An Efficient Preconditioner (LESP) for Hybrid Matrices Arising in RF MEMS Switch Analysis

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Abstract **–** The small dimensions of Radio Frequency Micro-ElectroMechanical Switches (RF MEMS) raise significant modeling challenges in terms of accuracy and solver efficiency. This paper introduces a practical RF MEMS switch analysis based on an extended finite element-boundary integral (EFE-BI) method with an iterative solver incorporating a new sparse-matrix preconditioner whose large eigenvalues are very close to those of the original matrix. This sparse preconditioner is key to successfully solving the ill-conditioned EFE-BI matrix. The smaller condition number and almost positive-definite eigenvalue spectrum after preconditioning leads to fast convergence. Specific RF MEMS simulations are presented to demonstrate the accuracy and effectiveness of the methodology and solution process.

I. INTRODUCTION

RF MEMS switches have demonstrated low on-state insertion loss, high off-state isolation, and very linear behavior over a broad frequency range [1] and [2]. Despite their excellent characteristics, they generally suffer from low power-handling capability, with most switches operating well below 1W [2]. This limitation is due to the complex interactions among electromagnetic losses, heat transfer, and mechanical deformations of the switch. To better understand the associated failures, a multiphysics model was proposed in [3]. However, the work in [3] employed an approximate two-dimensional modeling of the RF current though the switch. As such it was not sufficiently rigorous in characterizing the edge current behavior which is critical for the heat dissipation process. Toward the goal of developing a more accurate and reliable analysis of RF MEMS, we proposed in [4] and [5] a more robust and efficient analysis method referred to as the extended finite element-boundary integral (EFE-BI) method.

Of importance in our EFE-BI analysis was the treatment of very small features associated with the MEMS switches. For example, at 2 GHz, the beam length corresponds to an electrical size of $\lambda/1500$ to $\lambda/250$ and a gap of $\lambda/150,000$ to $\lambda/50,000$. Because of these small features, the resulting hybrid matrix system is highly ill-conditioned and the matrix entries (viz. the integrals defining the matrix entries) are difficult to be accurately evaluated. Standard implementations of the finite element (FEM) and moment methods (MoM) employ integrations based on the Gaussian quadrature formulae for evaluating the matrix entries. However, for the small RF MEMS dimensions, these standard integral treatments were found to lead to ill-conditioned matrices with erratic changes in the output of the observable quantities. In [6] we proposed a set of semi-analytic evaluations of the matrix entries for the resulting EFE-BI hybrid system. However, a good preconditioner is still needed to ensure convergence, especially for frequencies below *X* band (10 GHz).

Many authors have explored preconditioning matrices for ill-conditioned matrix systems [7], [8], and [9]. Although the standard diagonal (DP) and block-diagonal preconditioners (BDP) can partially overcome convergence issues, they are still not reliable for RF MEMS modeling. In this paper, we present a highly efficient and reliable analysis of RF MEMS systems based on a new preconditioner referred to as the Large-Eigenvalue-Sparse Preconditioner (LESP). This preconditioner is implemented within the Generalized Minimal Residual iterative solver (GMRES) and is shown to significantly reduce the condition number and lead to almost positive-definite preconditioned matrix for RF MEMS switches. The reader is referred to [4], [6] and [10] for details related to the formulation of the EFE-BI and the element evaluations. Here, we focus only on the preconditioning approach and the relevant results. The reader is also referred to [9] and [11] for a review of iterative solvers and pre-conditioners. Other preconditioners for RF applications are mentioned in [7] and [12]. However, our particular application relates to the unique issue of RF MEMS switches where the entire geometry is $\lambda/250$ or less in size.

