RCS Computation of Targets Using Three Dimensional Scalar Parabolic Equation

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Abstract **–** The parabolic equation (PE) method gives accurate results in calculation of scattering from objects with dimensions ranging from one to tens of wavelengths. Solving parabolic equation with the marching method needs limited computer storage even for scattering calculations of large targets. In this paper, the calculation procedure of radar cross section using scalar three dimensional parabolic equations is considered and the necessary equations are derived. In order to show the capabilities of the method two structures are analyzed. First scattered fields and RCS of an airplane in the forward direction are computed. Next, scattered fields and RCS of a reflector antenna in the backward direction are calculated. The obtained results are compared with physical optics results.

I. INTRODUCTION

Parabolic equation is an approximation of the wave equation which models energy propagating in a cone centered in a preferred direction, the paraxial direction. The parabolic equation was first introduced by Leontovich and Fock in order to study the diffraction of radiowaves around the earth [1]. By the advent of advanced computers closed form solution of the parabolic equation was replaced by numerical solutions. Since then, the parabolic equation is being applied to radar, sonar, acoustic and wave propagation. The parabolic equation has been recently used in scattering problems in acoustics [2] and electromagnetics [3].

II. THE PARABOLIC EQUATION FRAMEWORK

In this paper we concentrate on three dimensional analyses using parabolic equation. The time dependence of the fields is taken as $exp(-j\omega t)$. For horizontal polarization, the electric field $\vec{E}$ has only the non-zero component E_z, while for vertical polarization, the magnetic field $\vec{H}$, has the only H_z component. The reduced function u is defined as

$$u(x,y,z)=\exp(-ikx)\psi(x,y,z)\,. \qquad (1)$$

In which $\psi(x,y,z)$ is the E_z component for horizontal polarization and H_z component for vertical polarization. The paraxial direction is taken along the x axis. Assuming n, as the refractive index of the medium, the field component ψ satisfies the following three dimensional wave equation

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}+k^2n^2\psi=0\,. \qquad (2)$$

Using equations (1) and (2), the wave equation in terms of u is

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}+2ik\,\frac{\partial u}{\partial x}+k^2(n^2-1)u=0\,. \qquad (3)$$

Considering $Q=\sqrt{\frac{1}{k^2}\frac{\partial^2}{\partial y^2}+\frac{1}{k^2}\frac{\partial^2}{\partial z^2}+n^2}$, equation (3) is reduced to

$$\frac{\partial^2 u}{\partial x^2}+2ik\,\frac{\partial u}{\partial x}+k^2(Q^2-1)u=0 \qquad (4)$$

which can be written as

$$\left[\frac{\partial}{\partial x}+ik\,(1+Q)\right]\left[\frac{\partial}{\partial x}+ik\,(1-Q)\right]u=0\,. \qquad (5)$$

Decomposing equation (5), the following pair of equations is obtained

$$\frac{\partial u}{\partial x}=-ik\,(1-Q)u\,, \qquad (6\text{-}1)$$

$$\frac{\partial u}{\partial x}=-ik\,(1+Q)u\,. \qquad (6\text{-}2)$$

The solution to equation (6-1) corresponds to forward propagating waves, while that of equation (6-2) concerns the backward waves.

III. SCATTERED FIELD CALCULATION

The simplest approximation of equation (6-1) is obtained using few terms of the Taylor series expansion. Using this approximation, the standard parabolic equation is obtained. Assuming Q as

$$Q = \sqrt{Y + Z + 1} \quad (7)$$

in which

$$Y = \frac{1}{k^2}\frac{\partial^2}{\partial y^2}, \; Z = \frac{1}{k^2}\frac{\partial^2}{\partial z^2} + n^2 - 1,$$

and using the Feit and Fleck approximation to decouple Y and Z [4], one will have

$$\sqrt{Y + Z + 1} \sim \sqrt{Y + 1} + \sqrt{Z + 1} - 1. \quad (8)$$

Using the first order Taylor series of each square root of equation (8) and substituting in equation (6-1) one finds

$$\frac{\partial u}{\partial x} = \frac{ik}{2}(Y + Z)u. \quad (9)$$

With regard to the definition of Y and Z, equation (9) is reduced to

$$\frac{\partial u}{\partial x} - \frac{i}{2k}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{ik}{2}(n^2 - 1)u = 0. \quad (10)$$

This equation is the standard parabolic equation and is a narrow angle approximation of parabolic equation in three dimensions which calculates the total field in the forward direction. Integration domain is considered as a box which embraces the object. This domain must be truncated in the transverse plane. PML has been used as an absorbing boundary condition to solve parabolic equation by Collino [5]. The main advantage of PML is its efficiency for all incident angles by using it in few grid points of integration domain. Integration domain with a PML absorbing boundary condition is shown in Fig. 1.

