MoM Analysis of Apertures in Chiral Bodies of Revolution

Khaja Qutubuddin[1], Halid Mustacoglu[2], Joseph R. Mautz[1], and Ercument Arvas[1]

[1] Department of EECS
Syracuse University, Syracuse, NY 13244, USA
kqutubud@syr.edu, jrmautz@syr.edu, earvas@syr.edu

[2] Anaren Microwave, Inc.
East Syracuse, NY 13057, USA
hmustacoglu@anaren.com

Abstract **─** A chiral body of revolution (BOR) which is partially covered by a thin conducting shield is analyzed using the Method of Moments (MOM). The axisymmetric system is excited by a plane wave. The internal fields and the far scattered fields are computed. The problem is solved using the surface equivalence principle. The scattered fields outside the structure are assumed to be produced by an equivalent magnetic surface current that exists on the unshielded part of BOR surface and an external equivalent electric surface current that exists over the entire BOR surface. These two currents are assumed to radiate in the unbounded external medium. Similarly, the internal fields are assumed to be produced by the negative of the above magnetic current and an internal electric surface current that exists over the entire BOR surface, but is an independent unknown only on the shielded part of the BOR surface. These two currents radiate in the unbounded internal medium. Enforcing the boundary conditions at the surface of the BOR results in a set of coupled integral equations for the three equivalent surface currents. These equations are solved numerically using the MOM. The computed results for the partially shielded spherical chiral body are in excellent agreement with other data.

Index Terms **─** Aperture, body of revolution, chiral body, method of moments, equivalence principle.

I. INTRODUCTION

Figure 1 shows a chiral body of revolution that is partially covered by a perfectly conducting shield. The system is excited by a plane wave. We are interested in finding the field that penetrates into the chiral body through the apertures on its surface and the far field scattered by the structure. The problem of electromagnetic penetration into a regular dielectric body of revolution that is partially covered by a perfectly conducting shield is analyzed in [1] and [2]. The problem of electromagnetic transmission through an arbitrary aperture in an arbitrary 3-D conducting surface enclosing chiral material is analyzed in [3]. The electromagnetic analysis of general bodies of revolution is given in [4].

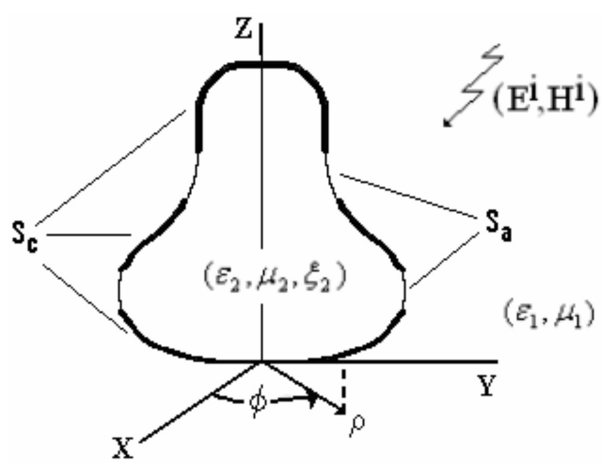

Fig. 1. A chiral body of revolution with two apertures.

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Penetration of electromagnetic waves through apertures has been studied extensively. Two dimensional apertures in thin infinite planes are studied in [5] and [6]. Apertures in arbitrarily shaped three dimensional objects are studied in [3] and [7]. In [3] the internal medium considered was chiral and in [7] both internal and external media were regular dielectrics. The research problem dealt with in [8] relates to a rotationally symmetric aperture on a perfectly conducting BOR containing the same homogeneous dielectric in the interior as well as the exterior. A boundary integral equation is used in [9] for dielectric objects partially coated with a perfectly conductive layer. Diffraction of an electromagnetic plane wave by a rectangular plate and a rectangular hole in the conducting plate [10] is rigorously tackled using the method of the Kobayashi potential (KP method).