II. PRECONDITIONING OF THE HYBRID MATRIX SYSTEM

A simplified RF MEMS switch is illustrated in Fig.1. As it is well known, the RF MEMS switch beam experiences shape deformation during its dynamic operation. The conventional FE-BI [13] with rectangular gridding cannot track this deformation with sufficient geometrical accuracy. For this purpose in [4], we introduced an extended FE-BI analysis method (EFE-BI) for RF MEMS switches. The EFE-BI employs the moment method to model the beam and the usual FE-BI for the substrate and conducting sections on the boundary of the same substrate. As a result, the beam mesh is separated from the FE-BI section of the model. It can therefore be readily re-meshed as the beam curves. This approach allows for full flexibility in modeling the deformed 3D surfaces while reducing the computational expense. The typical EFE-BI matrix takes the form [4] and [6]

$$\begin{bmatrix} \mathbf{A}^{FEM} + \mathbf{A}^{S_1S_1} & \mathbf{A}^{S_1S_2} \\ \mathbf{A}^{S_2S_1} & \mathbf{A}^{S_2S_2} \end{bmatrix} \begin{Bmatrix} \mathbf{E}_n^V \\ \mathbf{J}_n^{S_2} \end{Bmatrix} = \begin{Bmatrix} \mathbf{b}_m^V \\ \mathbf{0} \end{Bmatrix} \tag{1}$$

where $\mathbf{A}^{FEM}$ and $\mathbf{A}^{S_1S_1}$ represent the FE-BI system for the fixed volume V_1 enclosed by S_1 as shown in Fig. 1. As usual, $\mathbf{A}^{FEM}$ is a very sparse submatrix whereas $\mathbf{A}^{S_1S_1}$ is dense. Similarly, $\mathbf{A}^{S_1S_2}$ and $\mathbf{A}^{S_2S_1}$ are the dense matrices representing the interaction between the beam and the BI enclosing the substrate, whereas $\mathbf{A}^{S_2S_2}$ is a dense submatrix representing the discrete method of moments system. The small sizes discussed above lead to near-zone integrals in the various submatrices of equation (1). These integrals can be efficiently evaluated using the semi-analytic integrations [6]. However, the resulting matrices are still ill-conditioned (Fig. 2).

Given the small number of unknowns due to the electrically small size of RF MEMS switch, GMRES (without restart) [8] and [11] is a good choice for solving equation (1). A description of the GMRES algorithm is given in [11] and [14]. We also note that available commercial software typically converges rather slowly or never at frequencies below ~50 GHz due to the extremely small MEMS dimension. This highlights the need for a preconditioner, but also points to the need for improved methods to carry out a reliable analysis of RF MEMS switches. The next paragraphs describe the construction of the proposed LESP. We then proceed to demonstrate the solution effectiveness of the entire EFE-BI approach for RF MEMS analysis.

It is well known that a good preconditioner is sparse and should have eigenvalues close to the larger ones of the original matrix. This approach generates a preconditioner that is a highly sparse matrix, but incorporates the critical elements of the original matrix. A preconditioner A_{LESP} can be applied to equation (1) as

$$\mathbf{A}_{LESP}^{-1} \begin{bmatrix} \mathbf{A}^{FEM} + \mathbf{A}^{S_1S_1} & \mathbf{A}^{S_1S_2} \\ \mathbf{A}^{S_2S_1} & \mathbf{A}^{S_2S_2} \end{bmatrix} \begin{Bmatrix} \mathbf{E}_n^V \\ \mathbf{J}_n^{S_2} \end{Bmatrix} = \mathbf{A}_{LESP}^{-1} \begin{Bmatrix} \mathbf{b}_m^V \\ \mathbf{0} \end{Bmatrix} \tag{2}$$

with

$$\mathbf{A}_{LESP} = \begin{bmatrix} \mathbf{A}^{FEM} + \left(\mathbf{A}^{S_1S_1}\right)_{NZ1} & \left(\mathbf{A}^{S_1S_2}\right)_{NZ12} \\ \left(\mathbf{A}^{S_2S_1}\right)_{NZ21} & \left(\mathbf{A}^{S_2S_2}\right)_{NZ22} \end{bmatrix}. \tag{3}$$

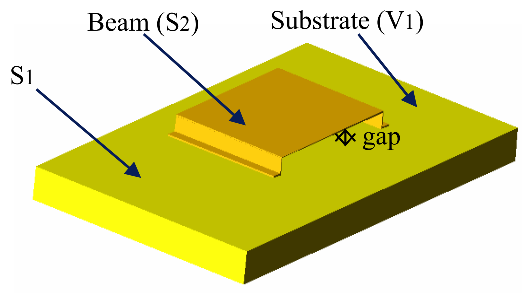


Fig. 1. RF-MEMS simplified model.

