THE USE OF SURFACE IMPEDANCE BOUNDARY CONDITIONS IN TIME DOMAIN PROBLEMS: NUMERICAL AND EXPERIMENTAL VALIDATION

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Abstract: This paper analyzes the limits of applicability of the time domain surface impedance concept. Numerical results obtained by the boundary element formulation employing time domain surface impedance boundary conditions (SIBCs) of different orders of approximation are compared with experimental data and numerical results obtained using the finite element method. An analytical formula for evaluation of the error due to application of the various SIBCs is proposed.

Keywords: Time Domain Solution, Surface Impedance Boundary Conditions, BEM method.

1. Introduction

Transient analysis of skin effect eddy current problems is of significant interest in practice. There are two basic approaches to solve transient problems: (1) by obtaining the solution in the frequency domain for the time-harmonic exciting source and using inverse Fourier transform techniques to calculate the required transient data and (2) by formulating the problem directly in the time domain. In [1, 2] the arguments in favor of the second method are discussed. However, time domain techniques remain computationally expensive in most cases. The problem is simplified if the electromagnetic penetration depth in the conducting body is so short that the variation of the field in the direction tangential to the body's surface is much less than the field variation in the normal direction, so that the complete equation of the electromagnetic field diffusion into the body can be replaced by a one-dimensional equation in the direction normal to the surface of the body. The solution of the reduced equation can be then used to derive the so-called surface impedance boundary conditions (SIBC) involving only the external fields imposed at the outer surface to simulate the material properties of the body and thereby to convert a two (or more) media problem into a one medium problem.

Existence of such conditions follows directly from Snell's law of refraction: if the electromagnetic wave propagates from a low-conductive medium to a high-conductive medium, the reflection angle is about 90 degrees and it practically does not depend on the incident angle. Suppose the conducting region is so large that the wave attenuates completely inside the region. Then the electromagnetic field distribution in the conductor's skin layer can be described as a damped plane wave propagating into the depth of the conductor, normal to its surface. In other words, the behavior of the electromagnetic field in the conducting region may be assumed known *a priori*. The electromagnetic field is continuous across the real conductor's surface so the intrinsic impedance of the wave remains the same at the interface. Therefore, the ratio E_x/H_y at the *xy*-plane of the dielectric/conductor interface is assumed to be equal to the intrinsic impedance of the plane wave propagating in the homogeneous conducting body in the positive *z*-direction:

$$\left.\frac{E_x}{H_y}\right|_{\text{interface}} = Z_\omega = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} \approx \left.\frac{1+j}{2}\mu\omega\delta\right|_{\sigma>>\omega\varepsilon};$$
$$\delta = \sqrt{\frac{2}{\omega\sigma\mu}} \tag{1}$$

where ω is the angular frequency of the field source, δ is the electromagnetic penetration depth, and σ, ε and μ are the electrical conductivity, permittivity and magnetic permeability of the body, respectively.

The SIBC for planar surfaces can be applied as long as the smallest radius of curvature of the surface is much larger than the wavelength inside the conductor. Leontovich developed the SIBC with a first order correction term that accounted for the curvature of the interface [3]. However, usually only the simplified form (1) of his condition is quoted so the SIBC for the planar surface is also called Leontovich's condition. A further correction has been introduced by Mitzner [4], who developed the conditions, now known by his name, for any smooth surface of a conducting body. The Mitzner's SIBC is written in the form:

$$\left.\frac{E_x}{H_y}\right|_{\text{interface}} = \frac{1+j}{2}\mu\omega\delta\left[1+\frac{1-j}{4}\delta\left(d_y^{-1}-d_x^{-1}\right)\right] \quad (2)$$

where d_x and d_y are the local radii of curvature of the coordinate lines. More information about origins of the surface impedance concept can be found in [5-7].

