On the Direct Computation of the Time-Domain Plane-Wave Reflection Coefficients

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Abstract **—** This communication compares different approximate techniques to calculate the transient reflection coefficient for TE and TM plane waves from a lossy half space in terms of accuracy, computational costs and area of validity. By varying angles of incidence and constitutive parameters of the half spaces, it is shown that approximate equations can efficiently and accurately calculate the reflection coefficients in the time domain, if the right equation and enough terms in the approximate series are chosen. To confirm these conclusions, it is considered, as a practical example, short pulses reflected by an operational Ground-Probing Radar from different half spaces.

I. INTRODUCTION

The calculus of the reflection coefficients (RC) for a plane-wave incident on an interface directly in the time domain has been a matter of interest for numerous practical applications, such as the characterization of materials of Ground-Penetrating Radar [1], non-destructive testing [2] and material characterization [3], and it is attracting renewed interest for estimating the multi-path propagation in UWB communication channel [4]. Efficient and accurate calculations are needed in these cases, in which the complexity of the case can require multiple computations of the RC. The first approach to solve this problem was based on the numerical evaluation of the inverse Fourier transform of the TE and TM Fresnel reflection coefficients [5, 6]. To reduce the high computational resources required by this approach, Barnes and Tesche [7] derived an approximate analytical expression for the time-domain reflection coefficients (TD-RC). This analytical expression assumes some restrictions related to the constitutive parameters of the media and angles of incidence of the plane wave. Later, Rothwell [8] and Rothwell and Suk [9] inferred other closed-forms of the TD-RC by means of a mathematical procedure involving analytic inverse Laplace transforms thereby avoiding the restrictions set by [7]. Starting from these closed-form solutions, recent papers [9, 10] have proposed expressions which can be numerically computed with efficiency. The accuracy of different approximations and related computational costs were, however, not compared.

In this paper, the aforementioned approaches are compared in terms of accuracy and computational costs. Also, some conclusions about the applicability areas for each method are drawn, by considering different media and angles of incidence for TE as well as TM polarizations. These inferences are confirmed in the last section of the work calculating the electric field radiated by a dielectric wedge antenna (DWA), which was designed for ground-penetrating radar applications [11], and the electric field reflected from lossy half-spaces.

II. TIME-DOMAIN REFLECTION COEFFICIENTS

The oblique incidence of a plane-wave from a general dielectric media onto a lossy frequency-independent half-space, of constitutive parameters (ε,σ), is solved in the frequency domain by using the Fresnel reflection coefficients [5]. Using θ_o as the angle between the incident wave and normal vector to the interface, the problem can be considered as TM or vertical-polarization incidence if the magnetic field is polarized parallel to the interface, or as TE or horizontal-polarization incidence in case of a parallel electric field to interface. The TD-RC is defined from the inverse Fourier transform of the Fresnel reflection coefficients [6]. Analytical expressions of the TD-RC are in the form,

$$\Gamma(t) = \Gamma_{die}\delta(t) + \Gamma_{con}(t)u(t) \tag{1}$$

where Γ_{die} is equal to the frequency-domain reflection coefficient for the lossless case, and $\Gamma_{con}(t)$ is a exponentially decreasing-in-time function associated with conductive processes in the lossy media. As this conductive term is composed by a sum of terms involving integrals of the modified first-class Bessel functions [8], its numerical evaluation is in general computationally expensive, and thus approximate forms of this term are often used in practice. As Γ_{die} is identical in all the approximations [7, 9, 10], this paper will focused on comparisons of $\Gamma_{con}(t)$.