In [11], there is a method applicable to arbitrarily-shaped apertures (in particular those not axially symmetric in bodies of revolution) employing the method of moments. The problem of the scattering of an electromagnetic plane wave with arbitrary polarization and angle of incidence from a perfectly conducting spherical shell with a circular aperture [12] is solved with a generalized dual series approach. In [13], the problem of scattering from a spherical shell with a circular aperture symmetrically illuminated by a plane electromagnetic wave is solved by expanding the fields inside and outside the cavity in terms of spherical vector wave functions. In [14], a hybrid FE−BI method that combines the finite element (FEM) and boundary integral (BI) methods is used to analyze electromagnetic scattering from structures consisting of an inhomogeneous dielectric body attached to perfectly conducting bodies. A new variational direct boundary integral equation approach is presented in [15] for solving the scattering and transmission problem for dielectric objects partially coated with a perfect electric conducting (PEC) layer. The absorption cross section of a dielectric sphere partially covered by a thin perfectly conducting spherical surface is calculated in [16]. Two-dimensional electromagnetic scattering by a dielectric cylinder partially covered by zero-thickness perfect conductors is treated in [17]. In [18], an axisymmetric chiral radome is analyzed via the method of moments. In [19], a method of moments solution is presented for electromagnetic scattering by a three-dimensional (3-D) inhomogeneous chiral scatterer illuminated by an arbitrary incident field.

This research work is important because of its significance for numerous applications in radar techniques and for tracking and discriminating between space vehicles and objects.

II. ANALYSIS

Let S_c be the part of the surface of the chiral body (scatterer) covered by rotationally symmetric perfect conductors, let S_a be the part of the surface of the scatterer not covered by the rotationally symmetric conductors, and let S be the entire surface of the scatterer as shown in Fig. 1. Here, the subscript "*c*" stands for conductor and the subscript "*a*" stands for aperture.

The surface equivalence principle is used to separate the problem of Fig. 1 into two simpler parts, namely, the region external to surface S and the region internal to S. The scattered fields in the external region are produced by an equivalent magnetic surface current **M** and an equivalent electric surface current $\mathbf{J}_e$ radiating in the unbounded external medium. The current **M** exists on only S_a and the current $\mathbf{J}_e$ exists on the whole surface S. The requirement that the tangential electric field of the external equivalence be zero just inside S is expressed as

$$-\frac{1}{\eta_e}\left[\mathbf{E}_e\left(\mathbf{J}_e,\mathbf{M}\right)\right]_{S^-}=\frac{1}{\eta_e}\left[\mathbf{E}^{inc}\right]_S, \tag{1}$$

where $\eta_e=\sqrt{\mu_e/\varepsilon_e}$ is the intrinsic impedance of the homogeneous achiral medium outside the scatterer in the original problem. Also, $\mathbf{E}_e(\mathbf{J}_e,\mathbf{M})$ is the electric field of the combination of $\mathbf{J}_e$ and **M**, both radiating in all space filled with the homogeneous achiral medium that is outside the scatterer in the original problem. The subscript S^- denotes evaluation on the side of S facing inside the scatterer of the tangential part of the enclosed vector. In (1), $\left[\mathbf{E}^{inc}\right]_S$ is the tangential part of the incident electric field on S.

The requirement that the tangential magnetic field of the external equivalence be zero just inside S is expressed as

$$-\left[\mathbf{H}_e\left(\mathbf{J}_e,\mathbf{M}\right)\right]_{S^-}=\left[\mathbf{H}^{inc}\right]_S, \tag{2}$$

where $\mathbf{H}_e(\mathbf{J}_e, \mathbf{M})$ is the magnetic field of the combination of $\mathbf{J}_e$ and $\mathbf{M}$, both radiating in all space filled with the homogeneous achiral medium that is outside the scatterer in the original problem.

The field in the chiral medium is produced by the equivalent magnetic surface current $-\mathbf{M}$, and an equivalent surface electric current $-\mathbf{J}_i$ radiating in the unbounded chiral medium. $\mathbf{J}_i$ exists on the whole surface S. The requirement that the tangential electric field of the internal equivalence be zero just outside S is expressed as

$$-\frac{1}{\eta_e}\left[\mathbf{E}_i\left(\mathbf{J}_{ie},\mathbf{M}\right)\right]_{S^+}=0, \tag{3}$$

where $\mathbf{E}_i(\mathbf{J}_{ie}, \mathbf{M})$ is the electric field of the combination of $\mathbf{J}_{ie}$ and $\mathbf{M}$, both radiating in all space filled with the homogeneous chiral medium that is inside the scatterer in the original problem. Here, $\mathbf{J}_{ie}$ is the combination of $\mathbf{J}_i$ on S_c and $\mathbf{J}_e$ on S_a. The subscript S^+ denotes evaluation on the side of S facing outside the scatterer of the tangential part of the enclosed vector.