Note that the condition (2) includes the term containing δ^2 whereas the condition (1) contains δ only. It is natural to expect that the SIBC of the approximation order exceeding the order of the Mitzner's approximation should include terms containing δ^3 and higher. The way to obtain these terms was suggested by Rytov [8] more than sixty years ago. He applied the perturbation method and used the following time-harmonic solution of the one-dimensional equation of the magnetic field diffusion into a perfect conductor as an initial approximation

$$H_y(z) = \left.H_y\right|_{z=0}\exp(-z/\delta)$$

where $\left.H_y\right|_{z=0}$ is the tangential magnetic field at the surface of the body. By substituting the solution into Maxwell's equations for the conducting region, Rytov derived the boundary conditions at the planar surface of a highly conducting body in the following form of asymptotic expansions in the skin depth taken as a small parameter

$$\left.E_x\right|_{\text{interface}} = \frac{1+j}{2}\mu\omega\delta\cdot$$
$$\cdot\left.\left[H_y+\frac{\delta^2}{4j}\left(-\frac{\partial^2 H_y}{\partial x^2}+\frac{\partial^2 H_y}{\partial y^2}+2\frac{\partial^2 H_x}{\partial x\partial y}\right)+\ldots\right]\right|_{\text{interface}} . \quad (3)$$

The main advantage of the expression in (3) is that the variation of the magnetic field in the direction *tangential* to the body surface is taken into account under the concept of the surface impedance based on the solution of the reduced 1-D problem in the direction *normal* to the body surface. The generality of the condition (3) is not appreciated since only the SIBC of lower order of approximation were used until recently.

The SIBC concepts can also be used in transient problems, when, for instance, the duration of the incident pulse is so short that the field has no time to diffuse deeply into the body and remains concentrated in the thin layer near the body surface. The simplest SIBC in the time domain is obtained directly from (1) by using the inverse Laplace transformation and written in the form of the convolution with respect to time:

$$\left.E_x\right|_{\text{interface}} = -\sqrt{\mu/(4\pi\sigma t^3)} * \left.H_y\right|_{\text{interface}} \quad (4)$$

Although condition (4) is mostly applied in analysis of high-frequency problems using the finite difference time domain method [9-11], it was also used in combination with the finite element method [12] and the boundary integral equation method [13-15].

Following the perturbation approach proposed by Rytov, the following time domain SIBC of high order of approximation has been developed [16]:

$$\left.E_x\right|_{\text{interface}} = \left\{\hat{T}_1 * H_x + \frac{d_x-d_y}{2d_x d_y}\hat{T}_2 * H_x + \frac{3d_x^2-d_y^2-2d_x d_y}{8d_x^2 d_y^2}\hat{T}_3 * H_x\right.$$
$$\left.\left.+\frac{\hat{T}_3}{2} * \left(-\frac{\partial^2 H_y}{\partial x^2}+\frac{\partial^2 H_y}{\partial y^2}+2\frac{\partial^2 H_x}{\partial x\partial y}\right)\right\}\right|_{\text{interface}} . \quad (5a)$$

Here * denotes a time domain convolution product $U(t)$ is the unit step function and time-dependent functions $\hat{T}_k$ are defined as follows:

$$\hat{T}_1(t) = -(4\pi\sigma/\mu)^{-1/2}t^{-3/2}\,;\ \hat{T}_2(t) = U'(t)/\sigma$$
$$\hat{T}_3(t) = \left(\pi\sigma^3\mu\right)^{-1/2}t^{-1/2} \quad (5b)$$

where $U'(t)$ is the delta function.

In some cases it is more convenient to use another SIBC relating normal and tangential components of the magnetic field on the conductor's surface:

$$\left.H_z\right|_{\text{interface}} = \left\{T_1 * \left(\frac{\partial H_x}{\partial x}+\frac{\partial H_y}{\partial y}\right)+\frac{d_y-d_x}{2d_x d_y}T_2 * \left(\frac{\partial H_x}{\partial x}-\frac{\partial H_y}{\partial y}\right)+\right.$$
$$T_3 * \left(\frac{3d_y^2-d_x^2-2d_x d_y}{8d_x^2 d_y^2}\frac{\partial H_x}{\partial x}+\frac{3d_x^2-d_y^2-2d_x d_y}{8d_x^2 d_y^2}\frac{\partial H_y}{\partial y}\right)$$
$$\left.\frac{T_3}{2} * \left(-\frac{\partial^2 H_x}{\partial x\partial y^2}-\frac{\partial^2 H_y}{\partial y\partial x^2}+\frac{\partial^3 H_x}{\partial x^3}+\frac{\partial^3 H_y}{\partial y^3}+2\frac{\partial^3 H_x}{\partial x\partial y^2}+2\frac{\partial^3 H_y}{\partial y\partial x^2}\right)\right\} \quad (6a)$$