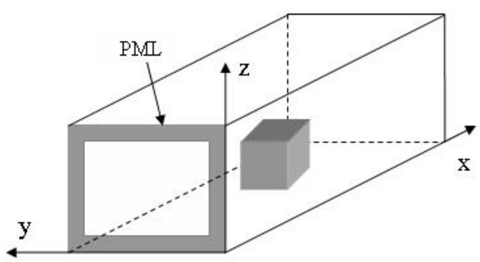

Fig. 1. Integration domain with a PML absorbing boundary condition.

We discretize equation (10) on rectangular grid by using finite difference method. In order to discretize parabolic equation usually Crank-Nicolson scheme is utilized. In this paper we use another scheme which shows better stability compared to the Crank-Nicolson scheme [6]. We define region $(m\triangle x, y, z)$ as range m. Despite Crank-Nicolson's scheme, in which second order derivatives with respect to y and z are calculated by averaging between range m and range m-1, this scheme calculates second order derivatives just in range m. This process decreases the accuracy of discretizing scheme with respect to the Crank-Nicolson and requires a smaller $\triangle x$, however the stability of the scheme is improved [6], [8]. The boundary of the object must be modeled accurately in scattering problems, therefore a smaller Δx is needed. Discretizing equation (10) for free space yields

$$\frac{u_{i,j}^m - u_{i,j}^{m-1}}{\triangle x} = \frac{i}{2k}\left(\frac{u_{i-1,j}^m - 2u_{i,j}^m + u_{i+1,j}^m}{\triangle y^2} + \frac{u_{i,j-1}^m - 2u_{i,j}^m + u_{i,j+1}^m}{\triangle z^2}\right). \quad (11)$$

By using equation (11), we can calculate fields in range m versus range m-1. Positions of grid points, while can be determined regarding equation (11), are shown in Fig. 2. In a two dimensional analysis by parabolic equation, we have to invert a triangular matrix to obtain u at range x_m. In the three dimensional case, the coefficient matrix is a very large sparse matrix and can not be solved with direct inversion. The conjugate gradient method is used to calculate u at range x_m in our work [7].

In order to calculate fields in all points of integration domain, first, the fields should be determined at range x_0. The incident field is assumed as a plane wave with unit amplitude as

$$u(x,y,z) = e^{(ik(x(\cos\theta - 1) + y\sin\theta\cos\varphi + z\sin\theta\sin\varphi))}, \quad (12)$$

in which θ and φ are the angles of incident plane wave with x and y axes, respectively.

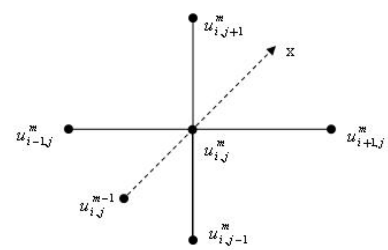

Fig. 2. Positions of grid points associated with equation (11).

The incident wave can be calculated at all points of integration domain using equations (11) and (12). Total fields in the forward direction can be calculated from equations (11) and (12) when the object is within the integration domain. Subtracting incident fields from total fields yields scattered fields in the forward direction. To compute the backward scattered fields we develop a two dimensional version in which the object is treated as a sequence of reflecting facets [3]. Modeling of the object boundary within the parabolic equation for scattering calculations has been shown in figure 3. In this case, the analysis is initiated from some range beyond the object, setting the initial scattering fields to zero. Boundary conditions on each facet are given by the appropriate polarization dependent reflection coefficients, which may vary along the scattering object.

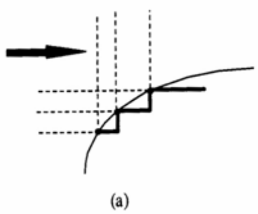

Fig. 3. Modeling of the object boundary within the parabolic equation for scattering calculations a) forward and b) backward [3].