A. Approximate expressions of TD-RC

Several computationally affordable approximations of $\Gamma_{con}(t)$ have been proposed in the literature in the form of infinite expansion series. Barnes and Tesche [7] considered those cases where the condition $\sin^2\theta_0/\varepsilon_r$ <<1 is satisfied, which leads to an approximate equation of the Fresnel reflection coefficient in which an analytical inverse Laplace transform can be developed to derive expressions in the form,

$$\Gamma_{con}^{TE}(t) = K_1^{TE}\frac{e^{-\frac{\sigma t}{2\varepsilon}}}{t}\sum_{n=1}^{\infty} n\left(K_2^{TE}\right)^n I_n\left(\frac{\sigma t}{2\varepsilon}\right) \quad (2)$$

and

$$\Gamma_{con}^{TM}(t) = K_1^{TM}\frac{e^{-\frac{\sigma t}{2\varepsilon}}}{t}\sum_{n=1}^{\infty} n\left(K_2^{TM}\right)^n I_n\left(\frac{\sigma t}{2\varepsilon}\right), \quad (3)$$

where K_i^{TE} and K_i^{TM} are functions exclusively of ε_r and θ_0, while $I_n(x)$ represents the *n*-order modified Bessel function of the first class.

Another approximation arises by considering a Taylor series of the function $Q(x) = e^{-x}I_0(x) + e^{-x}I_1(x)$. Substitution of this series in the closed form of the TD-RC simplifies the integral terms, and expressions found are [9, 10],

$$\Gamma_{con}^{TE}(t) = \sum_{n=1}^{\infty} C_1^{TE}\left(C_2^{TE}\right)^n Q^{(n)}\left(C_0^{TE}t\right) \quad (4)$$

and

$$\Gamma_{con}^{TM}(t) = \sum_{n=1}^{\infty}\left[C_1^{TM}\left(C_2^{TM}\right)^n + C_3^{TM}\left(C_4^{TM}\right)^n\right]Q^{(n)}\left(C_0^{TM}t\right) \quad (5)$$

where C_i^{TE}, and C_i^{TM} depend only on ε, σ and θ_0; and $Q^{(n)}(x)$ represents the *n*-order derivative of $Q(x)$.

Despite that equations (4) and (5) are as exact as the closed-form TD-RC analytical expressions, the truncation of the infinite series carries loses accuracy for small *t*. In fact, this initial period of time produces the main contribution to the reflected field, and improvements of the truncation error at early times are achieved by using [9, 10],

$$\Gamma_{con}^{TE}(t) = \left[1 + C_3^{TE}e^{-C_0^{TE}t}\right]\sum_{n=1}^{\infty} C_1^{TE}\left(C_2^{TE}\right)^n Q^{(n)}\left(C_0^{TE}t\right) \quad (6)$$

and

$$\begin{aligned}\Gamma_{con}^{TM}(t) &= C_5^{TM}e^{-2C_0^{TM}t}\left(C_6^{TM}C_0^{TM}t + 1\right) + \\ &+ \sum_{n=1}^{\infty}\left[C_1^{TM}\left(C_2^{TM}\right)^n + C_3^{TM}\left(C_4^{TM}\right)^n\right]Q^{(n)}\left(C_0^{TM}t\right).\end{aligned} \quad (7)$$

Detailed expressions of K_i^{TE}, K_i^{TM}, C_i^{TE}, C_i^{TM} are given in the appendix.

B. Convergence and Accuracy of the Approximations

Prior to comparing the approximations, it is performed a preliminary study on the convergence of the series for different truncations. By varying the angles of incidence for different soils (lossy ground with ε_r=10 and σ =0.01 and sea water with ε_r =72 and σ =4) and considering TE and TM polarizations, it is found that good rates of convergence are assured by using series of N=5 for the equations (2) and (3), N=10 for equations (4) and (5), and N=3 for equations (6) and (7). Figure 1 illustrates this convergence for the particular case of normal incidence on sea water. Furthermore, relative computational times for each method are compared in table 1. Computational times are normalized taking as a reference the time to calculate the closed-form solution by using a numerical integration based on the Simpson rule over 100 points [12]. Due to the high degree of convergence of the equations (6) and (7), an approximation of these equations by using only one term is included in the results, as a faster approximation of TD-RC.

Table 1. Relative computational time for equations (2) and (7).

