Finite Element Modeling of Dual-Core Photonic Crystal Fiber

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Abstract − In this paper, coupling characteristics of dual-core photonic crystal fiber (PCF) are studied extensively using vector finite element method, which has the potential to realize wavelength selective MUX-DEMUX for wavelength division multiplexing (WDM) application. Dispersion characteristic is also reported and demonstrates the wavelength region where it can support short duration soliton like pulses.

Keyword: Photonic Crystal, Waveguide, and FEM.

I. INTRODUCTION

Photonic crystal fiber (PCF) has recently attracted a considerable amount of attention, because of their unique properties that are not present in conventional optical fibers. A PCF has a central region of pure silica (core) surrounded by air holes. It is a regular morphological microstructure incorporated into the material to radically alter its optical properties [1]. Here a wavelength dependant effective volume average index difference between the defect regions will form the core, and the surrounding region, which contains air holes will be acting as the cladding. This effective-index guidance does not depend on having a periodic array of holes. Even other arrangements could serve a similar function [1, 2]. Index-guiding PCF guides light by total internal reflection between a solid core and a cladding region with multiple air-holes [1]. On the other hand, a perfectly periodic structure exhibiting a photonic band gap (PBG) effect at the operating wavelength to guide light in a low index core-region [1]. In this paper, we will focus on index-guiding PCFs, also called holey fibers (HFs).

HF possess numerous unusual properties such as wide single-mode wavelength, bend-loss edge at short wavelength, controlled effective-core-area at single-mode region, and anomalous group-velocity dispersion at visible and near-infrared wavelengths [2, 3]. It has been shown that the PCF with two adjacent defect area (served as two core), can be used as an optical fiber coupler [4-6]. These PCF couplers have the possibility of realizing a multiplexer-demultiplexer (MUX-DEMUX). In this paper, wavelength dependent coupling characteristics of dual-core PCF couplers are evaluated by using a vector finite element method (FEM) [7, 8]. This gives understanding of the PCF based MUX-DEMUX for wavelength selective application such as WDM.

II. FINITE ELEMENT FORMULATION FOR GUIDED MODE

The vector finite element method is used to compute the mode spectrum of an electromagnetic waveguide with arbitrary cross section [9, 10]. It eliminates the disadvantages of the scalar finite element approach of having undesired spurious modes or non-physical solutions and is characterized by easy implementation of boundary conditions at material interfaces [9]. Recent study shows that some double curl finite element formulations are not immune to spurious modes even though they are not observed frequently. It is due to the fact that the initial conditions (forcing term) corresponding to the physical situation eliminates frequent observation of spurious modes [9,10]. We discretized the continuous spectrum by enclosing the structure with an electrical wall as shown in Fig. 1. We have studied dual core PCF and evaluate coupling characteristics using vector FEM. Figure 1 shows the dual core PCF geometry. Some dimension of PCF geometry such as air hole diameter d, pitch between two adjacent holes Λ and core separation C were adjusted to obtain the desired values for coupling length and dispersion.

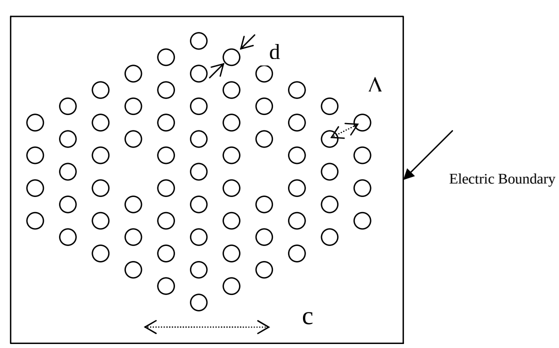

Fig. 1. Arbitrary shaped waveguide with electrical wall.