IV. COMPUTATION OF RADAR CROSS SECTION

After the calculation of fields over the entire computational domain, we can compute the fields within any arbitrary domain x as a function of the fields in x_0 in free space as follows [8]

$$u(x,y,z) = -\frac{1}{2\pi} \times \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u(x_0,y',z')\left[\frac{ik(x-x_0)}{d(y',z')} - \frac{1}{d(y',z')}\right] \times \frac{e^{ikd(y',z')}}{d(y',z')} dy'dz' \tag{13}$$

in which

$$d(y',z') = \sqrt{(x_0 - x')^2 + (y - y')^2 + (z - z')^2} \ .$$

The radar cross section is defined as

$$\sigma(\theta,\varphi) = \lim_{r\to\infty} 4\pi r^2 \left|\frac{u^s(x,y,z)}{u^i(x,y,z)}\right|^2 \tag{14}$$

in which we have

$$x = r\cos\theta \ , y = r\sin\theta\cos\varphi \ , z = r\sin\theta\sin\varphi \ .$$

Tending (x,y,z) to infinity along a given direction in equation (13), and assuming a unit amplitude for the incident wave, equation (14) yields [8]

$$\sigma(\theta,\psi) = \frac{k^2\cos^2\theta}{\pi} \times \left|\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} u_s(x_0,y',z') e^{-ik\sin\theta(y'\cos\varphi + z'\sin\varphi)} dy'dz'\right|^2, \tag{15}$$

in which $u_s(x,y,z)$ is the scattered field.

V. NUMERICAL RESULTS

In this section, scattered fields and RCS of an airplane and a reflector antenna, which have been computed in forward and backward directions respectively, are presented. The incident wave is a plane wave with horizontal polarization and a wavelength equal to 1 meter. Moreover the angle of incident wave is assumed to be zero.

In order to calculate the scattered fields from the airplane, its staircase model is utilized. Dimensions of airplane and its staircase model are shown in Figs. 4 and 5, respectively.

In order to calculate scattered fields from the airplane the size of the integration domain has been considered as 40λ, 30λ and 30λ in the *x*, *y*, and *z* directions respectively. In addition the grid spacing in the *x*, *y* and *z* directions are assumed to be $\lambda/4$, $\lambda/5$, and $\lambda/5$, respectively. The run time of the program on a Pentium IV with 256 MB of RAM was 15 minutes. Near field results in planes $z = 11.3m$ and $x = 25m$ are shown in Figs. 6 and 7, respectively. RCS results of airplane in $\varphi = 0°$ plane are presented in Fig. 8. Solid lines represent parabolic equation results and diamond markers represent physical optics (PO) data.

Backward scattered fields from the reflector antenna have been calculated using a staircase model for the antenna within the parabolic equation. Dimensions of reflector antenna and its staircase model are shown in Figs. 9 and 10, respectively. The size of integration domain has been considered as 3.5λ, 10λ, and 10λ in the *x*, *y* and *z* directions respectively. Moreover the grid spacing in the *x*, *y*, and *z* directions are assumed to be 0.01λ, 0.05λ, and 0.05λ, respectively. The run time of the program in this case was about 2 hours. Near field results in $y = 5m$ and $x = 0$ planes are shown in Figs. 11 and 12, respectively. RCS results of reflector antenna in $\varphi = 0°$ plane are presented in Fig. 13. Solid lines represent parabolic equation results and circle markers represent physical optics data.

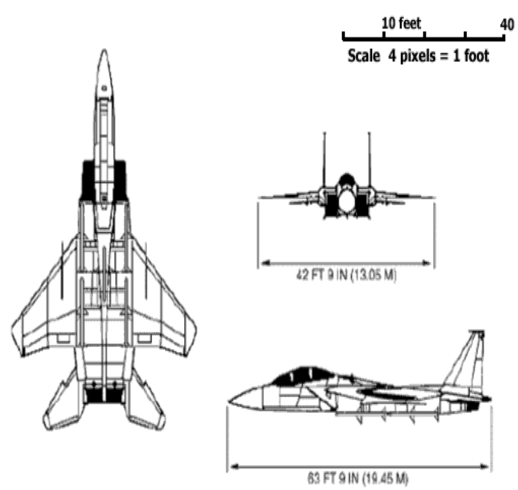

Fig. 4. The geometry of an actual airplane and its dimensions.