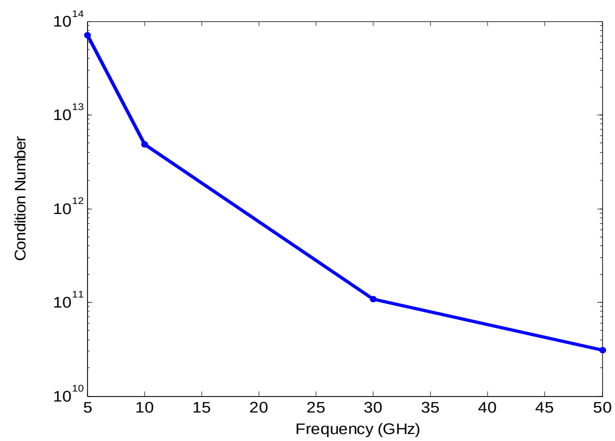


Fig. 2. Matrix condition number versus frequency (75*50*2 um).

In this, $\{\mathbf{A}^{S_1S_1}\}_{NZ1}$ contains an optimal number of the strongest coupling elements in each row of $\{\mathbf{A}^{S_1S_1}\}$. To actually generate $\{\mathbf{A}^{S_1S_1}\}_{NZ1}$, the matrix elements within each row of $\{\mathbf{A}^{S_1S_1}\}$ are sorted with respect to their modulus and the n_{NZ1} elements with the largest modulus are included to form the preconditioning matrix $\{\mathbf{A}^{S_1S_1}\}_{NZ1}$. Typically, most elements of $\{\mathbf{A}^{S_1S_1}\}_{NZ1}$ are located in a band around the main diagonal, but edge numbering can make some of the large elements distributed over the entire extent of the square matrix. A similar procedure is applied to submatrices $\mathbf{A}^{S_1S_2}$, $\mathbf{A}^{S_2S_1}$, and $\mathbf{A}^{S_2S_2}$. Unlike the conventional preconditioners, our approach includes the high modulus elements from the submatrices $\mathbf{A}^{S_1S_2}$ and $\mathbf{A}^{S_2S_1}$. For simplicity, in this paper, the same NZ from each row of the original matrix is selected to construct the preconditioner matrix, and an optimal NZ is found to achieve the best compromise between convergence versus CPU cost.

III. NUMERICAL APPLICATION

In this section, we present examples that demonstrate the efficiency of the LESP preconditioner. As a solver we used the general minimal residual algorithm (GMRES) with Krylov subspace methods [13] because it converges monotonically and (generally) gives the smallest residual errors among other Krylov subspace methods. The dimensions of the considered example are given in Fig. 3, and we note that the glass substrate was meshed using brick elements to reduce the number of unknowns. However, triangular surface (S2) elements were used to model the MEMS beam to accurately represent of the deformed beam surface. Beam thickness and conductivity were modeled using the resistive sheet model [13].

Figure 4 shows the construction of LESP. Specifically the original EFE-BI matrix is shown at the top of the figure with the corresponding preconditioner given at the bottom. We also remark that the elements in the beam are all in the near zone with respect to each other and are therefore strongly coupled. Thus, we found it necessary to include the entire BI matrix (marked in black in Fig. 4 (b)) to construct the preconditioner. This process was later found to ensure convergence in all cases.

A convergence rate comparison using different preconditioners with GMRES is shown in Fig. 5. We observe that the matrix condition number is very high (3.694×10^{10}) and therefore LESP preconditioner is needed to obtain fast convergence.