EXACT	*2-3, N=5*	*4-5, N=10*	*6-7, N=1*	*6-7, N=3*
1	0.011	0.168	0.008	0.030

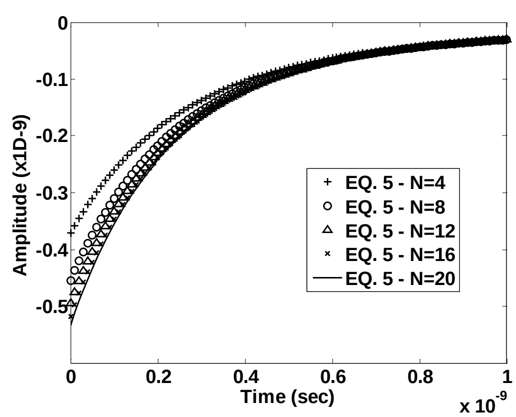

Fig. 1. Plots of the TM TD-RC for an incident plane wave forming θ_0=0° with sea water (ε_r =72, σ =4), calculated by using different truncations of equation 5.

Once the truncation of the different series is established, a study on the accuracy provided by each equation is carried out. Again, lossy ground and sea water are used as half-space, and different angles of incidence between the normal incidence and the Brewster angle of each interface are considered. Tables 2 and 3 summarize the results from both the TE and TM polarizations, taking as parameter of the comparison the maximum relative error over time made by each approximation. On examining Tables 2 and 3, it can be appreciated that equations (2) and (3) improve in performance when the condition required for their use is satisfied for cases where this approximation is poor (lossy ground and high angles of incidence), equations (5) and (6), with N=3, are closer to the exact solution. This fact is better seen both in TE polarization (Fig. 2 where marks correspond to equations (2) and (6), with N=3), and in TM polarization (Fig. 3, equations (3) and (7), with N=3). In both figures, all the approximations are very close to the exact solution, and little improvement is achieved by each one, depending on the aforementioned condition. Another remarkable fact is that Figs. 2 and 3 show that maximum relative errors appear in equations (6) and (7), with N=1 at late time. For short pulses, such as those used in UWB applications, these late-time errors have a minor effect on the calculus of the reflected field, and in principle the implementation of the computationally fastest solution should not be ruled out. A final conclusion related to these Tables is that poorest results are given by equation (3), and its use seems not to be advisable, either in terms of computational cost or in terms of accuracy.

Table 2 (a). Maximum relative error (%) for TM plane waves over ground.

	3, N=5	*5, N=10*	*7, N=1*	*7, N=3*
0º	1.74	4.69	11.90	2.31
10º	0.30	3.35	11.83	2.41
20º	1.19	0.52	11.50	2.69
30º	2.63	6.48	11.75	3.06
40º	4.50	13.18	12.69	3.02
50º	6.64	17.80	12.90	1.76
60º	8.82	16.61	11.20	0.38

Table 2 (b). Maximum relative error (%) for TE plane waves over ground.

	2, N=5	*4, N=10*	*6, N=1*	*6, N=3*
0º	1.74	4.69	13.55	4.31
10º	0.30	4.88	13.60	4.36
20º	1.16	5.47	13.74	4.54
30º	2.49	6.57	13.08	4.82
40º	4.13	8.35	13.28	4.85
50º	5.86	11.01	13.42	5.27
60º	7.49	14.79	13.42	5.71

Table 3 (a). Maximum relative error (%) for TM plane waves over sea.

	3, N=5	*5, N=10*	*7, N=1*	*7, N=3*
0º	2.36	19.39	3.74	1.73
10º	2.28	19.03	3.50	1.65
20º	0.16	17.96	3.24	1.35
30º	0.34	16.12	2.80	0.95
40º	0.58	13.45	12.96	2.30
50º	0.82	9.78	12.69	2.41
60º	1.06	4.86	12.23	2.51
70º	1.25	0.94	11.44	2.76
80º	1.38	2.75	8.55	0.57

Table 3 (b). Maximum relative error (%) for TE plane waves over sea.