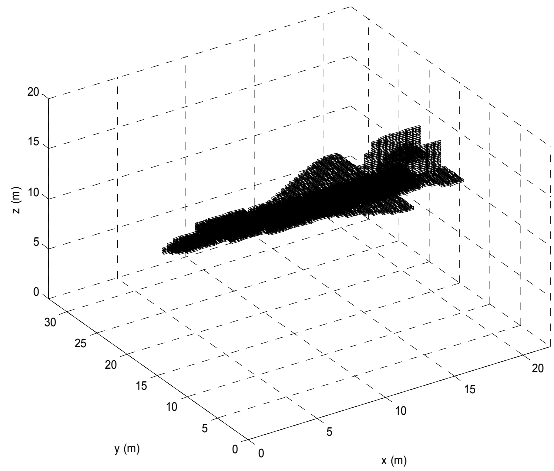

Fig. 5. Airplane staircase model (dimensions are in meters).

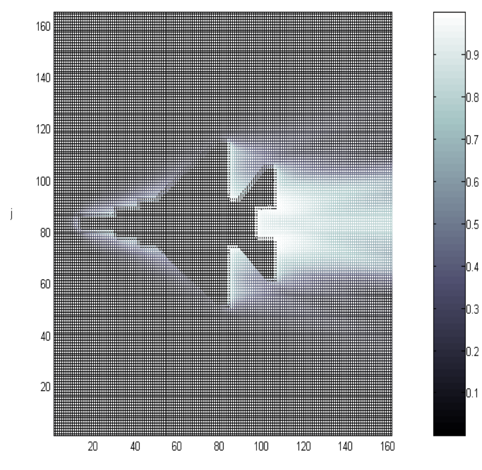

Fig. 6. Amplitude of the forward scattered field $u_s(m,i,j)$ from the airplane for z = 11.3 m and f = 300 MHz.

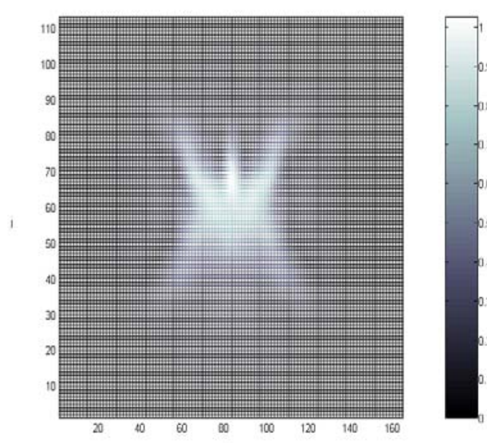

Fig. 7. Amplitude of the scattered field $u_s(m,i,j)$ from the airplane for x = 25m and f = 300 MHz.

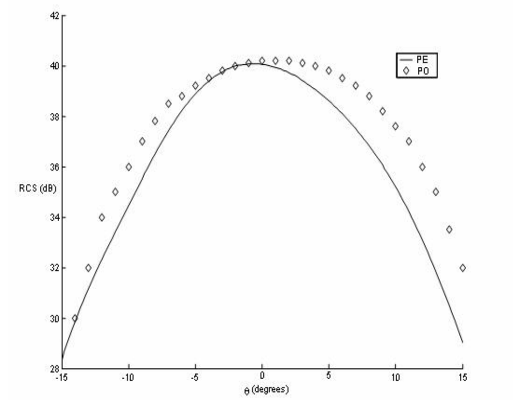

Fig. 8. Forward RCS of airplane in $\varphi = 0^\circ$ plane for f=300 MHz.

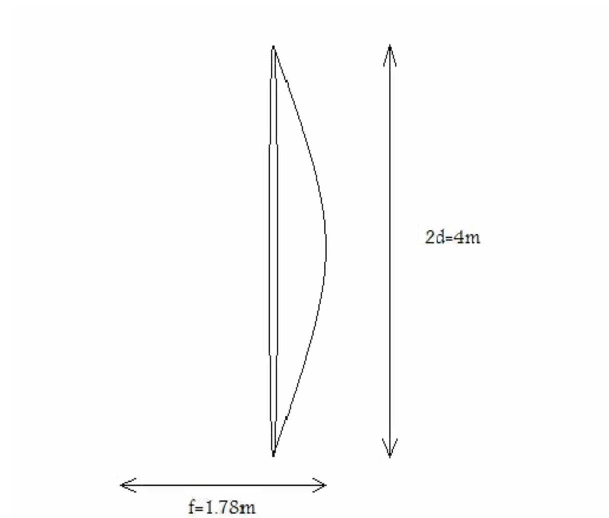

Fig. 9. The actual structure of the reflector antenna and its dimensions.

