An Improved Time-Domain Near-Field to Far-Field Transform in Two Dimensions

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Abstract – Computing the far-field transient response of a two-dimensional geometry requires a convolution of near-field currents with a two-dimensional far-field impulse response. In this work, a purely time domain implementation is derived and its accuracy is demonstrated. This method is applicable to EMI, radiation, and scattering problems.

I. INTRODUCTION

The finite difference time domain technique (FDTD) is a robust and proven technique for full-wave electromagnetic analysis of complex microwave, antenna, and scattering geometries. Often, the quantity of interest is not near-field quantities, which are directly computed by the time marching scheme. Instead, far-field radiation or scattering quantities are desired.

In order to compute these far-field quantities, various transform techniques have been successfully applied in both three dimensions [1-4] and two dimensions [5-6]. When angular patterns at a few discrete frequencies are of interest, a single frequency transform is appropriate and easily computed in either two or three dimensions using a running Fourier transform and proper weightings. Far-field patterns are then computed using a post processing operation.

When a broadband or transient far-field result is desired at a few discrete angles, a purely time-domain near-field to far-field transform is appropriate. In three dimensions, this process is straight forward in theory, though implementation requires a bit of bookkeeping. In two dimensions, the time-domain near-field to far-field transform is complicated by the two-dimensional Green's function which is no longer of a simple exponential form.

In [6], a hybrid time/frequency domain approach is derived for the two-dimensional time-domain transform. In this hybrid method, post processing accomplishes the complex Green's function convolution. Specifically, a multi-step procedure consisting of a Fourier transform, a frequency domain multiplication, and finally an inverse Fourier transform produces the final time-domain result.

In this paper, a more elegant approach is formulated which is accomplished entirely in the time domain. Efficiency is gained by using an extremely accurate approximation of the Green's function's time-domain impulse response and subsequent application of an efficiently implemented convolution sum. The accuracy of the method is proven against known analytic solutions. Furthermore, the discrete impulse response derived in this work has applications in other FDTD applications where $1/\sqrt{j\omega}$ type impulse responses are present.

II. FREQUENCY DOMAIN FORMULATION

It is widely known that if a radiator or scatterer is surrounded by a closed surface S, the far-fields may be computed from vector magnetic and electric surface currents $\vec{\mathbf{M}}_s$ and $\vec{\mathbf{J}}_s$ defined along the enclosing surface by $-\hat{\mathbf{n}}\times\vec{\mathbf{E}}$ and $\hat{\mathbf{n}}\times\vec{\mathbf{H}}$, respectively. The fields are most easily defined in terms of the intermediate radiation vectors defined as follow,

$$\vec{\mathbf{N}} = \oint_S \vec{\mathbf{J}} e^{-jk\left(\vec{\mathbf{r}}'\cdot\vec{\mathbf{r}}\right)} ds , \tag{1}$$

$$\vec{\mathbf{L}} = \oint_S \vec{\mathbf{M}} e^{-jk\left(\vec{\mathbf{r}}'\cdot\vec{\mathbf{r}}\right)} ds \tag{2}$$

where $\vec{\mathbf{r}}'$ is the vector from the origin to the surface current, and $\vec{\mathbf{r}}$ is the unit vector to the far-field point.

The vector components of the electric field in the far-field are then given by,

$$E_\theta^{3D} = \frac{-j}{2\lambda R} e^{-jkR}\left(\eta N_\theta + L_\phi\right), \tag{3}$$

$$E_\phi^{3D} = \frac{j}{2\lambda R} e^{-jkR}\left(-\eta N_\phi + L_\theta\right) \tag{4}$$

where λ is the wavelength, k is the wave number $\omega\sqrt{\mu\varepsilon}$, and η is the free-space wave impedance.

III. HYBRID TIME 2D FORMULATION

In a three-dimensional FDTD code the implementation of equations (3) and (4) is typically accomplished by introducing the temporary summation variables $\vec{W}, \vec{U}$ defined as,

$$\vec{\mathbf{W}}^{3D} = je^{-jkR}\,\vec{\mathbf{N}}\big/2\lambda R \tag{5}$$

$$\vec{\mathbf{U}}^{3D} = je^{-jkR}\,\vec{\mathbf{L}}\big/2\lambda R\,. \tag{6}$$

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Applying the Laplace transform, these become

$$\vec{\mathbf{W}}^{3D}(t)=\left(\frac{1}{4\pi Rc}\right)\frac{\partial}{\partial t}\left\{\int_S \vec{\mathbf{J}}\left[t+\left(\vec{r}^{\,'}\bullet\hat{r}\right)\Big/c-R\big/c\right]ds'\right\},(7)$$

$$\vec{\mathbf{U}}^{3D}(t)=\left(\frac{1}{4\pi Rc}\right)\frac{\partial}{\partial t}\left\{\int_S \vec{\mathbf{M}}\left[t+\left(\vec{r}^{\,'}\bullet\hat{r}\right)\Big/c-R\big/c\right]ds'\right\}\quad(8)$$

