Analysis of Electromagnetic Scattering Problems by Means of a VSIE-ODDM-MLFMA Method

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Abstract **─** The hybrid volume-surface integral equation (VSIE) method has the advantage of solving electromagnetic scattering problems involving complex structure mixed metal with dielectric. In this paper, a method combining VSIE with overlapped domain decomposition method (ODDM) is used to analyze electromagnetic scattering problems successfully. To further improve efficiency, the multilevel fast multipole algorithm (MLFMA) is adopted, then a novel VSIE-ODDM-MLFMA is proposed. Numerical results show that the proposed method has low memory requirement, fast convergence, and accurate simulation result. It indicates that the proposed method has the ability to analyze complicated electromagnetic problems.

Index Terms **─** volume-surface integral equation, electromagnetic scattering, overlapped domain decomposition method, multilevel fast multipole algorithm.

I. INTRODUCTION

The solution of the hybrid volume-surface integral equation (VSIE) is based on the method of moments (MoM) which has been widely used for numerical analysis of electromagnetic radiation and scattering problems [1-3]. For direct solver, the memory requirement is $O(N^2)$ and the CPU time is $O(N^3)$ in MoM. However, both of them in MoM are $O(N^2)$ for iterative solvers, where N denotes the number of unknowns. For the electrically large problems, it is difficult to fulfill the requirement of memory and efficiency on single personal computer presently. The overlapped domain decomposition method (ODDM) has the function of decomposing the computed domain into several subdomains. Each subdomain is extended with a buffer domain. The solution of the whole domain could be completed by solving the extended subdomains circularly. Due to introducing buffer domain, the current edge-effect in each subdomain could be depressed effectively and the convergence of the outer iteration is much fast. For solving only one subdomain at a time, the ODDM has the advantage of saving computing resources. The fast multipole method (FMM) can accelerate the matrix vector product with complexity of $O(N^{1.5})$ and its extension, the multilevel fast multipole algorithm (MLFMA) [4-7], further reduces the complexity to $O(N\log N)$ [8]. In this paper, a new VSIE-ODDM-MLFMA is proposed which combines both ODDM and MLFMA with VSIE. Numerical results show the accuracy and efficiency of the proposed method. It demonstrates that the proposed method has the ability to analyze complicated electromagnetic problems.

II. FORMULATION

The section presents the VSIE-ODDM-MLFMA solver and its computational complexity analysis.

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A. Outline of the VSIE

Using the equivalence principle, the conducting bodies are replaced by equivalent surface currents and the dielectric materials are replaced by equivalent volume currents [9]. Above is the basic idea of the VSIE [10-11] method.

For the electromagnetic scattering problems involving complex structure mixed metal with dielectric, the integral equations can be expressed by mathematical relationship with corresponding magnetic vector and electric scalar potentials. As follows, the volume integral equation (VIE) and the surface integral equation (SIE) are given by

$$\mathbf{E}^i = \frac{\mathbf{D}}{\hat{\varepsilon}(\mathbf{r})} + j\omega\mathbf{A}_V(\mathbf{r}) + \nabla\Phi_V(\mathbf{r}) + j\omega\mathbf{A}_S(\mathbf{r}) + \nabla\Phi_S(\mathbf{r}) \quad \mathbf{r}\in V, \tag{1}$$

and

$$\mathbf{E}^i{}_{\tan} = [j\omega\mathbf{A}_V(\mathbf{r}) + \nabla\Phi_V(\mathbf{r}) + j\omega\mathbf{A}_S(\mathbf{r}) + \nabla\Phi_S(\mathbf{r})]_{\tan} \quad \mathbf{r}\in S \tag{2}$$

where $\hat{\varepsilon}(\mathbf{r})$ is the permittivity of the dielectric material, $\mathbf{A}_V(\mathbf{r})$, $\mathbf{A}_S(\mathbf{r})$, $\Phi_V(\mathbf{r})$, and $\Phi_S(\mathbf{r})$ are vector and scalar potentials produced by the volume and surface current, respectively, given by

$$\mathbf{A}_u(\mathbf{r}) = \mu_0 \int_u \mathbf{J}_u(\mathbf{r}') g(\mathbf{r},\mathbf{r}') du' \quad u = S, V \tag{3}$$

$$\Phi_u(\mathbf{r}) = -\frac{1}{j\omega\varepsilon_0} \int_u \nabla\bullet\mathbf{J}_u(\mathbf{r}') g(\mathbf{r},\mathbf{r}') du' \quad u = S, V. \tag{4}$$