	2, N=5	*4, N=10*	*6, N=1*	*6, N=3*
0º	2.36	19.39	12.39	5.88
10º	2.43	19.57	12.38	5.90
20º	0.16	20.14	12.34	5.93
30º	0.34	21.08	12.27	5.97
40º	0.57	22.39	12.16	6.03
50º	0.81	24.06	12.01	6.08
60º	1.04	26.07	11.79	6.13
70º	1.22	28.38	11.05	6.15
80º	1.34	30.93	10.72	5.88

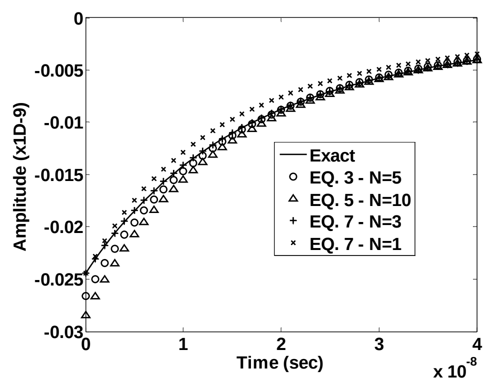

Fig. 2. TM TD-RC for an incident plane wave forming θ_0=60º with a lossy ground (ε_r=10, σ =0.01).

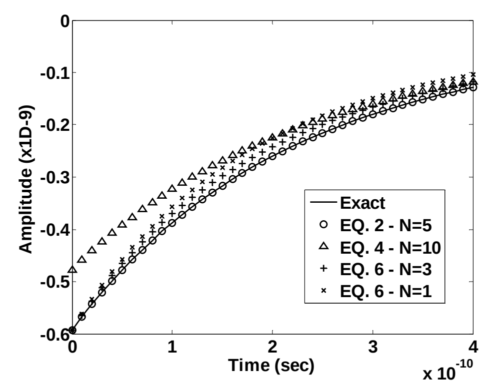

Fig. 3. TE TD-RC for an incident plane wave forming θ_0=0º with sea water (ε_r =72, σ =4).

Figure 4 shows the behaviour of the transient reflection coefficient for an incidence angle near the Brewster angle. For angles above the Brewster case, equations (5) an (7) are no longer valid [8], and significant variations compared with the exact solution appear. Nevertheless, restrictions in the applicability of equation (3) do not depend specifically on the proximity to the Brewster angle, offering thus better results.

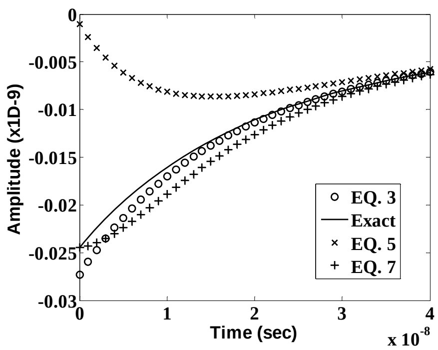

Fig. 4. TM TD-RC for an incident plane wave forming θ_0=78º with a lossy ground (ε_r =10, σ =0.01). N=10 for approximate equations.

III. EXAMPLE OF APPLICATION: DWA ANTENNA

As a practical example of application, the incidence of the electric field radiated by a DWA (Fig. 5) on different kind of soils is considered. Figure 6 depicts a comparison of the main peak of $E^r_{con}(t)$ for the incidence of the pulse of Fig. 5 in the same conditions as those of the Fig. 3 (normal incidence and sea water). As an illustration of cases where the condition for applying equation (2) does not strictly hold, the incidence of the DWA radiated pulse at a interface of a lossy concrete (ε_r =3 , σ=0.01), with an angle of 60º, is simulated (Fig. 7). To estimate the accuracy of the different approximations, the reflected electric field is compared. According to the definition of Fresnel RC, the reflected field is calculated numerically by applying,

$$E^r(t) = E^r_{die}(t) + E^r_{con}(t) = \Gamma(t) * E^i(t) \\ = \Gamma_{die} E^i(t) + \int_0^t \Gamma_{con}(\xi) E^i(t-\xi) d\xi \qquad (8)$$

where the TE or TM reflection coefficient has to be used in accordance with the polarization of the electric field.