The requirement that the tangential magnetic field of the internal equivalence be zero just outside S is expressed as

$$-\left[\mathbf{H}_i\left(\mathbf{J}_{ie},\mathbf{M}\right)\right]_{S^+}=0, \tag{4}$$

where $\mathbf{H}_i(\mathbf{J}_{ie}, \mathbf{M})$ is the magnetic field of the combination of $\mathbf{J}_{ie}$ and $\mathbf{M}$, both radiating in all space filled with the homogeneous chiral medium that is inside the scatterer in the original problem.

In view of (1) – (4), the equivalent currents $\mathbf{J}_e$ and $\mathbf{M}$ of the external equivalence and the equivalent currents $-\mathbf{J}_{ie}$ and $-\mathbf{M}$ of the internal equivalence assure that there is no electric field on both sides of S_c and that the tangential electric and magnetic fields are continuous across S_a.

The product of an arbitrary constant α with (3) is added to (1) to obtain

$$-\frac{1}{\eta_e}\left[\mathbf{E}_e\left(\mathbf{J}_e,\mathbf{M}\right)\right]_{S^-} -\frac{\alpha}{\eta_e}\left[\mathbf{E}_i\left(\mathbf{J}_{ie},\mathbf{M}\right)\right]_{S^+}=\frac{1}{\eta_e}\left[E^{inc}\right]_S, \tag{5}$$

and the product of an arbitrary constant β with (4) is added to (2) to obtain

$$-\left[\mathbf{H}_e\left(\mathbf{J}_e,\mathbf{M}\right)\right]_{S^-}-\beta\left[\mathbf{H}_i\left(\mathbf{J}_{ie},\mathbf{M}\right)\right]_{S^+}=\left[\mathbf{H}^{inc}\right]_S. \tag{6}$$

The method of moments as applied to bodies of revolution is used to solve (5) and (6) numerically. Piecewise linear variation of the currents is assumed along the generating curve of the BOR. The variation of the currents along the circumferential direction is represented by Fourier series. An approximate Galerkin's method is used for testing. If αβ* is real and positive where * denotes the complex conjugate, then it can be shown that (5) and (6) imply (1)–(4) [20, Section 2]. Equations (5) and (6) are two vector equations on S where the unknowns in (5) and (6) are $\mathbf{J}_e$ on S and the composite unknown consisting of $\mathbf{J}_i$ on S_c and $\mathbf{M}$ on S_a.

A. Expansion functions and testing functions

Let electric and magnetic currents $\mathbf{J}_e$, $\mathbf{J}_{ie}$, and $\mathbf{M}$ be expanded as

$$\mathbf{J}_e=\sum_{n=-N}^{N}\sum_{j=1}^{N_t}\left(I_{nj}^t\mathbf{J}_{nj}^t+I_{nj}^\phi\mathbf{J}_{nj}^\phi\right) \tag{7}$$

$$\mathbf{J}_{ie}=\sum_{n=-N}^{N}\sum_{j=1}^{N_t}\left(\left(L_j'V_{nj}^t+L_jI_{nj}^t\right)\mathbf{J}_{nj}^t+\left(L_j'V_{nj}^\phi+L_jI_{nj}^\phi\right)\mathbf{J}_{nj}^\phi\right) \tag{8}$$

$$\mathrm{M}=\eta_e\sum_{n=-N}^{N}\sum_{j=1}^{N_t}L_j\left(V_{nj}^t\mathbf{J}_{nj}^t+V_{nj}^\phi\mathbf{J}_{nj}^\phi\right), \tag{9}$$

where I_{nj}^t, I_{nj}^ϕ, V_{nj}^t, and V_{nj}^ϕ are complex constants to be determined and $\mathbf{J}_{nj}^t$ and $\mathbf{J}_{nj}^\phi$ are expansion functions given by

$$\mathbf{J}_{nj}^t=\hat{\mathrm{t}}\frac{T_j\left(t\right)}{\rho}e^{jn\phi} \tag{10}$$

$$\mathbf{J}_{nj}^\phi=\hat{\phi}\frac{T_j\left(t\right)}{\rho}e^{jn\phi}, \tag{11}$$