$$T_1(t) = (\pi\sigma\mu)^{-1/2}t^{-1/2},\ T_2(t) = U(t)/(\sigma\mu),$$
$$T_3 = \left(\pi\sigma^3\mu^3\right)^{-1/2}t^{1/2}. \quad (6b)$$

Although conditions (5) and (6) allow for such effects as curvature of the surface and variation of the field in the tangential direction, the SIBC (4) of lowest (Leontovich's) order of approximation only has been used until now in the time domain calculations (current situation in frequency domain analysis is better:

Mitzner's SIBC (2) is widely used). The matter has uncertain limits of applicability of the surface impedance concept. Indeed, under definition the surface impedance boundary conditions can be used when the skin depth δ is much less than characteristic size D of the conductor's surface:

$$\delta << D\,; \quad D = \min\left(d_x, d_y, R_{source}, R_{cond}\right) \tag{7}$$

where R_{source} and R_{cond} are the distances to the source and or neighboring conductor (if the system of conductors is considered), respectively. Condition (7) is usually used to check applicability of the concept. But it does give us neither an approximation error due to application of SIBC nor the rule which SIBC (for example, (4) or (5)) should be used in a given problem. In addition, SIBCs have been originally derived for smooth surfaces whereas real geometries include corners and edges. Although rigorous and practical technique to extend the concept to this kind of problems has not been developed so far, in practice, SIBCs are frequently applied to all kinds of bodies supposing that the errors due to singularities near edges are local. The situation is worse in the time domain due to lack of accurate mathematical definition for the skin depth in the transient case. Thus detailed validation of the SIBCs is of great importance for the concept. This problem has been frequently considered in the past [17-19], but almost all reported works are focused on the frequency domain SIBCs of low order of approximation. According to our knowledge, time domain SIBCs of high orders have not been validated using experimental methods so far. In the present paper limits of applicability of the low-frequency high order time domain SIBCs for homogeneous conductors are investigated by using experimental and numerical techniques.

2. Statement of Transient Problem

Consider a pair of identical long parallel aluminum ($\sigma = 3.82\times10^{7}\ (\Omega m)^{-1}$) conductors of circular cross section of the radius D equal to 30 mm. Distance between centers of the conductors is equal to 120 mm. Conductors are connected in series and the circuit is fed by a dc voltage source that provides equal and oppositely directed currents $I_1(t)$ and $I_2(t)$ flowing through the conductors:

$$I_1(t) = -I_2(t)\,. \tag{8}$$

The duration of the source current has been chosen so that

$$\tau << \sigma\mu D^2 \tag{9}$$

where τ is the pulse duration. Clearly, (9) is time domain analog of (7).

3. Boundary Element Formulation Employing Time Domain SIBC

Presence of condition (9) enables the surface impedance concept being applied. It is natural to consider the problem as two-dimensional in the plane of cross sections of the conductors. The magnetic scalar potential in free space can be introduced as follows:

$$\vec{H} = \vec{H}^{\,fil} - \nabla\phi \tag{10}$$

where $\vec{H}^{\,fil} = \sum_{i=1}^{N}\vec{H}_i^{fil}$ and $\vec{H}_i^{fil}$ is the magnetic field created by a filamentary conductor carrying the current I_i and placed at position $\vec{r}_i$ and it is expressed by the Biot-Savart law as

$$\left|\vec{H}_i^{fil}(\vec{r},t)\right| = \frac{I_i(t)}{2\pi\left|\vec{r}-\vec{r}_i\right|}\,. \tag{11}$$