The vector finite element formulation can be illustrated by using either the **E** or **H** field; here we explain the case for the **E** field, which is the same for the

H field. The vector wave equation for the **E** field is given by,

$$\nabla\times\left(\frac{1}{\mu_r}\nabla\times\mathbf{E}\right)-k_0^2\varepsilon_r\mathbf{E}=0 \qquad (1)$$

where μ_r and ε_r are, respectively, the permeability and permittivity of the material in the waveguide. k_0 is the free space wave number. The transverse and longitudinal components are separated and are written as,

$$\nabla_t\times\left(\frac{1}{\mu_r}\nabla_t\times\mathbf{E}_t\right)+\frac{1}{\mu_r}\left(k_z^2\nabla_t E_z+k_z^2\mathbf{E}_t\right)=k_0^2\varepsilon_r\mathbf{E}_t \qquad (2)$$

$$-\frac{1}{\mu_r}\left[\nabla_t\cdot\left(\nabla_t E_z+\mathbf{E}_t\right)\right]=k_0^2\varepsilon_r E_z\,. \qquad (3)$$

Since the vector Helmholtz equation is divided into two parts, equations (2) and (3), vector-based tangential edge elements, shown in Fig. 2 (a), can be used to approximate the transverse fields, and nodal-based elements, shown in Fig. 2 (b), can be used to approximate the longitudinal component.

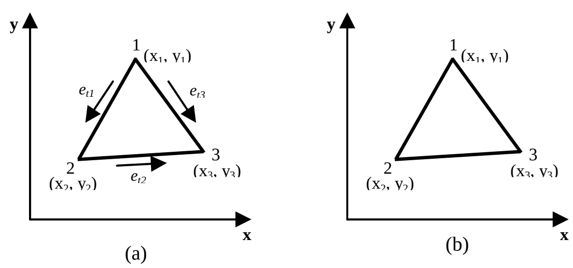

Fig. 2. Configurations of (a) tangential edge elements and (b) node elements.

For a single triangular element shown in Fig. 2, the transverse electric field can be expressed as a superposition of edge elements. The edge elements permit a constant tangential component of the basis function along one triangular edge while simultaneously allowing a zero tangential component along the other two edges [9]. Three such functions overlapping each triangular element provide the complete expansion, that is,

$$\mathbf{E}_t=\sum_{m=1}^{3}e_{tm}\mathbf{W}_{tm} \qquad (4)$$

where m indicates the m-th edge of the triangle and $\mathbf{W}_{tm}$ is the edge element for edge m given by,

$$\mathbf{W}_{tm}=L_{tm}\left(\alpha_i\nabla_t\alpha_j-\alpha_j\nabla_t\alpha_i\right) \qquad (5)$$

L_{tm} is the length of edge m connecting nodes i and j and α_i is the first-order shape function associated with nodes 1, 2, and 3. The longitudinal component is written as,

$$E_z=\sum_{i=1}^{3}e_{zi}\alpha_i\,. \qquad (6)$$

After simple manipulations the integral equation for each elements can be written in matrix form as,

$$\begin{bmatrix}S_{e(tt)} & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}e_t\\ e_z\end{bmatrix}=-k_z^2\begin{bmatrix}T_{e(tt)} & T_{e(tz)}\\ T_{e(zt)} & T_{e(zz)}\end{bmatrix}\begin{bmatrix}e_t\\ e_z\end{bmatrix} \qquad (7)$$

where

$$S_{e(tt)}=\frac{1}{\mu_r}\iint_\Delta\left(\nabla_t\times\mathbf{W}_{tm}\right)\cdot\left(\nabla_t\times\mathbf{W}_{tn}\right)ds-k_0^2\varepsilon_r\iint_\Delta\left(\mathbf{W}_{tm}\cdot\mathbf{W}_{tn}\right)ds\,, \qquad (8)$$

$$T_{e(tt)}=\frac{1}{\mu_r}\iint_\Delta\left(\mathbf{W}_{tm}\cdot\mathbf{W}_{tn}\right)ds\,, \qquad (9)$$

$$T_{e(tz)}=\frac{1}{\mu_r}\iint_\Delta\left(\mathbf{W}_{tm}\cdot\nabla\alpha_j\right)ds\,, \qquad (10)$$

$$T_{e(zt)}=\frac{1}{\mu_r}\iint_\Delta\left(\nabla\alpha_i\cdot\mathbf{W}_{tn}\right)ds\,, \qquad (11)$$

$$T_{e(zz)}=\frac{1}{\mu_r}\iint_\Delta\left(\nabla\alpha_i\cdot\nabla\alpha_j\right)ds-k_0^2\varepsilon_r\iint_\Delta\alpha_i\alpha_j\,ds\,. \qquad (12)$$