where t is the arc length along the generating curve C of the body of revolution (BOR), $\rho=\rho(t)$ is the distance from the z-axis of the BOR, ϕ is the angle measured from the positive x-axis toward the y-axis in the xy-plane, and $T_j(t)$ is the triangular function defined by

$$T_j\left(t\right)=\begin{cases}\dfrac{t-t_{2j-1}}{d_j}, & t_{2j-1}\le t\le t_{2j+1}\\ \dfrac{t_{2j+3}-t}{d_{j+1}}, & t_{2j+1}\le t\le t_{2j+3}\\ 0, & elsewhere\end{cases}, \tag{12}$$

where

$$d_j=\Delta_{2j-1}+\Delta_{2j} \tag{13}$$

$$\Delta_j = t_{j+1} - t_j \,. \tag{14}$$

The generating curve C consists of the straight line segment from $t=t_1$ to $t=t_2$, that from t_2 to t_3,…, that from $t_{2\mathrm{Nt}+2}$ to $t_{2\mathrm{Nt}+3}$ where, as in (7), N_t is the number of triangles on C. In (8) and (9),

$$L_j = \begin{cases} 1, \; T_j(t) \text{ is in an aperture} \\ 0, \; T_j(t) \text{ is on a conductor} \end{cases} \tag{15}$$

$$L'_j = \begin{cases} 1, \; T_j(t) \text{ is on a conductor} \\ 0, \; T_j(t) \text{ is in an aperture.} \end{cases} \tag{16}$$

Testing functions $\mathbf{J}^t_{-mi}$ and $\mathbf{J}^\phi_{-mi}$ are defined by

$$\mathbf{J}^t_{-mi} = \hat{\mathrm{t}} \frac{T_i(t)}{\rho} e^{-jm\phi} \tag{17}$$

$$\mathbf{J}^\phi_{-mi} = \hat{\phi} \frac{T_i(t)}{\rho} e^{-jm\phi} \,. \tag{18}$$

Henceforth, we assume that (7)–(9) have been substituted into (5) and (6). The symmetric product of two vector functions is the integration over S of their dot product. First taking the symmetric product of $\mathbf{J}^t_{-mi}$ with (5), then taking the symmetric product of $\mathbf{J}^\phi_{-mi}$ with (5), next taking the symmetric product of $\mathbf{J}^t_{-mi}$ with (6), and finally taking the symmetric product of $\mathbf{J}^\phi_{-mi}$ with (6), one obtains, for (i=1,2,…, N_t) and for (m= –N, –N+1,…,N), the following matrix equation.

$$\begin{bmatrix} Z^{tt}_n & Z^{t\phi}_n & C^{tt}_n & C^{t\phi}_n \\ Z^{\phi t}_n & Z^{\phi\phi}_n & C^{\phi t}_n & C^{\phi\phi}_n \\ D^{tt}_n & D^{t\phi}_n & Y^{tt}_n & Y^{t\phi}_n \\ D^{\phi t}_n & D^{\phi\phi}_n & Y^{\phi t}_n & Y^{\phi\phi}_n \end{bmatrix} \begin{bmatrix} I^t_n \\ I^\phi_n \\ V^t_n \\ V^\phi_n \end{bmatrix} = \begin{bmatrix} \vec{V}^t_n \\ \vec{V}^\phi_n \\ \vec{I}^t_n \\ \vec{I}^\phi_n \end{bmatrix} \tag{19}$$

For (n= –N, –N+1,…, N) where, for $q=t$ or $q=\phi$, I^q_n and V^q_n are column matrices whose j^{th} elements are I^q_{nj} and V^q_{nj}, respectively. The ij^{th} elements of the members of the 4×4 array in (19) are, for $p=t$ or $p=\phi$ and $q=t$ or $q=\phi$,

$$Z^{pq}_{nij} = \iint_S \mathbf{J}^p_{-ni} \cdot \left[-\frac{1}{\eta_e} \mathbf{E}_e\left(\mathbf{J}^q_{nj}, 0\right) \right]_{S^-} dS$$

$$+L_j \iint_S \mathbf{J}^p_{-ni} \cdot \left[-\frac{\alpha}{\eta_e} \mathbf{E}_i\left(\mathbf{J}^q_{nj}, 0\right) \right]_{S^+} dS \tag{20}$$