In (3) and (4), $g(\mathbf{r},\mathbf{r}') = \frac{e^{-jk_0|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}$, the Green's function of free space, $\mathbf{J}_S$ is the surface current, $\mathbf{J}_V$ is the volume current which is related to the total electric flux density $\mathbf{D}$ in equation (1).

To solve the equations (1) and (2), the conducting surface is discretized into small triangular patches, while the dielectric region is divided into tetrahedral elements [9]. Employing both the Schaubert-Wilton-Glisson (SWG) [12] and the Rao-Wilton-Glisson (RWG) [1] basis functions in equations (1) and (2), then testing (1) with SWG basis function and testing (2) with RWG basis function, we can get a matrix equation which could be written as a submatrix form in the following:

$$\begin{bmatrix} Z^{DD} & Z^{DM} \\ Z^{MD} & Z^{MM} \end{bmatrix} \begin{bmatrix} I_{Dn} \\ I_{Mn} \end{bmatrix} = \begin{bmatrix} E^D \\ E^M \end{bmatrix}, \tag{5}$$

where the first matrix is impedance matrix, I_{Dn} and I_{Mn} are the unknown expansion coefficients, E^D and E^M denote the excitation vectors. The more details of the VSIE can be found in [9].

B. Basic principle of ODDM

When decomposing the whole computed domain to several subdomains, the corresponding impedance matrix $[Z]$ will be decomposed into several submatrices. The solution of the whole domain could be got by solving submatrix equations circularly. Above-mentioned process is the idea basis of domain decomposition method (DDM) [13]. For solving only one submatrix equation at a time, the memory requirement can be reduced. However, as the whole matrix equation need to be solved by iterative solvers, the computing time would become longer usually. Using parallel computation could improve efficiency. Similarly, employing preconditioned techniques can reduce iteration number and CPU time.

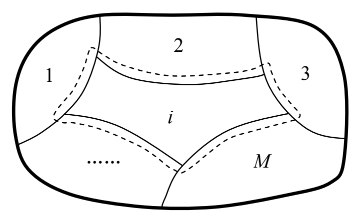


Fig. 1. The illustration of the DDM model.

As is shown in figure 1, if the domain is decomposed along the thin solid lines, there would be no common elements among the submatrices. In this way, the electric current of subdomain boundary would have singularity which can lead to the problem of low efficiency and slow convergence, even no convergence. In order to restrain electric current singularity, Brennan proposed a forward and backward buffer region (FBBR) iterative scheme which regards the forward or backward domain of subdomain boundary as buffer domain of the subdomain [14]. However, only the current edge-effect in one side of each subdomain is depressed. On the basis of the idea presented in [14], the ODDM was proposed in [15]. The dotted line in Figure 1 is the boundary of the extended subdomain whose solution is restricted to the original subdomain by discarding the currents in buffer domain. The whole domain current could be got by solving circularly. Above paragraph has illustrated the basic principle of ODDM.

The ODDM involves twofold iterations including inner iteration and outer iteration. The iteration solving subdomain is inner iteration while the process of solving all the subdomains once is called as an outer iteration, in which the current in the whole domain is updated once by the inner iteration. The following figure has explained the relation between inner iteration and outer iteration.