$\vec{\mathbf{W}}$ and $\vec{\mathbf{U}}$ result in a discrete binning operation whereas contributions from each surface patch on the far-field transform surface is time-delayed and added appropriately to the appropriate time bin. At the end of the FDTD computation, the far-field components of the electric field are then simply given as,

$$E_\theta^{3D}=-\eta W_\theta^{3D}-U_\phi^{3D}\qquad(9)$$

$$E_\phi^{3D}=-\eta W_\phi^{3D}+U_\theta^{3D}\,.\qquad(10)$$

In [6] it was shown that a simple relation exists between equations (9), (10), and their two dimensional counterparts,

$$\vec{E}^{2D}(\omega)=\sqrt{\frac{2\pi c\rho}{j\omega}}\vec{E}^{3D}(\omega)\quad.\qquad(11)$$

The implementation of equation (11) in conjunction with equations (9) and (10) was accomplished in [6] as a three-step post processing operation. Specifically, once the far-field components $E_z(t)$ and $E_\phi(t)$ are computed for all times of interest, a Fourier transform is applied to the time waveforms rendering a discrete frequency spectrum for the fields. Next, equation (11) is applied to each frequency component of interest. If a frequency spectrum alone is needed, no further processing is necessary. However, if the time-domain far-field is the quantity of interest, equation (11) must be applied to the entire frequency spectrum of the signal. Subsequently, the weighted frequency spectrum is transformed back to the time domain using the inverse Fourier transform.

IV. A FULLY TIME-DOMAIN FORMULATION

While the hybrid approach presented in the last section is simple and effective, a fully time-domain approach can be beneficial, particularly if the time-domain far-field is of interest. To this end, consider the time-domain representation of equation (11),

$$\begin{aligned}E_z^{2D}(t)&=\int_0^T\sqrt{\frac{2c\rho}{\tau}}E_\theta^{3D}(t-\tau)\,d\tau\\ E_\phi^{2D}(t)&=\int_0^T\sqrt{\frac{2c\rho}{\tau}}E_\phi^{3D}(t-\tau)\,d\tau\end{aligned}\quad.\qquad(12)$$

These convolutions are quite costly in the present form which of course is the reason this form has been avoided. To minimize this cost, the impulse response of $1/\sqrt{\tau}$ is required in a form amenable to a more efficient implementation. Specifically, an accurate exponential representation of the discrete impulse response is needed. In [2], Prony's method was used to approximate this time domain discrete impulse response. However, in the course of the present work, it was found that this published expansion was not adequate. Therefore, a new expansion was generated using the TLS (total least squares) Prony method [7-8]. Using this approximation, equation (12) is accomplished in discrete form as,

$$E_{z,\phi}^{(n+1)2D}=2\sqrt{2\Delta_t c_0}\,E_{\theta,\phi}^{(n+1)3D}+\sum_{i=1}^{10}\psi_i^{(n+1)}\qquad(13)$$

where

$$\psi_i^{(n+1)}=e^{\omega_i}\psi_i^{(n)}+a_iE_{\theta,\phi}^{(n)3D},\qquad(14)$$

$$a_i=\sqrt{2\Delta_t c_0}\,C_i\qquad(15)$$

the variables C_i and ω_i are defined in Table 1 and Δ_t is the time step used in the FDTD computation

Equation (13) is applicable to each field component with the summation variable of equation (14) is executed in tandem. Note that this operation is accomplished after the simulation has been completed and does not add additional cost to the FDTD time stepping algorithm.

Table 1. Time domain approximate expansion coefficients for the discrete impulse response.

C_i	w_i
2.8127012947e-002	-1.5342833368e-004
3.0430608890e-002	-1.4566027270e-003
3.6162010199e-002	-4.5530898790e-003
4.7669826308e-002	-1.0919385763e-002
6.7935155570e-002	-2.4433346192e-002
9.8433658984e-002	-5.4569291616e-002
1.3696192243e-001	-1.2389528094e-001
1.6810710585e-001	-2.8723727545e-001
1.5152191381e-001	-6.8432281745e-001
6.3077757874e-002	-1.7514471961e+000

The accuracy of this technique is directly impacted by the accuracy of this rational approximation. This accuracy is demonstrated in Fig. 1 where the impulse response for the time domain expression $1/\sqrt{\tau}$ is demonstrated. This formulation is independent of time step. Note that the late time response (large *n*) is much improved over previously published approximations of the underlying discrete impulse response.

V. RESULTS

Initially, the accuracy of this method was proven by computing the monostatic scattering from a two-dimensional perfectly conducting cylinder with a radius of 0.25 meters. The FDTD problem size was 500 by 500 cells and the cell size was 1.25 mm. These results were validated against a Mie series solution for the geometry. The accuracy of the method in this paper is demonstrated in Figs. 2 and 3. Note that the new formulation and the previous combined frequency/time domain method give near identical results. Figure 4 demonstrates the accuracy of the method across the entire bistatic angular sweep at 300 MHz for the TM_z polarization. Again the agreement is excellent.

