Computer Tool for ATU Loss Estimation

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***Abstract*— Antenna tuning units (ATUs) using Tee networks are in widespread use by practical radio communicators throughout the HF (3-30 MHz) and VHF (30-300 MHz) spectrum. A computer software tool for estimating the power losses in such networks in general impedance matching applications has been developed. The program is described here, and illustrative case study results are reported.**

I. Introduction

The Tee-network ATU, shown in Fig. 1 with antenna feedpoint load $Z_{in} = R_A + jX_A$, is a classic approach to antenna impedance matching in practical HF and VHF radio applications. For discussion here, the coaxial line from the transmitter is assumed to be of characteristic impedance $Z_0 = Z_{line} = 50\Omega$. Further, it is assumed that all network voltages and currents are specified with RMS values.

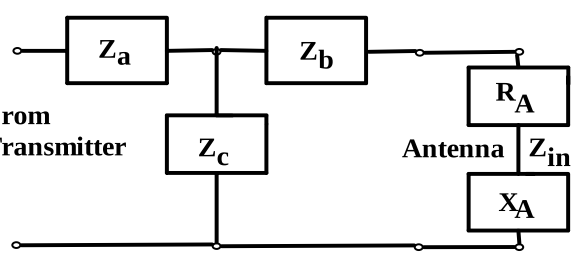

Figure 1. Tee-network Antenna Tuning Unit.

If the load (that is, antenna feed point) is purely resistive with value R_A, a $\frac{\lambda}{4}$ transmission line section of charactistic impedance $Z_{xfmr} = \sqrt{Z_0 \cdot R_A}$ will produce the desired impedance match at the design frequency. The Tee network components, idealized for the lossless case, are then pure reactances

$$|Z_a| = |Z_b| = |Z_c| = \sqrt{Z_0 \cdot R_A} \tag{1}$$

[1] with Z_c of opposite sign from Z_a and Z_b; this results in a lumped element circuit equivalent to a $\frac{\lambda}{4}$ section of transmission line of appropriate characteristic impedance. Interested readers can find more complete background in [2], where it is shown that the impedance matrix for a section of lossy transmission line with propagation constant $\gamma = \alpha + j\beta$ and length d is

$$Z = \begin{bmatrix} Z_a + Z_c & Z_c \\ Z_c & Z_b + Z_c \end{bmatrix} = \begin{bmatrix} Z_0 \coth \gamma d & \frac{Z_0}{\sinh \gamma d} \\ \frac{Z_0}{\sinh \gamma d} & Z_0 \coth \gamma d \end{bmatrix} \tag{2}$$

Simplifying to the lossless case,

$$Z_a = Z_b = jZ_0 \tan \beta \frac{d}{2}; \quad Z_c = -jZ_0 \csc \beta d. \tag{3}$$

With $\beta = \frac{2\pi}{\lambda}$ and $d = \frac{\lambda}{4}$, the results in [1] follow directly.

Fig. 2 shows more Tee-network ATU circuit detail, for the case of inductive input and output legs and a capacitive shunt leg reactance. It is also possible to have the input/output legs capacitive with the shunt leg inductive as an alternative, but the Fig. 2 configuration is preferred for use with radio transmitters because its low-pass filter behavior attenuates the harmonic output produced (to varying degrees) by all high-power rf amplifiers.

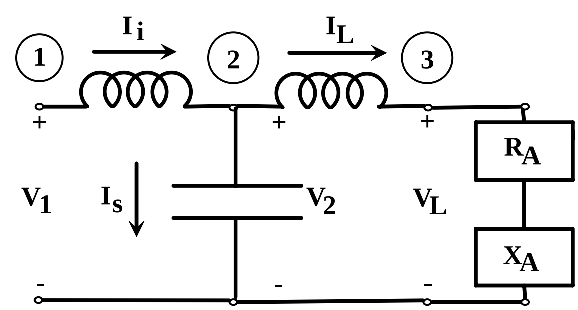

Figure 2. More detailed ATU circuit.

For the moment, the premise is continued that $X_A = 0$, so antenna $Z_{in} = R_A$ is purely resistive. Analysis of the Fig. 2 network with conventional circuit theory to obtain voltage and current expressions is then straightforward. However, the results are of limited utility because, at this point, the reactances are lossless and the load purely real.