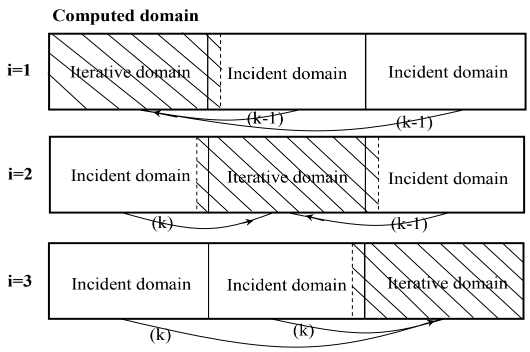


Fig. 2. The relation between inner iteration and outer iteration.

In figure 2, the iterative domain denotes the extended subdomain that need to be computed and its complementary domain is the corresponding incident domain, the thin solid line is subdomain boundary, the dotted line is the boundary of the extended subdomain with buffer domain, i represents the sequence number of the solved subdomain and k denotes kth outer iteration.

C. FMM and MLFMA

Based on FMM, the MLFMA has gained great success in solving electromagnetic problems with electrically large size [16-17]. MLFMA is the promotion of FMM in hierarchical structure. The basic principle of FMM is to divide scattering units which derived by discretizing scattering object into groups. The mutual coupling of any two scattering units is calculated by different methods according to the relative position of their groups. When they are adjacent, we use direct calculation method, otherwise, separate into three steps containing aggregation, translation and disaggregation. As shown in Fig.3, for a given group of field point, firstly, the contributions of all scattering units in its non-adjacent group would be aggregated to the center of each group, secondly, the contributions of these groups would be translated from each center to the center of the given group, finally, all the contributions of the non-adjacent groups would be disaggregated from the center of the given group to each scattering unit in the group. For a group of source point, the group center represents the contributions of all scattering units in this group to its non-adjacent groups. For a group of field point, the group center represents the contributions of all non-adjacent groups to this group. In this way, the number of scattering center is considerably reduced [18-19].

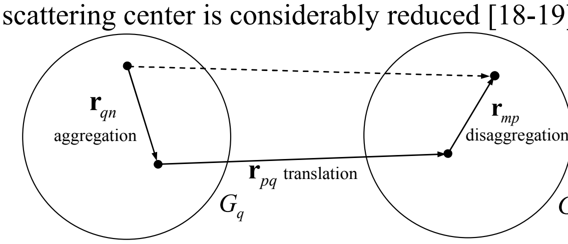


Fig. 3. The direct interaction between two far-field elements is separated into three steps containing aggregation, translation and disaggregation.

The expression for matrix vector product in FMM is written as

$$\sum_{n=1}^{N} Z_{mn} I_n = \sum_{q \in B_p} \sum_{n \in G_q} Z_{mn} I_n$$
$$+ \oiint \mathbf{R}_{mp}(\hat{k}) \cdot \sum_{q \notin B_p} \Gamma_{pq}(\mathbf{k}, \hat{r}_{pq}) \sum_{n \in G_q} \mathbf{F}_{qn}(\hat{k}) I_n d^2\hat{k}. \quad (6)$$

The first term in (6) denotes the contribution from nearby groups (including the self-group) which is represented by the symbol B_p, the second term is the far-field interaction calculated by FMM, the $\mathbf{F}_{qn}(\hat{k})$, $\Gamma_{pq}(\mathbf{k}, \hat{r}_{pq})$, and $\mathbf{R}_{mp}(\hat{k})$ donate the aggregation, translation, and disaggregation factor, respectively.

D. The VSIE method combined with ODDM

As is shown in (3) and (4), using the volume current $\mathbf{J}_V$ and surface current $\mathbf{J}_S$ to respectively denote magnetic vector and electric scalar potentials in equation (1) and equation (2), we get the incident field expression. To derive the formulation of VSIE-ODDM conveniently, we define one F linear operator as

$$F_u(\mathbf{J}_u(\mathbf{r}')) = j\omega\mu_0 \int_u \mathbf{J}_u(\mathbf{r}') g(\mathbf{r},\mathbf{r}') du'$$
$$-\frac{\nabla}{j\omega\varepsilon_0}\int_u \nabla\bullet\mathbf{J}_u(\mathbf{r}')\, g(\mathbf{r},\mathbf{r}') du' \quad u = S, V. \quad (7)$$