Figure 6 show that both equations (2) and (3), as well as equations (6) and (7) (with N=1 and N=3) give an accurate representation of the reflected field. As pointed out above, errors made by equation (6) with N=1 do not lead to any appreciable difference for this UWB pulse. In Fig. 7, we clearly appreciate errors in plot corresponding to equation (2), as was expected taking into account that $\sin^2\theta_0/\varepsilon_r = 0.25$ for the lossy concrete with an incidence of 60º. The rest of the equations, which are not restricted by this condition, reproduce more accurately the exact solution. Therefore, it can be concluded that equations (2) and (3) are more sensible to the effect of the medium parameters than the others, as long as the angle of incidence remains enough low to hold the accuracy of equations (4) to (7).

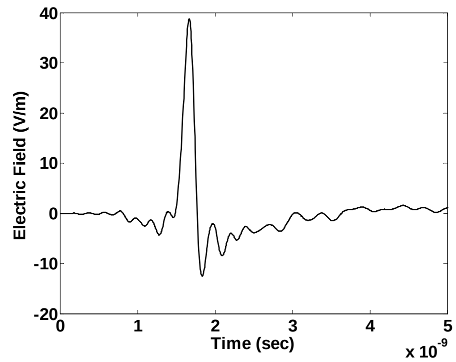

Fig. 5. Electric field radiated by a dielectric wedge antenna.

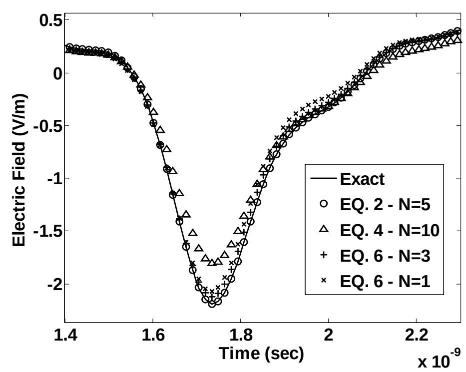

Fig. 6. Conductive part of the reflected electric field for the normal incidence of the DWA radiated pulse at sea water (ε_r =72, σ =4).

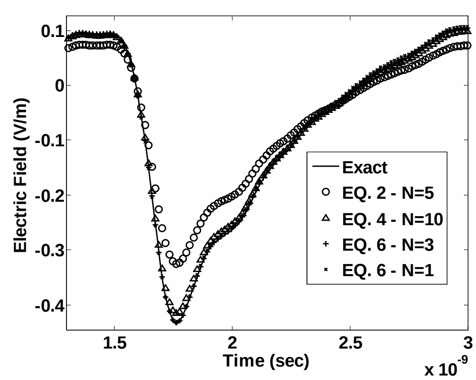

Fig. 7. Conductive part of the reflected electric field for the incidence with θ_0=60º of the DWA radiated pulse at lossy concrete (ε_r =3, σ =0.01).

IV. CONCLUSIONS

A comparison of the different approximations to estimate the TD-RC reveals the relevance of their implementation for accurate simulation of the electromagnetic pulses reflected from homogeneous half-spaces. Equations (2) and (3) proved computationally faster than any other approximation and very accurate, but they have a limited area of validity and thus their use has to be restricted to cases which satisfy a specific condition. Equations (6) and (7) constitute another good choice. They are in general computationally fast, and their accuracy can be increased by adding only a few more terms to the series. They are not restricted to particular combinations of angles of incidence or constitutive soil parameters, and their use guarantees accurate results. Equations (4) and (5) are definitely poor choices in comparison with the other approximations. Therefore, a correct choice in these approximations enables faster and accurate simulations of interaction of electromagnetic waves with real media, which is a matter of interest in such fields as ground-penetrating radar and material characterization.

APPENDIX

Equations (2) to (7) have arisen to the set of functions K_i^{TE}, K_i^{TM}, C_i^{TE}, C_i^{TM} whose operational expressions are given below. Note that integer N represents the order of the truncation of the infinite series appearing in those equations, and $Q^{(n)}(0)$ can be derived from the recurrence relations of the modified Bessel functions $I_n(0)$ [13].

