMENT

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REFERENCES

[1] http://www.kiva.net/~sabbagh.

[2] R. K. Murphy, H. A. Sabbagh, J. C. Treece and L. W. Woo, ‘A Volume-Integral Code for Electromagnetic Nondestructive Evaluation,’ Conference Proceedings: 11th Annual Review of Progress in Applied Computational Electromagnetics, Monterey, CA, March 20-25, pp. 109-116, 1995.

[3] R. Murphy, H. Sabbagh, A. Chan, and E. Sabbagh, “A volume integral code for electromagnetic nondestructive evaluation”, *Proceedings of the 13th Annual Review of Progress in Applied Computational Electromagnetics*, Monterey, CA, March 1997.

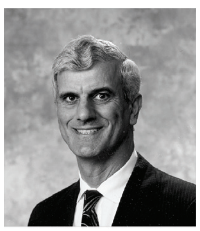
[4] J. S. Knopp, H. A. Sabbagh, J. C. Aldrin, R. K. Murphy, E. H. Sabbagh, J. Hoffmann and G. J. Steffes, ‘Efficient Solution of Electromagnetic Scattering Problems using Spatial Decomposition Algorithms,’ Conference Proceedings: Review of Progress in Quantitative Nondestructive Evaluation, Volume 25A, D. O. Thompson and D. E. Chimenti, eds., American Institute of Physics, pp. 299-306, 2006.

[5] J. R. Bowler, L. D. Sabbagh, and H. A. Sabbagh, “A Theoretical and Computational Model of Eddy-Current Probes Incorporating Volume Integral and Conjugate Gradient Methods,” IEEE Trans. Magnetics, vol. 25, no. 3, pp. 2650-2664, 1989.

[6] J. S. Knopp, J. C. Aldrin, and P. Misra, “Considerations in the Validation and Application of Models for Eddy Current Inspection of Cracks Around Fastener Holes”, Journal of Nondestructive Evaluation, vol. 25, no. 3, pp. 123-138, 2006.

[7] J. J. More, B. S. Garbow, and K. E. Hillstrom, *USER GUIDE FOR MINPACK-1*,

ANL-80-74, Argonne National Laboratory, August 1980.
[8] J. S. Knopp, J. C. Aldrin, H. A. Sabbagh, K. V. Jata, "Estimation Theory Metrics in Electromagnetic NDE", Electromagnetic Nondestructive Evaluation Workshop Proceedings, Soeul, Korea, June 10-12, 2008.
[9] S. M. Kay, *Fundamentals of Statistical Processing, Volume I: Estimation Theory*, Prentice Hall PTR, 1993.
[10] L. N. Trefethen and D. Bau, *Numerical Linear Algebra*, SIAM, 1997.
[11] N. J. Goldfine, "Magnetometers for Improved Materials Characterization in Aerospace Applications", *Mat. Eval*. vol. 51, pp. 396-405, 1993.
[12] O. Baltzersen, "Model-Based Inversion of Plate Thickness and Liftoff from Eddy Current Probe Coil Measurements", *Mat. Eval.*, vol. 51, pp. 72-76, 1993.



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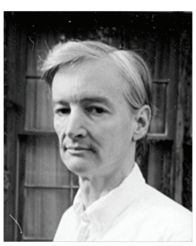

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Jeremy Knopp received the B. Sc. degree in engineering physics and the M.Sc. degree in electrical engineering from Wright State University, Dayton, OH, USA, in 2001 and 2005, respectively. Since 2002, he has worked as a researcher at the nondestructive evaluation (NDE) branch of the Air Force Research Laboratory (AFRL). His research interests include eddy current NDE, computational electromagnetics, inverse problems, and model-assisted probability of detection. In 2009, he won the Charles J. Cleary award for basic research at AFRL.

Mark Blodgett received the B. Sc. in metallurgical engineering from Iowa State University, Ames, Iowa, USA in 1985 and M.Sc. and Ph.D. degrees in materials engineering from University of Dayton, Dayton Ohio, USA in 1992 and 2000, respectively. Since 1986, he has worked as materials research engineer in the Nondestructive Evaluation Branch at the Air Force Research Laboratory. His research interests include developing electromagnetic and multi-modal NDE approaches for materials characterization.