Then the equation (1) and equation (2) could be expressed as

$$\mathbf{E}^i = \frac{\mathbf{D}(\mathbf{J}_V(\mathbf{r}))}{\hat{\varepsilon}(\mathbf{r})} + F_V(\mathbf{J}_V(\mathbf{r}'))$$
$$+F_S(\mathbf{J}_S(\mathbf{r}')) \quad \mathbf{r}\in V, \quad (8)$$

$$\mathbf{E}^i{}_{\tan} = [F_V(\mathbf{J}_V(\mathbf{r}')) + F_S(\mathbf{J}_S(\mathbf{r}'))]_{\tan} \quad \mathbf{r}\in S. \quad (9)$$

To build the integral iteration formula between subdomains, we define two linear operators $T^D(\mathbf{r},\mathbf{J})$ and $K^D(\mathbf{r},\mathbf{J})$ for the VIE (8). The two linear operators can be written as

$$T^D(\mathbf{r},\mathbf{J}) = \frac{\mathbf{D}(\mathbf{J}_V(\mathbf{r}))}{\hat{\varepsilon}(\mathbf{r})} + F_V(\mathbf{J}_V(\mathbf{r}'))$$
$$+F_S(\mathbf{J}_S(\mathbf{r}')) \quad \mathbf{r}'\in\Omega_i', \mathbf{r}\in V_i' \quad (10)$$

$$K^D(\mathbf{r},\mathbf{J}) = F_V(\mathbf{J}_V(\mathbf{r}'))$$
$$+F_S(\mathbf{J}_S(\mathbf{r}')) \quad \mathbf{r}'\in\bar{\Omega}_i', \mathbf{r}\in V_i' \quad (11)$$

where i is the sequence number of subdomain, the ith extended subdomain Ω_i' includes V_i' and S_i', $\bar{\Omega}_i'$ is the complementary domain of Ω_i'. Here, $\Omega_i' = \Omega_i + \Omega_{b(i)}$ where $\Omega_{b(i)}$ denotes the buffer domain of Ω_i. Similarly, $V_i' = V_i + V_{b(i)}$ and $S_i' = S_i + S_{b(i)}$. Combining equation (8) with equation (10) and equation (11), we get the VIE-ODDM iteration scheme which is expressed as

$$T^D(\mathbf{r},\mathbf{J}^{(k)}) = -K^D(\mathbf{r},\mathbf{J}^{(k-1)})$$
$$+\mathbf{E}^i(\mathbf{r}) \quad \mathbf{r}\in V_i'. \quad (12)$$

Comparing equation (12) with equation (8), the computed domain is reduced from V to V_i', and the excitation source in the computed domain includes both the information of incident plane wave and the coupling from other subdomains.

A similar procedure can be applied for the SIE (9). We define two linear operators $T^M(\mathbf{r},\mathbf{J})$ and $K^M(\mathbf{r},\mathbf{J})$, then we can get the SIE-ODDM iteration scheme, expressed as

$$T^M(\mathbf{r},\mathbf{J}^{(k)}) = -K^M(\mathbf{r},\mathbf{J}^{(k-1)})$$
$$+\mathbf{E}^i(\mathbf{r})_{\tan} \quad \mathbf{r}\in S_i'. \quad (13)$$