These element matrices are assembled over all the triangular elements in the cross section of the structure to obtain a global eigenvalue equation [9, 10]. Solving the above equation yields the eigenvalues or the longitudinal propagation constants k_z, from which the effective refractive index n_e is obtained by using the relation,

$$n_e=\left(\frac{k_z}{k_0}\right)\,. \qquad (13)$$

III. CHARACTERISTICS OF DUAL-CORE PCF

In brief signal power is exchanged between the coupled cores due to weak overlap of adjacent electric field. Here light confined into one of the PCF core moves to the other waveguide after propagating a distance known as coupling length Lc due to the different propagation constants of the even and odd modes of the coupler [11]. Coupling length Lc is determined by the following equation,

$$L_C = \frac{\pi}{\beta_e - \beta_o}. \quad (14)$$

Here β_e is the dispersion coefficient for even mode; β_o is the dispersion coefficient for odd mode. Figure 3 shows the coupling length L_c with the hole pitch for d/Λ =0.7. It is shown from numerical results that it is possible to realize significantly shorter MUX-DEMUX PCFs, compared to conventional optical fiber couplers. In the conventional fiber coupler with core spacing and core radius ratio of 3 we found that the coupling length of is 1cm. If the spacing between the cores increases then the coupling length will also increase [12]. On the other hand PCF coupler with d/Λ =0.7 has coupling length of few mm (at 1.50 um it is between 2-4 mm). The advantage of having short coupling length is that the device becomes more miniaturized.

Figure 4 shows the wavelength dependency of Lc. It is observed that the coupling length decreases sharply at shorter wavelength up to 0.5 um then the slope decreases up to 2 um then it get almost constant at higher wavelength which comparable to the pitch (in this case 2.5 um). This is because of the sharp change of material dispersion of silica glass at the short wavelength contributes in a higher extent in evaluation of even and odd wave number. The difference between these two wave numbers varies rapidly at short wavelength but at higher wavelength this remains fairly constant.

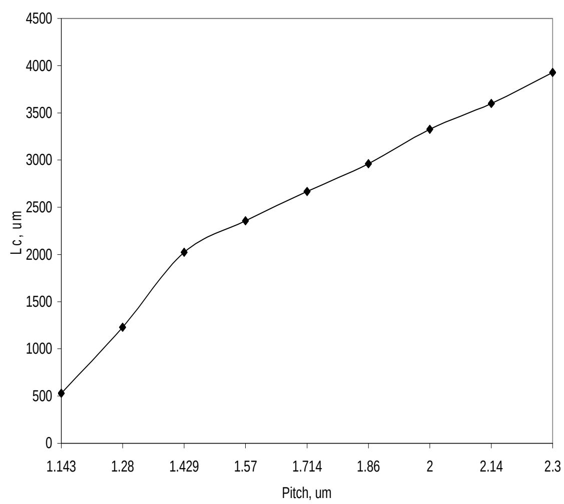

Fig. 3. Coupling lengths of PCF couplers with d/Λ =0.7 at 1500 nm.

It is also possible to significantly change the coupling length by altering the cladding geometry as well as the core separation. For the similar dual core PCF with d=0.8 um and d/Λ=0.4, as shown in Fig. 5, Lc shows similar characteristics with the wavelength but this time it is longer than the one of the PCF with d/Λ=0.7 shown in Fig. 4. For a constant d/Λ ratio we can vary Lc by varying d. Figure 6 shows that the Lc can be increased by increasing d.

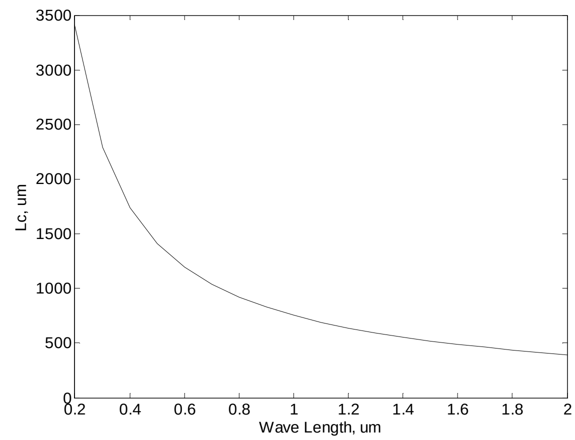

Fig. 4. Coupling lengths of PCF couplers (d=0.8 um and d/Λ =0.7).