II. Extension to Complex Load

Generally, $X_A \neq 0$ and antenna feed $Z_{in} = R_A + jX_A$. In this case, the standard practice is to use the driving-point reactance jX_A to make up part of $Z_b = jX_2$ from Fig. 1, with total value X_2 calculated according to Eqn. 1. Hence, the reactance actually placed in output leg "b" of the ATU is

$$X_2' = X_2 - X_A. \tag{4}$$

III. Extension to Lossy Reactances

With the above procedure of routinely incorporating X_A into the ATU's output leg reactance $Z_b = jX_2$, the impedance matching task is reduced to matching a real load to a real transmission line characteristic impedance Z_0, which has been specified to be 50Ω throughout this discussion. Denoting inductance Q-factor by Q_L and capacitor Q-factor by Q_C according to the most fundamental specification of Q

$$Q = \frac{\text{reactance in Ohms}}{\text{resistance in Ohms}} \tag{5}$$

allows the calculation of lossy reactive element resistances through

$$R_L = R_{ind} = \frac{jX_L}{Q_L} \text{ and } R_C = R_{cap} = \left|\frac{-jX_C}{Q_C}\right|. \tag{6}$$

In Fig. 3 below, a dashed line appears through the block previously occupied by X_A to represent replacement by a short-circuit connection, jX_2 in the ATU output leg has been changed to jX_2' to indicate incorporation of X_A into X_2, and the three lossy reactance resistances are denoted by R_1, R_2, and R_3.

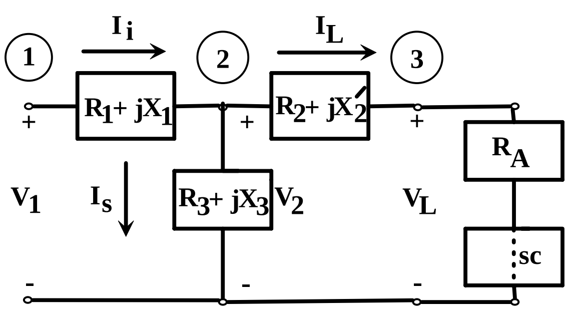

Figure 3. Lossy ATU.

The introduction of component losses requires a more robust solution strategy, as the application conditions for Eqn. 1 are now violated, and its guidance is now potentially highly unreliable and inaccurate. An analytical attempt at solution of the new, real-world problem quickly becomes egregiously heinous, and a computer-based numerical solution is highly preferable.

IV. Program Equations and Strategy

Even in the lossy element case, it remains practical to readily obtain 1:1 SWR at the connection of the transmitter output coax to the ATU input for the vast majority of, if not for all, complex antenna Z_{in} impedances. However, as practical radio communicators know, obtaining a matched-impedance condition now generally is an experimental adjustment procedure under human operator or microprocessor control, monitoring input SWR value as ATU reactance values are varied.

The computer program objectives are (1) given Q_L and Q_C, determine reactance values $X_1 - X_3$ that will give a matched impedance condition between antenna and coaxial rf feed, and (2) determine the percent power dissipated in each of the three ATU legs, as well as in the antenna feed resistance R_A. Note that R_A is actually a series combination of ohmic loss and radiation resistance, and separation of the two is beyond the scope of this study. The reader should note that, for this particular study, all ATU inductors are assumed to have the same Q_L, and all capacitors are assumed to have the same Q_C.

Since an accounting for the percent distribution of rf input power is sought, the numerical value of input power P_{in} is immaterial, and is set to 100 Watts in the code. Relations for the network voltages and currents are developed below.

Refer to Fig. 3, recalling Eqn. 4 said $X_2 = X_2' + X_A$ and keeping in mind that X_A physically is in the antenna feedpoint load. Assume (i) a matched condition to $Z_0 = 50$ is achieved, (ii) input power P_{in} is specified, (iii) RMS values of voltage and current are used, and (iv) the ATU shunt leg is capacitive while the input and output legs are inductive. By Ohm's Law,

$$P_{in} = I_i^2 Z_0 = \frac{V_1^2}{Z_0} \Rightarrow I_i = \sqrt{\frac{P_{in}}{Z_0}} \text{ and } V_1 = \sqrt{P_{in} \cdot Z_0}. \tag{7}$$

Then, by current division,

$$I_S = I_i \frac{(R_2 + R_A + jX_2)}{(R_2 + R_A + jX_2) + (R_3 - jX_3)} \tag{8}$$

and

$$V_2 = (I_S)(R_3 - jX_3). \tag{9}$$

Applying current division again, this time to the output leg feeding the antenna:

$$I_L = I_i \frac{(R_3 - jX_3)}{(R_2 + R_A + jX_2) + (R_3 - jX_3)} \tag{10}$$

and

$$V_L = I_L (R_A + jX_A). \tag{11}$$

Computed Z_{incalc} at the ATU input is from

$$Z_{incalc} = (R_1 + jX_1) + \frac{(R_2 + R_A + jX_2)(R_3 - jX_3)}{(R_2 + R_A + jX_2) + (R_3 - jX_3)} \tag{12}$$

Because the component resistances are now coupled into the equations and those resistances, in turn, depend on the corresponding component reactances, the cause for analytical difficulty and need for numerical aid is apparent.