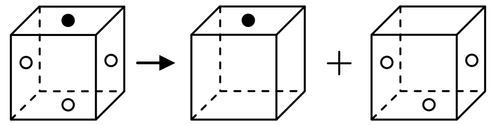
Expanding the equation (12) by RWG and SWG basis function and testing it with SWG basis function, the matrix form of the VIE-ODDM can be obtained. Similarly, for equation (13), selecting RWG basis function as testing function, we can get the matrix form of the SIE-ODDM. Combining the two matrix equations, a VSIE-ODDM iteration scheme is presented as

$$\begin{bmatrix} \tilde{Z}_{ii}^{DD} & \tilde{Z}_{ii}^{DM} \\ \tilde{Z}_{ii}^{MD} & \tilde{Z}_{ii}^{MM} \end{bmatrix}\begin{bmatrix} \tilde{I}_{Di}^{(k)} \\ \tilde{I}_{Mi}^{(k)} \end{bmatrix} = \begin{bmatrix} \tilde{E}_{V_i'}^{D} \\ \tilde{E}_{S_i'}^{M} \end{bmatrix}$$
$$-\sum_{j<i, c(j)\notin b(i)} \begin{bmatrix} \tilde{Z}_{ij}^{DD} & \tilde{Z}_{ij}^{DM} \\ \tilde{Z}_{ij}^{MD} & \tilde{Z}_{ij}^{MM} \end{bmatrix}\begin{bmatrix} I_{Dj}^{(k)} \\ I_{Mj}^{(k)} \end{bmatrix}$$
$$-\sum_{j>i, c(j)\notin b(i)} \begin{bmatrix} \tilde{Z}_{ij}^{DD} & \tilde{Z}_{ij}^{DM} \\ \tilde{Z}_{ij}^{MD} & \tilde{Z}_{ij}^{MM} \end{bmatrix}\begin{bmatrix} I_{Dj}^{(k-1)} \\ I_{Mj}^{(k-1)} \end{bmatrix}$$
$$i = 1, 2, \cdots, M. \quad (14)$$

Here, M denotes the number of subdomain, $\begin{bmatrix} \tilde{E}_{V_i'}^{D} & \tilde{E}_{S_i'}^{M} \end{bmatrix}^T$ represents the vector of incident field in the ith extended subdomain Ω_i'. $\tilde{Z}_{ii}^{DD}$, $\tilde{Z}_{ii}^{DM}$, $\tilde{Z}_{ii}^{MD}$, and $\tilde{Z}_{ii}^{MM}$ are the self-impedance matrices in Ω_i'. $\tilde{Z}_{ij}^{DD}$, $\tilde{Z}_{ij}^{DM}$, $\tilde{Z}_{ij}^{MD}$, and $\tilde{Z}_{ij}^{MM}$ are the mutual-impedance matrices between Ω_j and Ω_i'. By solving the equation (14) and discarding the current in the buffer domain $\Omega_{b(i)}$, the current in subdomain Ω_i could be updated. The process of solving the equation (14) is an inner iteration. By several outer iterations, we can get the current in the entire domain.

E. The VSIE-ODDM-MLFMA solver

The MLFMA may be employed to accelerate the matrix vector product. The entire object is first enclosed into a large cube, which is partitioned into eight smaller cubes. Each subcube is then recursively subdivided into smaller cubes until the edge length of the finest cube is about 0.1 wavelength. In ODDM, the whole computed domain needs to be decomposed into iterative domain and incident domain. However, when combining the ODDM and MLFMA, it is necessary to consider the problems of decomposition and grouping simultaneity.



(a) (b) (c)

Fig. 4. The cube belongs to iterative domain and incident domain simultaneity. (● , ○ represent basis function units in the ith iterative and incident domain, respectively.)

In VSIE-ODDM-MLFMA, for a certain cube of a certain MLFMA layer, if no less than one basis function unit is located in ith iterative domain or incident domain, the cube belongs to ith iterative domain or incident domain. As is shown in Fig. 4, the cube (a) belongs to ith iterative and incident domain simultaneity because it has both ● and ○ basis function units. As a cube in ith iterative domain, the cube (a) is equal to cube (b), while as a cube in ith incident domain, the cube (a) is equal to cube (c). Due to outside of the iterative domain, the ○ basis function units will contribute outgoing radiation (called aggregation) but do not receive updates via translation and disaggregation.