Hence in free space the governing equation is

$$\nabla^2\phi = 0 \tag{12}$$

and application of the boundary element method yields the following set of integral equations over the contours of cross sections of conductors:

$$\frac{1}{2}\phi + \sum_{i=1}^{N}\int_{L_i}\phi\frac{\partial G}{\partial \vec{n}'}ds' = \sum_{i=1}^{N}\int_{L_i}G\frac{\partial\phi}{\partial\vec{n}'}ds' \tag{13}$$

$$\frac{1}{2}\vec{n}\times\vec{H} + \vec{n}\times\sum_{i=1}^{N}\oint_{L_i}\int_0^t\vec{H}\frac{\partial T}{\partial\vec{n}'}dt'\,ds' = \vec{n}\times\sum_{i=1}^{N}\oint_{L_i}\int_0^t K\frac{\partial\vec{H}}{\partial\vec{n}'}dt'\,ds' \tag{14}$$

$$\frac{1}{2}\vec{n}\cdot\vec{H} + \vec{n}\cdot\sum_{i=1}^{N}\oint_{L_i}\int_0^t\vec{H}\frac{\partial T}{\partial\vec{n}'}dt'\,ds' = \vec{n}\cdot\sum_{i=1}^{N}\oint_{L_i}\int_0^t K\frac{\partial\vec{H}}{\partial\vec{n}'}dt'\,ds' \tag{15}$$

$$\partial\phi/\partial\vec{n} = \vec{n}\cdot\left(\vec{H}^{\,fil} - \vec{H}\right). \tag{16}$$

Here L_i is the contour of the cross section of the conductor *i*, *s=s(x,y)* is the coordinate directed along the contour of the conductor's cross section, $\vec{n}$ is the normal unit vector directed inside a conductor, c is the coefficient depending on the shape of the contour. G and K are the fundamental solutions of the two-dimensional Laplace and diffusion equations, respectively [20]:

$$G(s,s') = \frac{1}{2\pi}\ln\left(\frac{1}{|\vec{r}(s)-\vec{r}(s')|}\right)$$

$$K(s,s') = \frac{(\sigma\mu)^{1/2}}{4\pi(t_F - t)}\exp\left[-\frac{|\vec{r}(s)-\vec{r}(s')|^2(\sigma\mu)^{1/2}}{4(t_F - t)}\right] \quad (17)$$

with t_F the final time of analysis.

Solution of (13)-(16) yields distributions of ϕ, $(\vec{n}\times\vec{H})$, $(\vec{n}\cdot\vec{H})$ and $\partial(\vec{n}\times\vec{H})/\partial\vec{n}$ over the contour of the conductor's cross sections. Hence, for our problem, *ne* being the number of elements in which the contour of each conductor is discretized, the system to be solved is a square system of dimension 5·2*ne*. However, the number of unknowns can be reduced by application of SIBC (6) that in our 2-D case is written in the form:

$$H_n = \frac{\partial H_s}{\partial s} * T_1 + \frac{1}{2D}\frac{\partial H_s}{\partial s} * T_2 + \left(\frac{3}{8D^2}\frac{\partial H_s}{\partial s^3} + \frac{1}{2}\frac{\partial^3 H_s}{\partial s^3}\right) * T_3. \quad (18)$$

SIBC (18) can be used instead of integral equations (14)-(15) so the BEM-SIBC formulation consists of equations (13), (16) and (18) and can be solved with respect to ϕ, $(\vec{n}\times\vec{H})$, $(\vec{n}\cdot\vec{H})$. If *ne* is the total number of nodes, 3 linear systems of *ne* equations and *ne* unknowns must be solved in order to calculate the scalar potential over the nodes.

4. Experimental Setup

The experimental set-up is described in Fig. 1. The transient is obtained closing a circuit breaker so that the resulting source current is exponential (see Figure 2). Commercial magnetoresistive sensors (Philips KMZ10A) of nominal sensitivity S = 80 mV/(kA/m) lie over the conductors, as described in Fig. 1. Sensor No 1 is in position (-121 mm, 54.2 mm), sensor No. 2 is in position (-73 mm, 54.2 mm) and sensor No. 3 is in position (-25 mm, 54.6 mm). Measurement standard uncertainty of the positions has been estimated 0.5 mm. Sensors are oriented with their sensitivity axis parallel to *x*-axis. Six low drift, high accuracy instrument amplifiers (INA 128, Burr Brown) have been employed in order to process and amplify the signals generated by each magnetoresistive sensor. The output signals are sampled and acquired by an 8 channels data acquisition system, with 12-bit resolution and 32 ksamples/s rate for each channel