As noted earlier, Eqn. 1 is inaccurate and unreliable in the lossy case with significant antenna mismatch, but it does provide a useful initial estimate for the three ATU reactances. MATLAB [3] includes an optimization function *fminsearch.m* in its Optimization Toolbox library which can be employed to find the minimum of an unconstrained multivariable function $\min_X f(x)$, where x is a vector and $f(x)$ is a function that returns a scalar. The "multivariable" values to be optimized are those for X_1, X_2, and X_3, and the returned scalar is the absolute value of the difference between the desired 50Ω input and the calculated ATU input impedance at each iteration of the reactance values. For each iteration,

$$R_1 = \left|\frac{X_1}{Q_L}\right| \text{ or } R_1 = \left|\frac{X_1}{Q_C}\right|, \tag{13}$$

depending on whether X_1 is inductive/positive or capacitive/negative for that particular iteration. Similar arithmetic is also applied for R_3 and R_2, noting that X_2' and not X_2 is the numerator for calculating R_2 because the primed value is that actually placed in the ATU output leg. Note also that although we are starting with a network with positive input and output leg reactances and a negative shunt leg reactance, the matching optimization routine may occasionally change the sign of one or more of the components.

Default values of Q_L and Q_C are set in the computer tool to 100 and 1000, respectively, but the user is prompted and offered the opportunity to change either value when the program is executed. The code essentially implements the following sequence:

input Z_{in}
$\Downarrow$
accept default Q_L/Q_C, or change
$\Downarrow$
get initial X estimates from Eqn. 1
$\Downarrow$
get R values from Xs and Q
$\Downarrow$
compute Z_{incalc}
$\Downarrow$
optimize X_1, X_2, X_3 for match to Z_0
using *fminsearch* routine
$\Downarrow$
compute final, optimized Z_{incalc}
$\Downarrow$
compute voltages and currents
$\Downarrow$
compute power dissipated in ATU
components and delivered to antenna

Optimized ATU input impedances are not generally *exactly* $50 + j0\Omega$, but are so close that reflected power from the ATU input port is insignificant.

V. Illustrative Results

Case 1: Antenna $Z_{in} = 72 + j43\Omega$, typical of a $\frac{\lambda}{2}$ dipole. Using default Qs, program execution yields the following results:

Case 1: $Q_L = 100, Q_C = 1000$	
Initial $X_1 - X_3$	+60, +60, -60 Ω
Optimized $X_1 - X_3$	+60.507, +61.023, -59.708 Ω
Initial Z_{incalc}	50.501 + j0.099682 Ω
Optimized Z_{incalc}	50.000 + j3.6275e-6 Ω
Final $R_1 - R_3$	0.60507, 0.18023, 0.059708 Ω
ATU input leg power	1.2101 %
ATU shunt leg power	0.20436 %
ATU output leg power	0.24617%
Power delivered to antenna	98.33937 %
Total power	100.000%
Total ATU loss	0.073 dB

For the remaining case studies, power percents are rounded to the nearest tenth.

Case 2: Antenna $Z_{in} = 20 - j300\Omega$, a moderately mismatched small antenna.

Case 2: $Q_L = 100, Q_C = 1000$	
Initial $X_1 - X_3$	+31.62, +31.62, -31.62 Ω
Optimized $X_1 - X_3$	+29.986, +32.254, -34.177 Ω
Initial Z_{incalc}	43.2 + j0.086 Ω
Optimized Z_{incalc}	50.0 - j3.93e-7 Ω
Final $R_1 - R_3$	0.299, 3.323, 0.034 Ω
ATU input leg power	0.6 %
ATU shunt leg power	0.2 %
ATU output leg power	14.1 %
Power delivered to antenna	85.1 %
Total power	100.000%
Total ATU loss	0.70 dB

Case 3: Antenna $Z_{in} = 11.7 + j0\Omega$, a self-resonant normal mode helical antenna (NMHA) of length 0.05λ, as described on page 68 of reference [4].

Case 3: $Q_L = 100, Q_C = 1000$	
Initial $X_1 - X_3$	+24.19, +24.19, -24.19 Ω
Optimized $X_1 - X_3$	+24.07, +24.34, -24.4 Ω
Initial Z_{incalc}	49.15 + j 0.098 Ω
Optimized Z_{incalc}	50.00 + j4.75e-6 Ω
Final $R_1 - R_3$	0.241, 0.243, 0.024 Ω
ATU input leg power	0.5 %
ATU shunt leg power	0.3 %
ATU output leg power	2.0 %
Power delivered to antenna	97.2 %
Total power	100.000%
Total ATU loss	0.12 dB

Case 4: Antenna $Z_{in} = 0.49 - j900\Omega$ for a short dipole reported by [5] and further considered in [4].