F. Computational complexity analysis of the VSIE-ODDM-MLFMA

The computational complexity for the VSIE-ODDM-MLFMA is composed of the inner and outer iterations. We define the average number of unknowns in every iterative domain as $N_i=N/M+N_b$ where N is the total number of unknowns, N_b denotes the average number of unknowns in buffer domain. Suppose that the average number of inner iterations is ξ, the average CPU time of solving each iterative domain is $O(\xi N_i \log N_i)$. Assuming that the number of the outer iterations is ς, then the total CPU time of the inner iteration is about

$$\begin{aligned} &O(\varsigma \cdot M \cdot \xi \cdot N_i \log N_i) \\ &= O(\varsigma \cdot M \cdot \xi \cdot (\frac{N}{M} + N_b) \log(\frac{N}{M} + N_b)) \\ &= O(\varsigma \cdot \xi \cdot (N + MN_b) \log(\frac{N}{M} + N_b)). \end{aligned} \tag{15}$$

When the size of the whole coefficient matrix is fixed, the CPU time of inner iteration is mostly determined by $\varsigma \cdot \xi$. According to past experience, the accuracy of both electric current and radar cross section (RCS) could generally meet the requirements when ς is 3. Obviously, the memory requirement for the inner iteration is $O(N_i \log N_i)$. When using an iterative solver to solve the problems, only near-field matrix elements need to be stored. So the VSIE-ODDM-MLFMA is better than VSIE and VSIE-ODDM in the aspects of memory requirement or computational efficiency. In contrast to VSIE-MLFMA, the memory requirement is reduced significantly, which is very important to analyze electrically large problems.

III. NUMERICAL RESULTS

To demonstrate the accuracy and efficiency of the proposed method, several numerical examples are presented in this section.

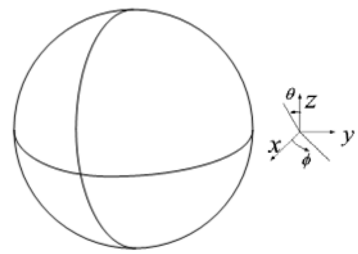


Fig. 5. Perfectly conducting sphere coated with dielectric layer.

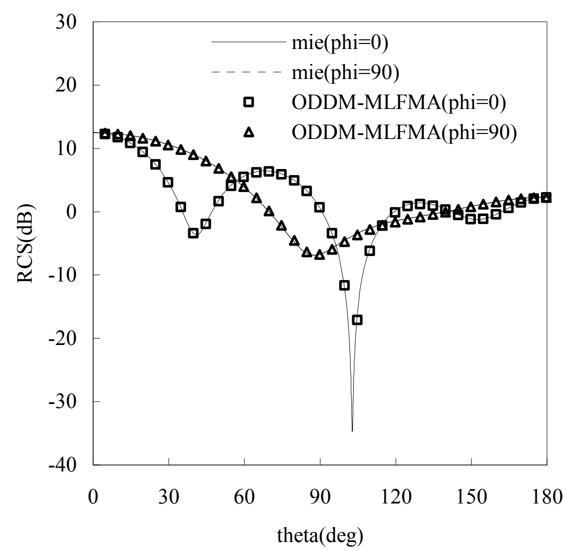


Fig. 6. Bistatic radar cross sections of a dielectric-coated sphere.

The first example is a perfectly conducting sphere with a diameter of $0.3423\lambda_0$, which is coated with a $0.1017\lambda_0$ thick dielectric layer whose relative dielectric constant is $\varepsilon_r=2$. The mixed structure is illuminated by a plane wave. In order to use the VSIE-ODDM-MLFMA, the target is decomposed into four domains. The dielectric domain is discretized into tetrahedrons and the metal domain is discretized into triangles, as a result, 14332 SWG basis functions and 963 RWG basis functions are generated.

The bistatic RCS computed by the VSIE-ODDM-MLFMA is shown in Figure 6. The comparison with the exact Mie series solution is given and excellent agreement is found. The example demonstrates the accuracy of VSIE-ODDM-MLFMA for analyzing the structure mixed metal with dielectric.