$$C^{pq}_{nij} = L_j \left(\iint_S \mathbf{J}^p_{-ni} \cdot \left[-\mathbf{E}_e\left(0, \mathbf{J}^q_{nj}\right) \right]_{S^-} dS \right.$$

$$\left. + \iint_S \mathbf{J}^p_{-ni} \cdot \left[-\alpha \mathbf{E}_i\left(0, \mathbf{J}^q_{nj}\right) \right]_{S^+} dS \right)$$

$$+L'_j \iint_S \mathbf{J}^p_{-ni} \cdot \left[-\frac{\alpha}{\eta_e} \mathbf{E}_i\left(\mathbf{J}^q_{nj}, 0\right) \right]_{S^+} dS \tag{21}$$

$$D^{pq}_{nij} = \iint_S \mathbf{J}^p_{-ni} \cdot \left[-\mathbf{H}_e\left(\mathbf{J}^q_{nj}, 0\right) \right]_{S^-} dS$$

$$+L_j \iint_S \mathbf{J}^p_{-ni} \cdot \left[-\beta \mathbf{H}_i\left(\mathbf{J}^q_{nj}, 0\right) \right]_{S^+} dS \tag{22}$$

$$Y^{pq}_{nij} = L_j \left(\iint_S \mathbf{J}^p_{-ni} \cdot \left[-\eta_e \mathbf{H}_e\left(0, \mathbf{J}^q_{nj}\right) \right]_{S^-} dS \right.$$

$$\left. + \iint_S \mathbf{J}^p_{-ni} \cdot \left[-\beta\eta_e \mathbf{H}_i\left(0, \mathbf{J}^q_{nj}\right) \right]_{S^+} dS \right)$$

$$+L'_j \iint_S \mathbf{J}^p_{-ni} \cdot \left[-\beta \mathbf{H}_i\left(\mathbf{J}^q_{nj}, 0\right) \right]_{S^+} dS \,. \tag{23}$$

For $p=t$ or $p=\phi$, the i^{th} elements of $\vec{V}^p_n$ and $\vec{I}^p_n$ are, respectively, $\vec{V}^p_{ni}$ and $\vec{I}^p_{ni}$ given by

$$\vec{V}^p_{ni} = \iint_S \mathbf{J}^p_{-ni} \cdot \left[\frac{1}{\eta_e} \mathbf{E}^{inc} \right]_S dS \tag{24}$$

$$\vec{I}^p_{ni} = \iint_S \mathbf{J}^p_{-ni} \cdot \left[\mathbf{H}^{inc} \right]_S dS \,. \tag{25}$$

The preceding discretization gives the 2N+1 small matrix equations {(19) for n=–N, –N+1,…, N} instead of one large matrix equation because, due to the rotational symmetry, an $e^{jn\phi}$ dependent current source produces only an $e^{jn\phi}$ dependent field.

III. COMPUTED RESULTS

Numerical results are given for the bodies shown in Fig. 2.

Figs. 3 to 8 are for a chiral sphere contained in a perfectly conducting thin metallic spherical shell with a single aperture of $\alpha_0 = 30°$ at its bottom that exposes the chiral material to the unit plane wave that illuminates the bottom of the sphere, as shown in Figure 2. The purpose of choosing this partially covered chiral sphere is to compare our results with those of early researchers, particularly with graphic results in [3]. The generating curve is approximated by 1200 straight line segments for Figs. 3 and 5.

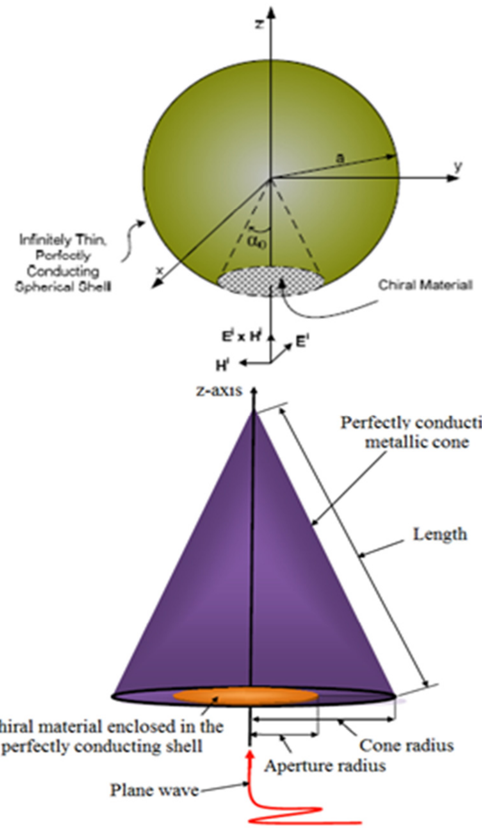

Fig. 2. Sphere and cone with single aperture.