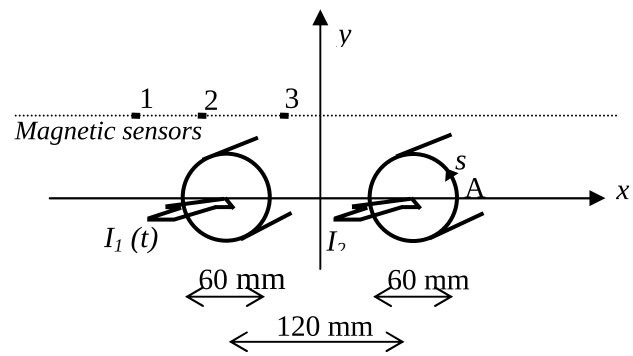

Fig. 1. Experimental set-up.

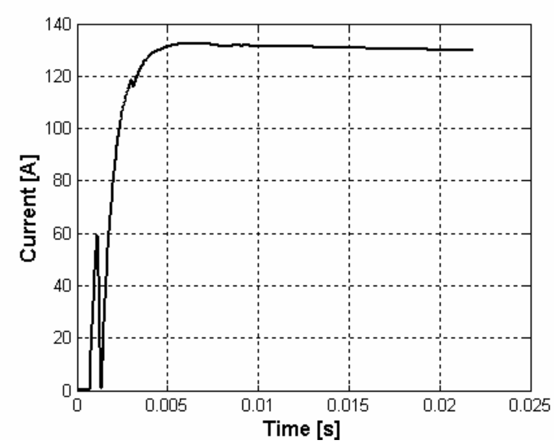

Fig. 2. Current waveform $I_1(t)$.

5. Comparison of Numerical and Experimental Results

In this section the experimental data together with numerical results obtained using BEM formulation employing SIBCs of different orders of approximation and commercial finite element software [21] are presented. Figures 3, 4 and 5 give the magnetic fields at the position of sensors 1, 2 and 3, respectively. Figures 6,7 and 8 report the difference between calculated and measured fields.

In Figures 9, 10 and 11 distributions of the tangential magnetic field over the surface of the conductor obtained using PEC, Leontovich's, Mitzner's and Rytov's boundary conditions are compared with data obtained using commercial FEM software. BEM code uses 80 nodes per conductor, numbered starting from A along *s*, and constant elements. From the results shown in Figures

3-11 it can be concluded that the SIBC formulation allows an efficient and accurate simulation of the test case. The hypothesis of perfect electric conductor gives definitely worse results. Increasing the order of the SIBC formulation, numerical results are closer and to the FEM solution and to the experimental measurements, considering uncertainty in the latter. However, it is unclear a priori, until which times BEM-SIBC formulation may be used. For this purpose an analytical formula giving approximate limit of applicability of the surface impedance concept is derived in the next Section. Note that the error (difference between results obtained using BEM-SIBC and FEM) is higher in Figures 9-11 than in Figures 3-8. It occurs because the field in free space has been calculated by performing integration of the scalar potential over the surface of the conductors that reduces computational error.

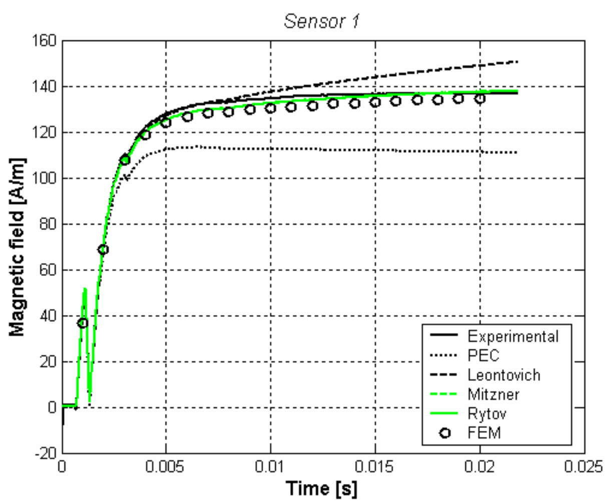

Fig. 3 The magnetic field near the sensor No. 1.