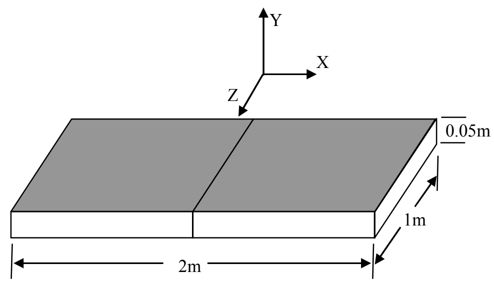
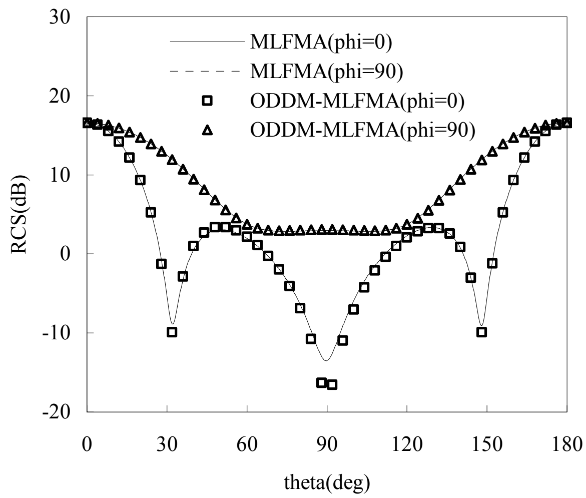


Fig. 7. A dielectric cuboid coated with a metal layer on the surface.

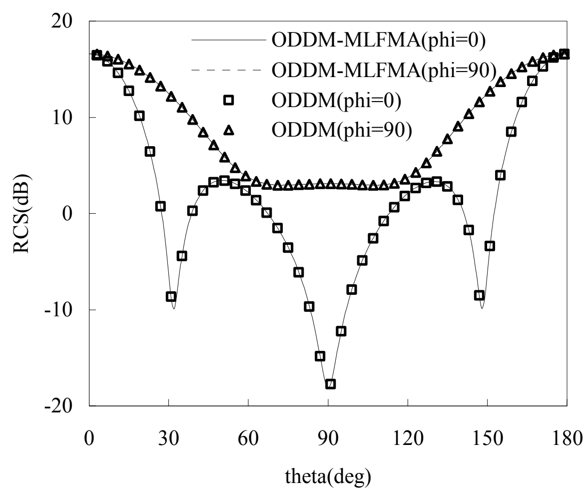
As shown in Figure 7, the second example is a dielectric cuboid coated with a metal layer on the surface whose specific size is marked. The relative permittivity of the dielectric cuboid is 1.6. The incident wave frequency is 300MHz. To apply the ODDM, first we divide the computed domain into two domains averagely in X direction. Then we discretize the dielectric and metal domains as the first example, as a result, 4959 basis functions are generated, which includes 4323 SWG basis functions and 636 RWG basis functions.

We use VSIE-MLFMA program, VSIE-ODDM-MLFMA program, VSIE-ODDM program and VSIE program to compute bistatic RCS of the target respectively. The comparative results are presented in Figure 8.

As shown in Table 1, the comparison of the total memory requirement and CPU time between VSIE-ODDM-MLFMA and VSIE-MLFMA is provided. In VSIE-ODDM-MLFMA, the total memory requirement is 42288KB, which is reduced by 35.3% significantly in contrast to 65364KB in VSIE-MLFMA. If we divide the computed domain into more subdomains, the memory requirement will be further reduced. This experiment results also tell us that the total memory requirement of VSIE-ODDM-MLFMA only accounts for 10.6% of VSIE, 20.4% of VSIE-ODDM, and the CPU time is reduced significantly in contrast to VSIE and VSIE-ODDM.



(a)



(b)

Fig. 8. Bistatic radar cross sections of a dielectric cuboid coated with a metal layer on the surface.