The K_i^{TE}, K_i^{TM} of equations (2) and (3) are,

$$K_1^{TE} = -\frac{4k_{TE}}{1-\left(k_{TE}\right)^2}$$
$$K_2^{TE} = \frac{1-k_{TE}}{1+k_{TE}},$$
$$K_1^{TM} = \frac{4k_{TM}}{1-\left(k_{TM}\right)^2},$$
$$K_2^{TM} = \frac{1-k_{TM}}{1+k_{TM}},$$

where

$$k_{TE} = \frac{\cos\theta_0}{\sqrt{\varepsilon_r - \sin^2\theta_0}},$$
$$k_{TM} = \frac{\sqrt{\varepsilon_r - \sin^2\theta_0}}{\varepsilon_r \cos\theta_0}.$$

Equations (4) and (6) are expressed in terms of C_i^{TE},

$$C_0^{TE} = \frac{\sigma}{2\varepsilon_0\left(\varepsilon_r - \sin^2\theta_0\right)}$$
$$C_1^{TE} = \frac{\sigma\cos\theta_0}{\varepsilon_0\left(\varepsilon_r - 1\right)\sqrt{\varepsilon_r - \sin^2\theta_0}},$$
$$C_2^{TE} = \frac{1-\varepsilon_r}{2\left(\varepsilon_r - \sin^2\theta_0\right)},$$
$$C_3^{TE} = \frac{c_{TE}^a}{C_1^{TE}\sum_{n=1}^{N}\left(C_2^{TE}\right)^n Q^{(n)}(0)},$$

where

$$c_{TE}^a = -\frac{\cos\theta_0}{\varepsilon_0\left(\cos\theta_0 + \sqrt{\varepsilon_r - \sin^2\theta_0}\right)^2\sqrt{\varepsilon_r - \sin^2\theta_0}}.$$

Finally, C_i^{TM} in equations (5) and (7) are,

$$C_0^{TM} = \frac{c_{TM}^b}{2}$$
$$C_1^{TM} = -4C_0^{TM}\frac{\left(c_{TM}^p + \frac{\sigma}{\varepsilon}\right)\sqrt{c_{TM}^d}}{\left(c_{TM}^p - c_{TM}^q\right)\left(c_{TM}^d - 1\right)}$$
$$C_2^{TM} = \frac{c_{TM}^b}{2c_{TM}^p}$$
$$C_3^{TE} = -4C_0^{TM}\frac{\left(c_{TM}^q + \frac{\sigma}{\varepsilon}\right)\sqrt{c_{TM}^d}}{\left(c_{TM}^q - c_{TM}^p\right)\left(c_{TM}^d - 1\right)}$$
$$C_4^{TM} = \frac{c_{TM}^b}{2c_{TM}^q}$$
$$C_5^{TM} = \frac{\sqrt{c_{TM}^d}}{\left(1+\sqrt{c_{TM}^d}\right)^2}\left(c_{TM}^b - 2\frac{\sigma}{\varepsilon}\right) - \sum_{n=1}^{N}\left[C_1^{TM}\left(C_2^{TM}\right)^n + C_3^{TM}\left(C_4^{TM}\right)^n\right]Q^{(n)}(0)$$
$$C_6^{TM} = \left|\frac{C_5^{TM}\left(1+\sqrt{c_{TM}^d}\right)^2}{\left(c_{TM}^b - 2\frac{\sigma}{\varepsilon}\right)\sqrt{c_{TM}^d}}\right|$$

where

$$c_{TM}^b = \frac{\sigma}{\varepsilon_0\left(\varepsilon_r - \sin^2\theta_0\right)}$$
$$c_{TM}^d = \frac{\left(\varepsilon_r\cos\theta_0\right)^2}{\varepsilon_r - \sin^2\theta_0},$$
$$c_{TM}^e = \frac{2\sigma c_{TM}^d - \varepsilon c_{TM}^b}{\varepsilon\left(1-c_{TM}^d\right)},$$
$$c_{TM}^f = \frac{\sigma^2 c_{TM}^d}{\varepsilon^2\left(c_{TM}^d - 1\right)},$$
$$c_{TM}^p = \frac{1}{2}\left(c_{TM}^b + \sqrt{\left(c_{TM}^b\right)^2 - 4c_{TM}^f}\right),$$
$$c_{TM}^p = \frac{1}{2}\left(c_{TM}^b - \sqrt{\left(c_{TM}^b\right)^2 - 4c_{TM}^f}\right).$$

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