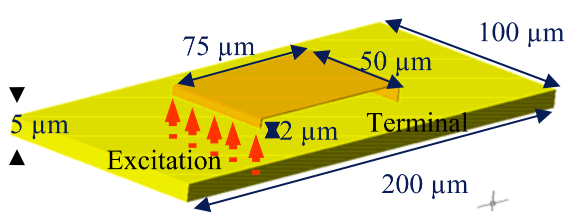
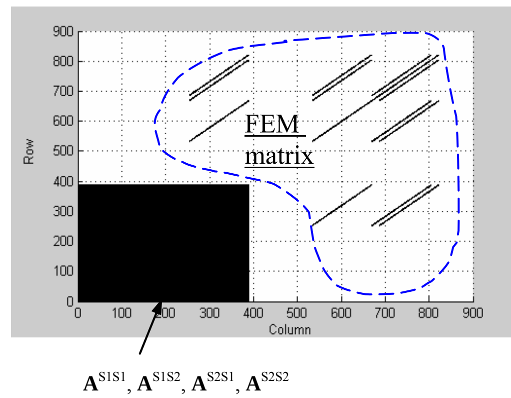


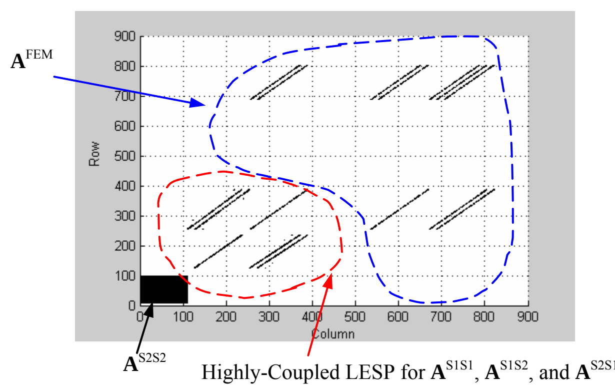
Fig. 3. RF-MEMS switch for our modeling.

From Fig. 5, it is seen that LESP leads to faster convergence as compared to the diagonal/block preconditioner. In addition, LESP has an optimized number of high-coupling terms which generate the best convergence (here NZ = 10 for the 50 GHz case). As can be expected, the value of NZ is dependent on the geometry. The mesh size and expansion function also affect the number of the near zone elements to be included in the preconditioner.

Figure 6 presents the convergence rate versus frequency. As seen, more iteration is needed to obtain the same convergence as the frequency is reduced. At the same time, the optimized NZ rises due to the much higher coupling among the matrix elements. It is also interesting to point out that the convergence rate is much better at the beginning of the iteration process. However, it reaches a relatively stable rate at lower frequencies. At higher frequencies, the convergence rate is slower at the start, but is more consistent and reaches the convergence criteria more quickly.



(a) Original EFE-BI matrix.



(b) Preconditioner.

Fig. 4. Profile of the EFE-BI and preconditioner matrices.

To better understand the preconditioner's influence on convergence, Fig. 7 shows the eigenvalue spectrum before and after preconditioning. Specifically, we show the spectrum when NZ = 1 (same as the diagonal preconditioner) and 15 (optimal) at 30 GHz. It is seen in Fig. 7 (a) that for NZ=15, most of the eigenvalues are closer to those of the original matrix. Nevertheless, of importance is that after preconditioning (Fig. 7 (b)): (1) the eigenvalue spectrum cluster becomes tighter and the convergence is faster since the condition number is proportional to the ratio of the maximum to minimum eigenvalues (as compared to the NZ = 1 case); (2) the preconditioned matrix with the optimized LESP leads to an almost all-real and positive eigenvalue spectrum (implying an almost positive-definite system).