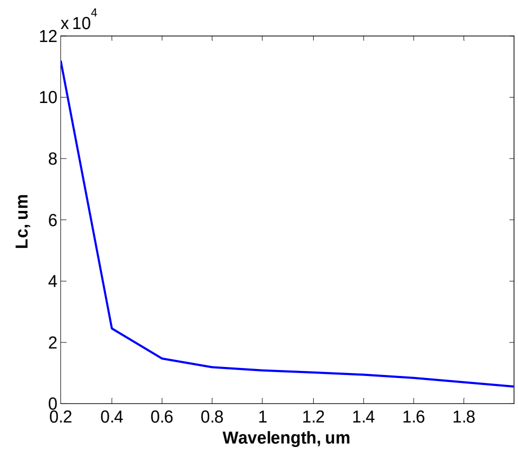

Fig. 5. Coupling length of dual core PCF coupler (d=0.8um and d/Λ=0.4).

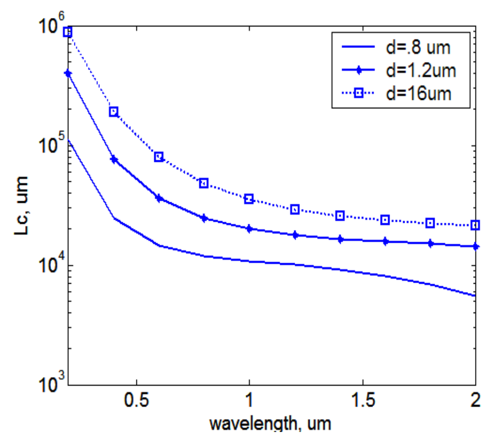

Fig. 6. Coupling length comparison for dual core PCF coupler (C=4 Λ, d/Λ=0.4).

IV. WAVEGUIDE DISPERSION

When electromagnetic wave interacts with the bound electron of the dielectric materials the medium response is frequency dependant and this manifests itself through the change of refractive index n(ω). It is due to the characteristic resonance frequency at which the bound electron oscillation of the dielectric medium absorbs the electromagnetic radiation [2, 13]. It is the dispersion of optical waveguide which is the most critical for short pulse propagation because the different spectral component associated with the pulse travels at different speed c/n(ω). The parameter D which is commonly used in optical fiber literature is called the group velocity dispersion and can be expressed as,

$$D = (\frac{d\beta_1}{d\lambda}) = -\frac{2\pi c}{\lambda^2}\beta_2 \qquad (15)$$

were β_1 and β_2 are the first and second derivative of wave number β with respect to ω . Fig 6 shows the dispersion profile for the same coupler. There is a sign change around 1.0 μm. These wavelength, when the group velocity dispersion shifts from normal (D is +ve) to anomalous (D is -ve), is called the zero dispersion wavelength and treated as a very important design parameter for device supporting short pulse propagation. Soliton pulse propagation through this optical waveguide is dependant on the delicate balance between nonlinearity and the dispersion. Nonlinear phase modulation tries to compress the pulses and the dispersion causes pulse broadening. If the power and dispersion is properly balanced then input pulse can propagates without any distortion. In order to support soliton pulses the dispersion has to be in the anomalous region [13, 14].

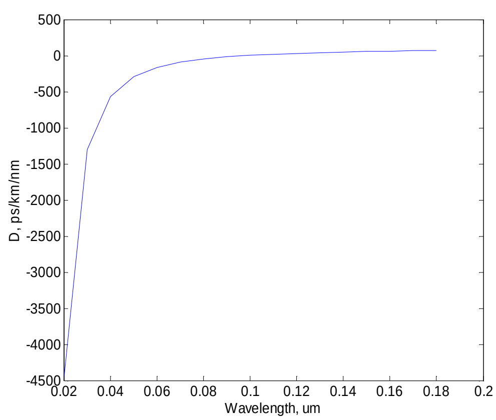

Fig. 7. Dispersion characteristics of dual core PCF (d=0.8 and d/Λ =0.7).

From the dispersion results shown in Fig. 7 we found that the wavelengths beyond 1.0 um have anomalous dispersion. At those wavelengths the waveguide is able to support soliton pulses. Therefore Lc along with the dispersion parameters determines the waveform while it propagates.

V. CONCLUSION

We have numerically demonstrated the coupling and dispersion characteristics of dual core PCF which have a potential application in the wavelength selective system. With short coupling length compared to regular fiber coupler the device can be used as a MUX-DEMUX or power coupler in the WDM system. Dispersion profile demonstrates the wavelength region where it can support short duration soliton like pulses.

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