We see marked resemblance between our graph of Fig. 3 and that of the insert from [3], Fig. 4. Both graphs indicate insignificant variations in the overall RCS values as relative chirality varies from ξ_r=0.2 to ξ_r=0.9 with parameters $k_e a$=1.5, ε_r=2, and μ_r=1.

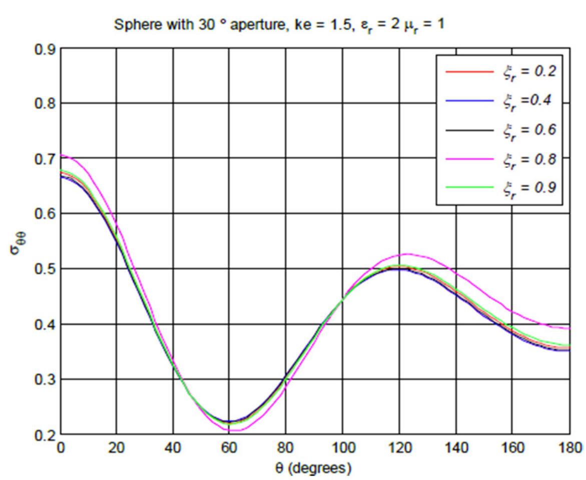

Fig. 3. $\sigma_{\theta\theta}/\lambda_0^2$ of the obstacle with 30° aperture at its bottom.

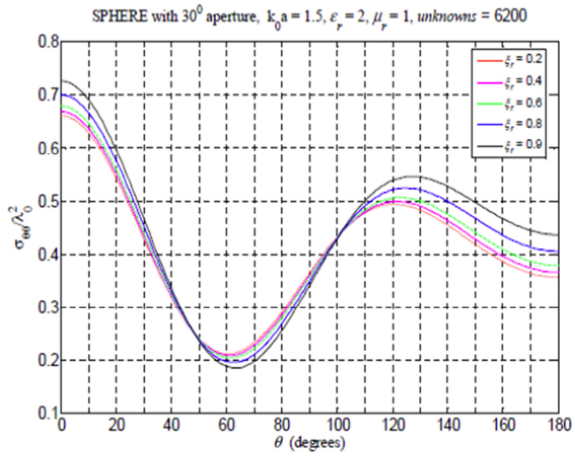

Fig. 4. $\sigma_{\theta\theta}/\lambda_0^2$ of the obstacle with 30° aperture at its bottom. (Insert taken from [3]).

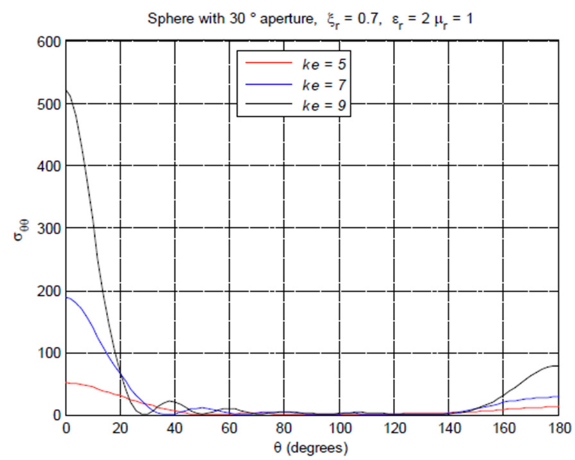

Fig. 5. $\sigma_{\theta\theta}/\lambda_0^2$ of the obstacle with 30° aperture at its bottom.