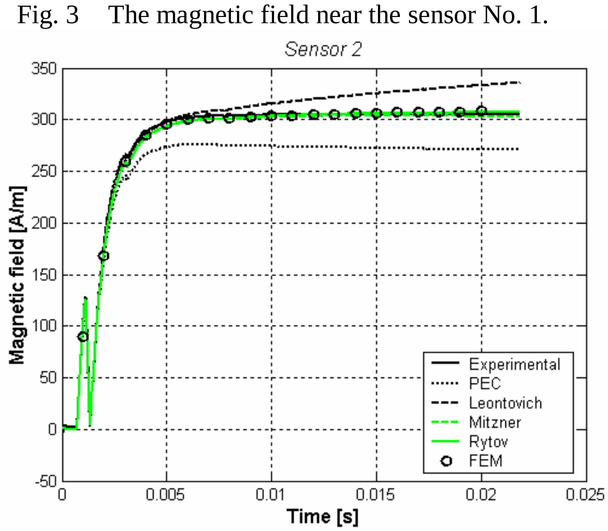

Fig. 4 The magnetic field near the sensor No. 2.

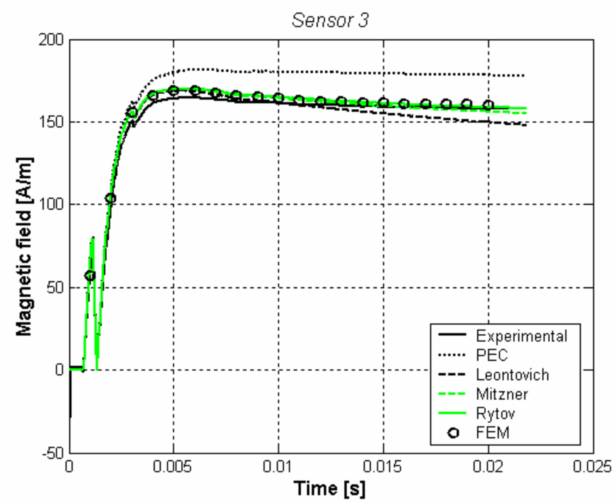

Fig. 5 The magnetic field near the sensor No. 3.

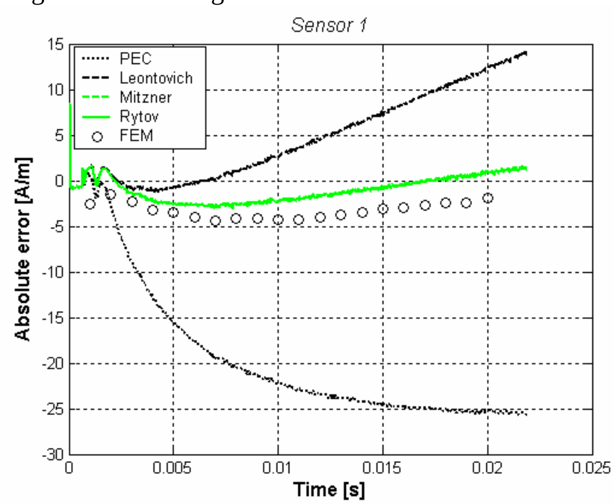

Fig. 6 Difference between computed and measured fields in the case of sensor No. 1.

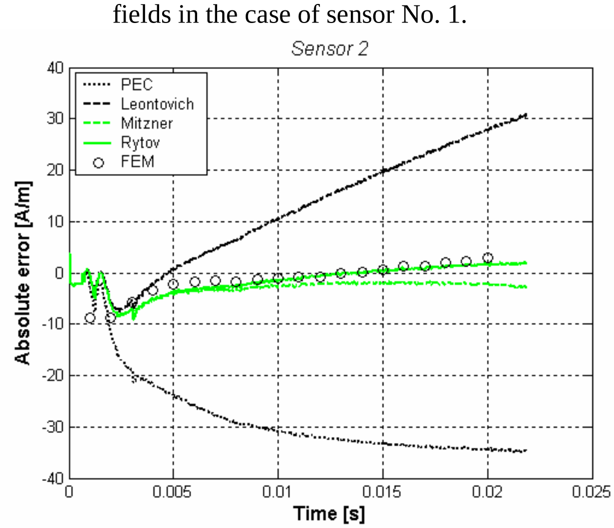

Fig. 7 Difference between computed and measured fields in the case of sensor No. 2.