Case 4: $Q_L = 100, Q_C = 1000$	
Initial $X_1 - X_3$	+49.5, +49.5, -49.5 Ω
Optimized $X_1 - X_3$	+0.12, +19.8, -24.5 Ω
Initial Z_{incalc}	2.62 + j5.13e-3 Ω
Optimized Z_{incalc}	50.0 +j1.18e-5 Ω
Final $R_1 - R_3$	0.0012, 9.198, 0.0245 Ω
ATU input leg power	0.0 %
ATU shunt leg power	0.3 %
ATU output leg power	94.7 %
Power delivered to antenna	5.0 %
Total power	100.000%
Total ATU loss	12.97 dB

The computer tool result of 5.0% power delivered to the antenna agrees with the tabulated value on p. 68 of [4]. NOTE: Case 4 was also run with $Q_L = 400$ for comparison with [4]. The full table of results is omitted in the interest of brevity, but the total power (efficiency) came out 18%, again in agreement with Fujimoto, and the total ATU loss was 7.54 dB.

Case 5: Antenna $Z_{in} = 0.001 + j11000\Omega$. This is an extreme case of a 1 mH inductor being driven at 1.8 MHz through an ATU. The resistance value is synthetic, for illustration purposes only, and the radiation resistance is likely even smaller. Turns of the coil are necessarily tightly wound, resulting in high proximity effect losses, and the coupling of the coil to ground will also cause a loss resistance to appear in series at the feed terminals. An actual inductor approximating this case has been constructed, requiring approximately 285 turns of insulated #14 electrical wire in a single layer, with adjacent turns touching, on a nominal 4-inch diameter PVC pipe core. The measured inductance was 1.2 mH. In a rudimentary experiment, the inductive load did radiate, at a level 50-60 dB down from a dipole but sufficient to establish an interstate radio link under favorable noise and interference conditions.

The cited coil terminal resistance of 0.001 Ohm is not a value realistically expected to be observed but, at the same time, is optimistic for a radiation resistance value in this case.

Case 5: $Q_L = 100, Q_C = 1000$	
Initial $X_1 - X_3$	+0.224, +0.224, -0.224 Ω
Optimized $X_1 - X_3$	+64.6, +36.8, -28.8 Ω
Initial Z_{incalc}	0.007 + j9.1e-6 Ω
Optimized Z_{incalc}	50.0 + j2.45e-5 Ω
Final $R_1 - R_3$	0.65, 10.96, 0.03 Ω
ATU input leg power	1.3 %
ATU shunt leg power	0.5 %
ATU output leg power	98.2 %
Power delivered to antenna	9e-3 %
Total power	100.000%
Total ATU loss	40.5 dB

As expected, the ATU loss is enormous. For a real load device similar to that described, again, the observed input resistance would be much higher and the resulting ATU loss in dB much lower. However, this would be deceptive as nearly all the power delivered to the antenna terminals in that instance would be actually dissipated in ohmic loss versus radiation.

VI. Program Availability

Copies of the MATLAB code are available from the author on request by email. Please enter "ATU MATLAB code" in the email subject line. Further, please be advised that prospective users must have not only base MATLAB, but also the Optimization Toolbox, available to them.

VII. Concluding Remarks

Given accurate Q_L and Q_C values, the computer tool for ATU loss estimation described here has produced useful results in numerous test applications. Clearly, however, the reliability of the output depends directly on the precision of Q specifications. It has proved challenging to discern better "typical" Q_L and Q_C values for real components operating in the HF-VHF spectrum than those entered as the default numbers in the present code. Equipment is relatively available for measuring Q values and, because they are so important, ATU designers and users are urged to expend the time and effort necessary to obtain measured data. Individuals willing to share their experiences, data, and/or conclusions about appropriate default inductor and capacitor Q values are encouraged to contact the author.

References

[1] Layton, Jack, *Directional Broadcast Antennas: A Guide to Adjustment, Measurement, and Testing*, TAB Books, Blue Ridge Summit, PA, 1974, p. 123.

[2] Brown, R.G., Sharpe, R.A., Hughes, W.L., and Post, R.E., *Lines, Waves, and Antennas - The Transmission of Electric Energy, 2^{nd} Edition*, John Wiley & Sons, New York, 1973, pp. 160 and 112.

[3] MATLAB is a registered trademark of The Mathworks, Natick, MA.

[4] Fujimoto, K., Henderson, A., Hirasawa, K., and James, J.R., *Small Antennas*, John Wiley & Sons, New York, 1987, p. 68.

[5] King, R.W.P.,*The Theory of Linear Antennas*, Harvard University Press, Cambridge, MA, 1950.