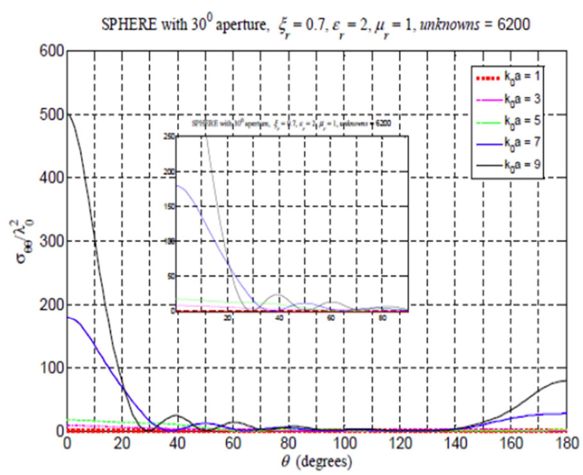

Fig. 6. $\sigma_{\theta\theta}/\lambda_0^2$ of the obstacle with 30° aperture at its bottom. (Insert taken from [3]).

We see marked resemblance between our graph of Fig. 5 and that of the insert from [3], Fig. 6.

Figures 7 and 8 show the internal electric fields along the *z*-axis of the body with varying chiralities. The generating curve is approximated by 3132 straight line segments and 102 points on the *z*-axis were used to obtain the graphs.

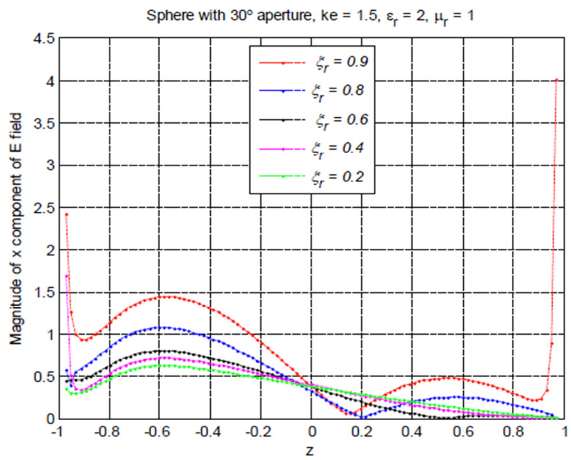

Fig. 7. Magnitude of *x*-component of internal electric field along *z*-axis.

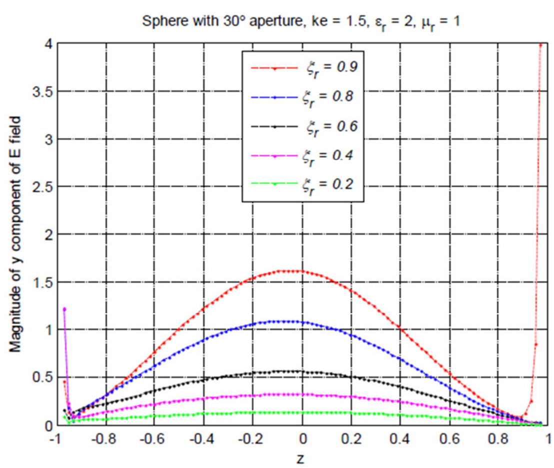

Fig. 8. Magnitude of *y*-component of internal electric field along *z*-axis.

Figs. 9 and 10 are for a conical-shaped chiral BOR contained in a perfectly conducting thin metallic shell with a single aperture at its bottom that exposes the chiral material to the plane wave that illuminates the BOR along the *z*-axis from the bottom of the conical shell, as shown in Figure 2. The radius of the aperture is 0.5m, the radius of the cone is 1m, and the length of the cone is 2m. The generating curve is approximated by 3132 straight line segments.

More results for these structures are available in [21].

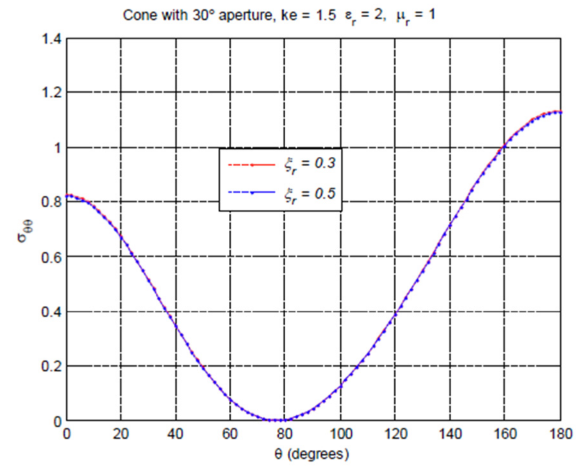

Fig. 9. $\sigma_{\theta\theta}/\lambda_0^2$ of the obstacle with single aperture at its bottom.