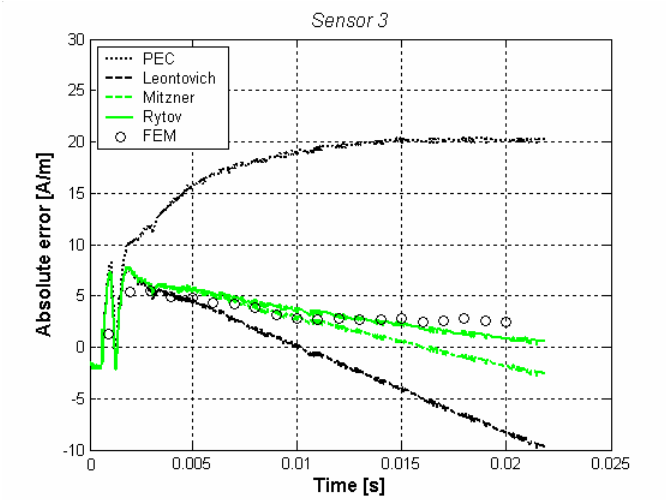

Fig. 8 Difference between computed and measured fields in the case of sensor No. 3.

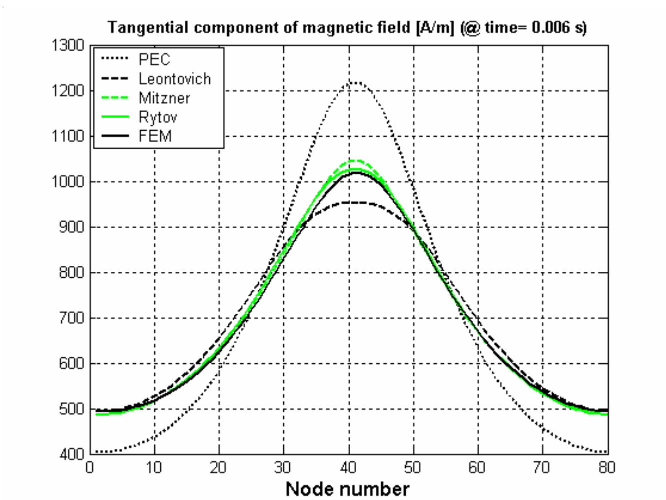

Fig. 9 Distribution of the tangential magnetic field over the conductor's surface at 0.006 s.

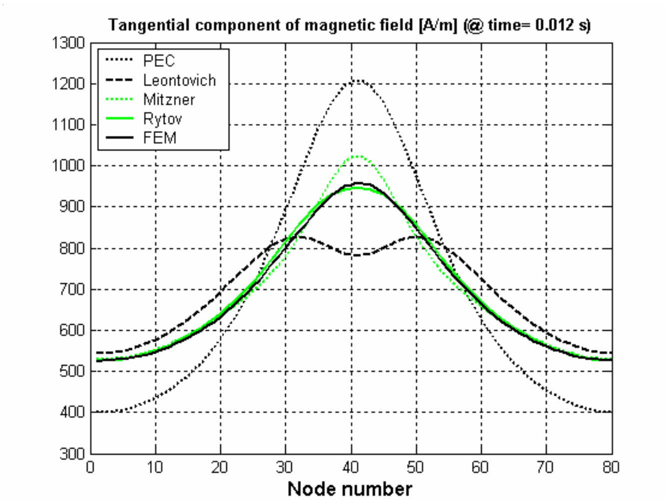

Fig. 10 Distribution of the tangential magnetic field over the conductor's surface at 0.012 s.