Table 1: The comparison of the total memory requirement and CPU time between VSIE-MLFMA and VSIE-ODDM-MLFMA

Method	Memory requirement(KB)	CPU time(s)
VSIE-MLFMA	65364	88
VSIE-ODDM-MLFMA	42288	132

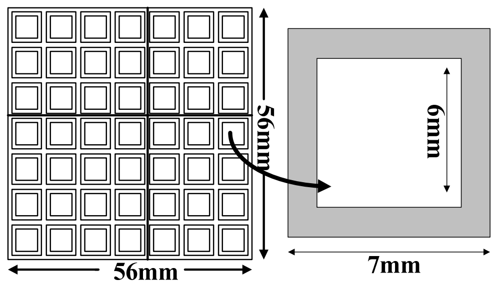
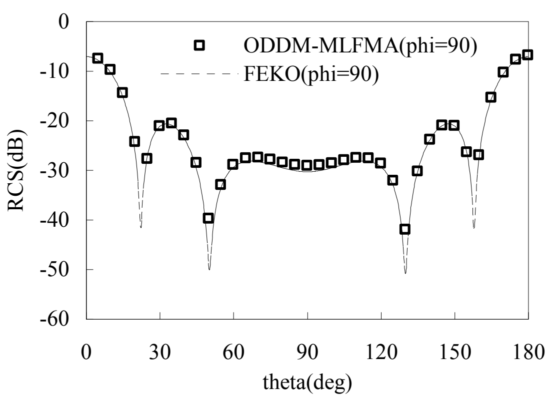
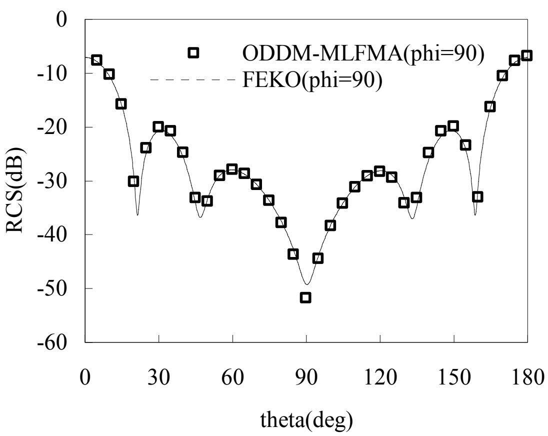


Fig. 9. The illustration of a 7×7 planar FSS array and the square ring unit.



(a)



(b)

Fig. 10. Bistatic radar cross sections computed by the VSIE-ODDM-MLFMA and FEKO.

In order to further demonstrate the VSIE-ODDM-MLFMA has the ability to solve the problems of complicated structure, we take the third example of a 7×7 FSS array whose dimension is shown in Figure 9. The FSS unit is square ring whose outside edge length D1 is 7mm, inside edge length D2 is 6mm, and cycle is 8mm.The dielectric substrate has 56mm in length, 56mm in width, and 0.5mm in thickness with the relative permittivity ε_r=3.0. To use the ODDM, we first divide the computed domain into four inhomogeneous domains, then discretize them into tetrahedrons in dielectric domain and triangles in metal domain, respectively. As a result, 30213 SWG basis functions and 1372 RWG basis functions are generated. The FSS array is illuminated by a TM polarization wave from the vertical direction at 14 GHz. The RCS results computed by the proposed method are plotted in Figure 10. The comparison with the FEKO is given and the good agreement between the two methods is obtained.

The experimental results in Figure 10 have demonstrated the proposed method has the ability to solve electromagnetic scattering problems of complicated structure accurately.

IV. CONCLUSION

In this paper, the ODDM and MLFMA are introduced to the VSIE simultaneity, which could solve the problem of insufficient hardware sources and improve the efficiency. Numerical results of the presented examples demonstrate the accuracy and efficiency of this proposed method. It shows that the VSIE-ODDM-MLFMA can solve complicated electromagnetic problems successfully.

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