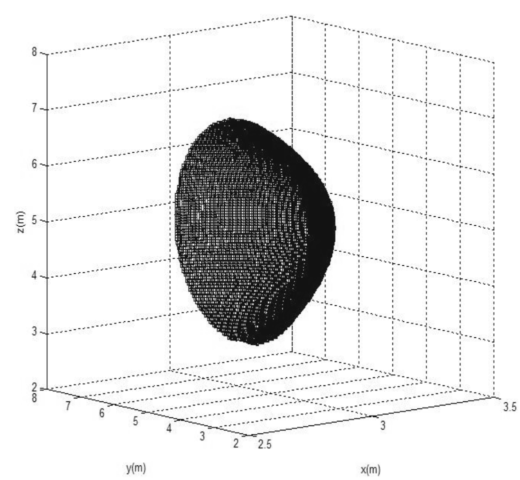

Fig. 10. Reflector antenna staircase model (dimensions are in meters).

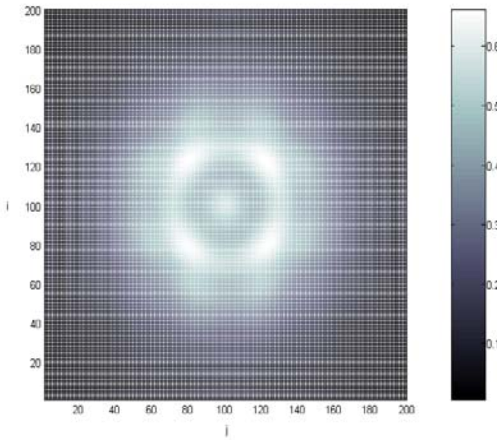

Fig. 11. Amplitude of the backward scattered field $u_s(m,i,j)$ from the reflector antenna for $x = 0$ and f = 300 MHz.

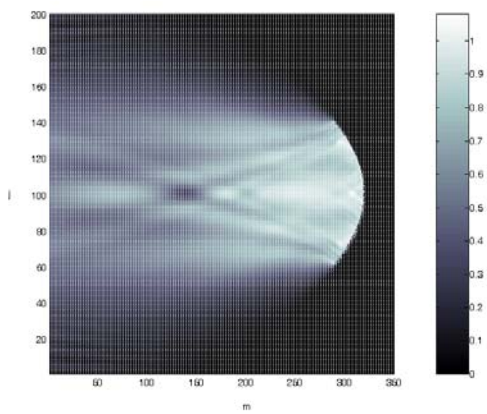

Fig. 12. Amplitude of the backward scattered field $u_s(m,i,j)$ from the reflector antenna for $y = 5\ m$ and f = 300 MHz.

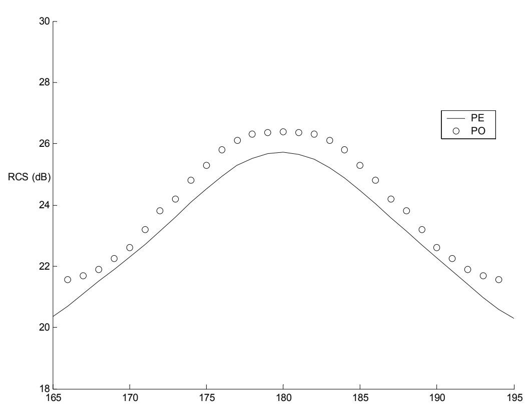

Fig. 13. Backward RCS of the reflector antenna in $\varphi = 0°$ plane for f = 300 MHz.

VI. CONCLUSION

Parabolic equation method is an efficient tool in calculating scattered fields from objects with dimensions large compared to the wavelength. A three dimensional scalar parabolic equation technique was used for RCS calculation of two structures in this paper. First scattered fields and RCS of an airplane in the forward direction were computed and the results were compared with physical optics results. There is a good agreement between the physical optics results and the parabolic equation data. Next, scattered fields and RCS of a reflector antenna in the backward direction were calculated and the results were compared with physical optics data. Close agreement between the two results is observed.

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