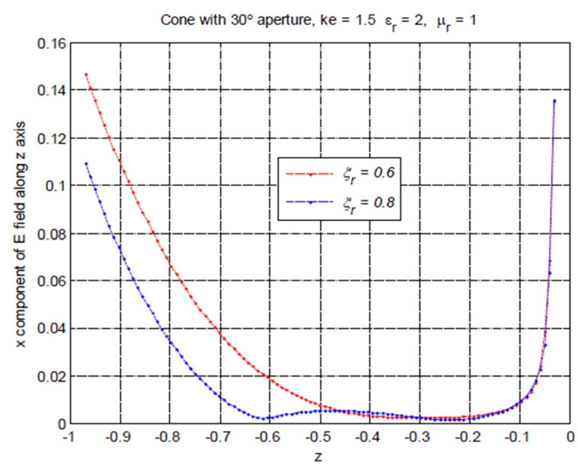

Fig. 10. Magnitude of *x*-component of internal electric field along *z*-axis.

VI. CONCLUSION

In this paper, plane wave incidence on a homogeneous chiral body partially covered by a thin perfectly conducting surface is investigated using the surface equivalence principle and MoM.

The body is replaced by equivalent electric and magnetic surface currents, which produce the correct fields inside and out. The application of the boundary conditions on the tangential components of the electric and the magnetic fields results in a set of two equations to be solved. Triangular expansion functions are used for both *t*-directed and ϕ-directed currents. The unknown coefficients of these expansion functions are obtained using the method of moments.

The inside fields and the scattering cross section are computed. The results are generated by a computer code, which produces agreement with available published results.

The theoretical framework presented in this paper can be used to obtain results that are not available elsewhere.

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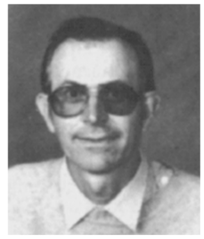
Khaja Qutubuddin received his M.S. degree in Engineering Science from Florida State University in 1968 and Computer Engineer degree from Syracuse University, in 2002. He worked from 1968 until 2006 as Associate Professor and later as Professor in the Technology Department of Onondaga Community College, Syracuse, NY. His primary fields of interest are electromagnetic waves and fields and microwave engineering.

Halid Mustacoglu was born in Kocaeli, Turkey, in 1980. He received the B.S. degree in electronics and communication engineering from Yildiz Technical University, Istanbul, Turkey, in 2002 and the M.S. and Ph.D. degrees in electrical engineering from Syracuse University, Syracuse, NY, in 2005 and 2011, respectively.

From 2002 to 2003, he was a Research Assistant at Syracuse University and between 2003 and 2004 he was an intern at Herley Microwave, working on microwave filter designs. From 2004 to 2011, he was a co-op at Anaren Microwave, Inc., where he worked on passive component designs. Since 2011, he has been an RF/Microwave Engineer at Anaren Microwave, Inc., where he designs RF/Microwave components. His research interests include RF/Microwave design, scattering problems, and computational electromagnetics.



Joseph R. Mautz was born in Syracuse, NY, in 1939. He received the B.S., M.S., and Ph.D. degrees in electrical engineering from Syracuse University in 1961, 1965, and 1969, respectively. Until July 1993, he was a Research Associate with the Electrical Engineering and Computer Science Department, Syracuse University, where he worked on radiation and scattering problems. Currently, he is affiliated with the Electrical Engineering and Computer Science Department at the same university. His primary fields of interest are electromagnetic theory and applied mathematics.

Ercument Arvas received the B.S. and M.S. degrees from the Middle East Technical University, Ankara, Turkey, in 1976 and 1979, respectively, and the Ph.D. degree from Syracuse University, Syracuse, NY, in 1983, all in electrical engineering.

From 1984 to 1987, he was with the Electrical Engineering Department, Rochester Institute of Technology, Rochester, NY. In 1987, he joined the Electrical Engineering and Computer Science Department, Syracuse University, where he is currently a Professor in the Electrical Engineering and Computer Science Department. His research and teaching interests are in electromagnetic scattering and microwave devices.

Prof. Arvas is a Member of the Applied Computational Electromagnetics Society (ACES).