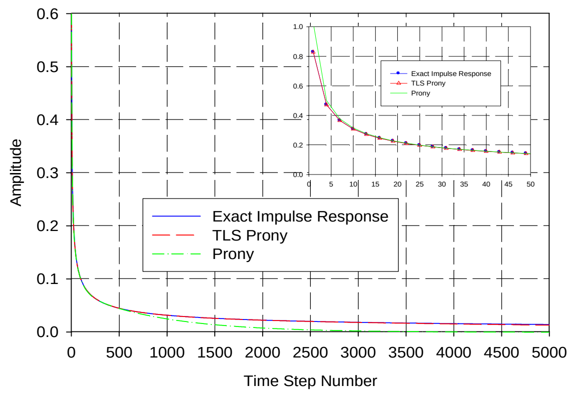

Fig. 1. Discrete impulse response for far-field convolution.

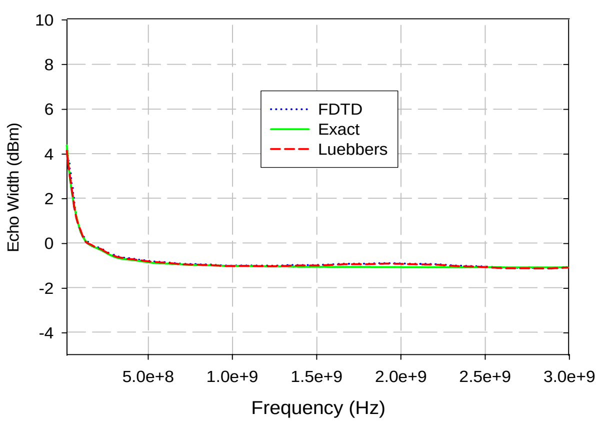

Fig. 2. TM_z monostatic backscatter for 0.25 meter PEC cylinder.

Finally, the far-field radiation from an infinitely thin wire was computed. This geometry has an analytic solution also which is well known. Figure 5 demonstrates the accuracy of the present method for this practical geometry.

VI. CONCLUSION

In this paper, an efficient and compact method for computing the two-dimensional far-fields from an FDTD simulation was presented. The accuracy was demonstrated for a simple canonical test case. The coefficients generated for the time-domain solution of this problem are also applicable to commonly used high frequency surface impedance formulations and provide an accurate alternative to previously published approximations.

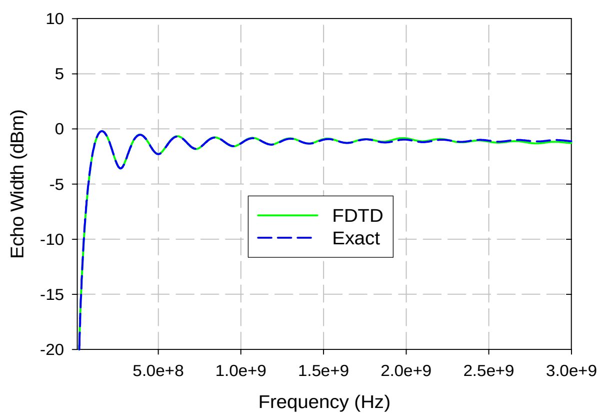

Fig. 3. TE_z monostatic backscatter for 0.25 meter PEC cylinder.

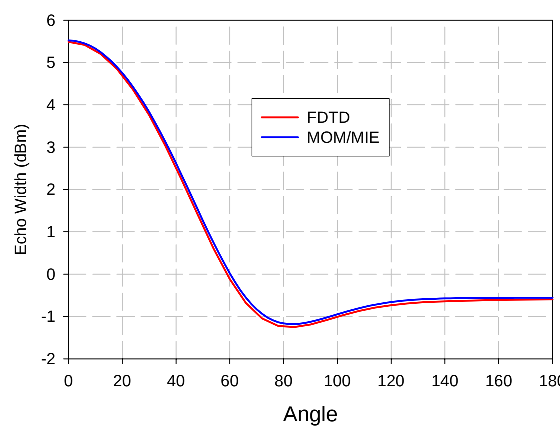

Fig. 4. Bistatic TM_z scattering from a PEC cylinder.

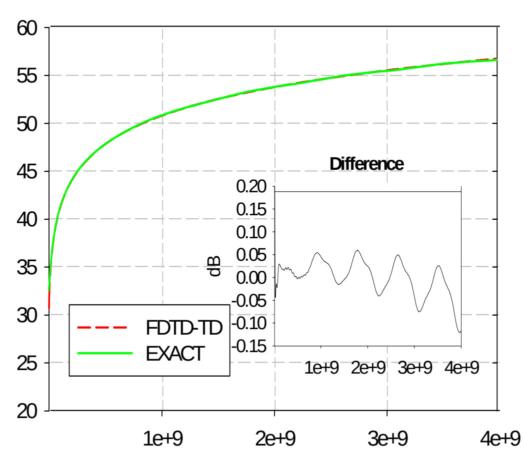

Fig. 5. Far-field of a long thin wire.

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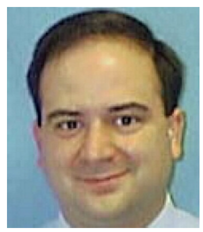

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