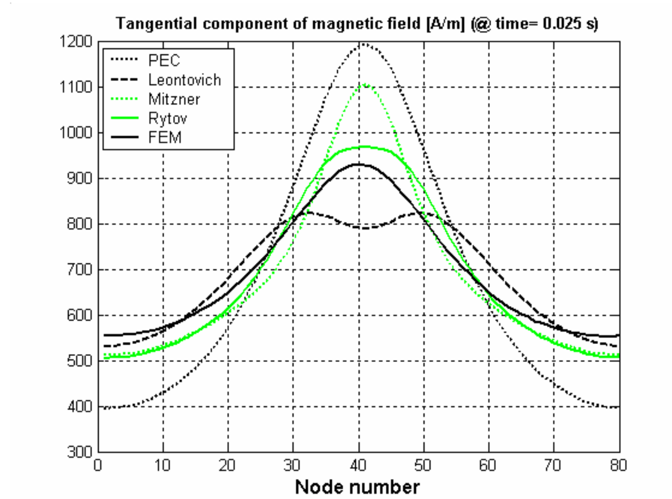

Fig. 11 Distribution of the tangential magnetic field over the conductor's surface at 0.025 s.

6. Conditions of Applicability

Since the surface impedance approach gives the solution in the form of asymptotic expansions, a natural question is limits of their applicability. Basic condition giving an error of approximation of the surface impedance boundary condition is derived from (9) and written in the form:

$$p^k = \left[\tau/(\sigma\mu D^2)\right]^{k/2} << 1 \qquad (19)$$

where values of k equal to *1,2,3* correspond to PEC-limit, Leontovich's SIBC, Mitzner's SIBC and Rytov's SIBC respectively. Small parameter p is combination of two values, τ and D. In our experimental setup the duration τ of the pulse may vary whereas the conductor's radius D is constant. Thus condition (19) can be represented in the form:

$$\varepsilon_k = \alpha^k \tau^{k/2}, \ \alpha = (\sigma\mu D^2)^{-1/2}, \quad k=1,2,3 \qquad (20)$$

where ε_k is the error of approximation k. Figure 10 shows distribution of the errors corresponding to PEC-limit, Leontovich's, Mitzner's and Rytov's approximations. For example, application of Leontoivh's SIBC for simulations with the pulse duration equal to 0.0065 s leads to the 10% error. Use of Rytov's SIBC allows to perform simulations for longer pulse of the duration equal to 0.021 s with the same error.

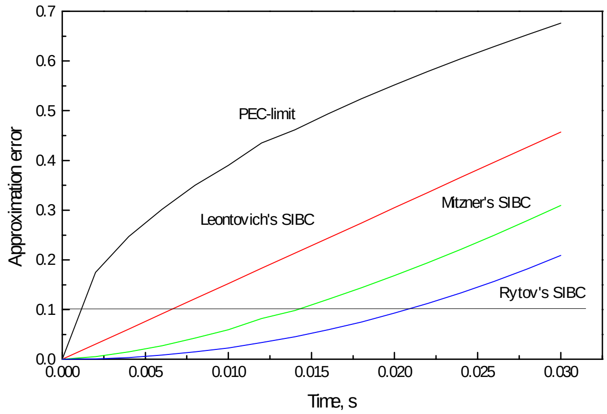

Fig. 12. The approximation error as a function of the pulse duration.

Note that the disagreement between results obtained using the SIBCs and measured data is actually less than the error predicted by the formula (20) since it does not take into account such effects as symmetry of the problem, shape of the pulse and the proximity effect. Nevertheless, (20) gives quick evaluation of the applicability of the surface impedance concept for a given problem and can be used for selection of the approximation order.

7. Conclusions

Experimental and numerical verification of the time domain surface impedance concept has been performed by simulation and measurement of the transient electromagnetic field around a system of two long parallel conductors with oppositely directed currents. The time domain surface impedance boundary conditions of different orders of approximation have been coupled with the boundary element code based on the fundamental solution in free space (the Laplace equation). The results have been compared with measured data and numerical results obtained using the boundary element code employing the fundamental solutions of the Laplace and diffusion equations. A formula for quick evaluation of applicability of the surface impedance concept for a given problem has been proposed and analyzed.

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