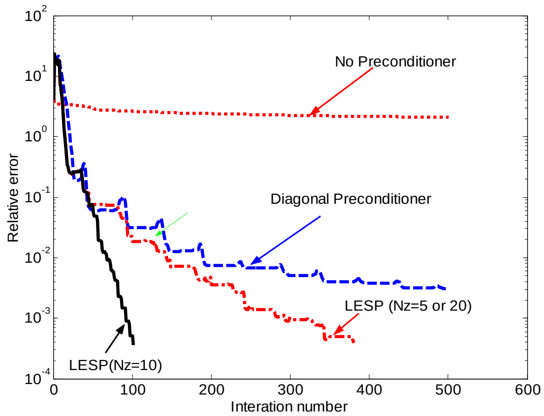


Fig. 5. Convergence versus iteration number for the preconditioned EFE-BI matrix.

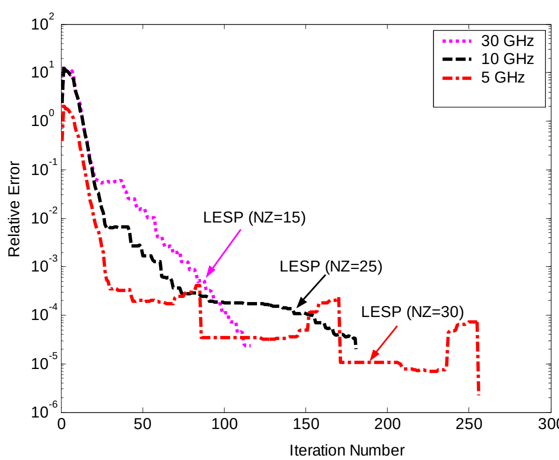
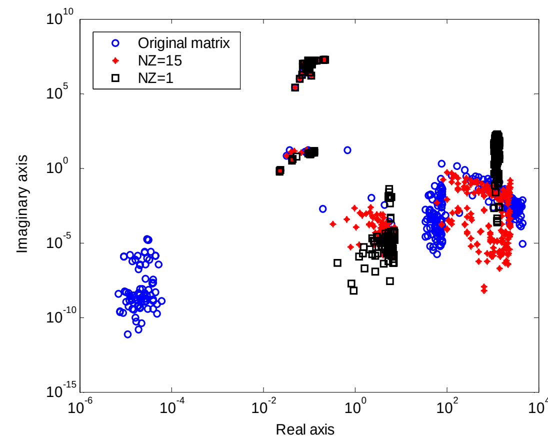


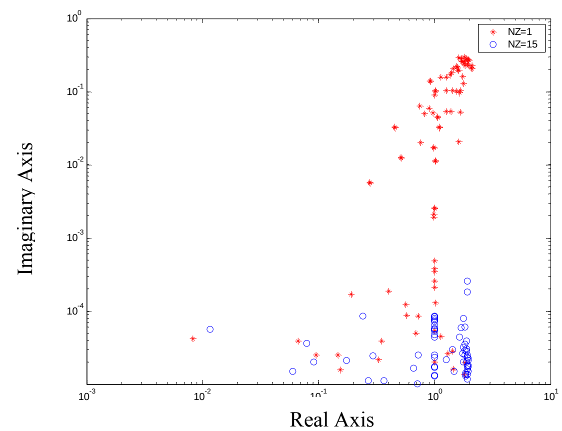
Fig. 6. Convergence versus frequency using an optimal number of non-zero rows (NZ is given in the parenthesis).

To compare the proposed LESP with the diagonal and block preconditioner, we repeated the example at 50 GHz (1241 unknowns) on an Intel Pentium-IV® [2-9]. It was found that at each iteration, LESP (NZ = 10) took 1.92 sec, whereas the diagonal preconditioner took about the same time of 1.914 sec. However, LESP (NZ = 10) was 4.2 times faster in reaching the normalized residual norm (set to 0.005) as compared to the diagonal preconditioner and 3 times faster as compared to the block preconditioner (NZ = 20) due to the fewer iterations. At the same time, the memory requirements were reduced dramatically since the needed storage per iteration rises linearly with the iteration count [15].

Using the preconditioner discussed above, we simulated the model in Fig. 3 at 5 GHz. The current is shown in Fig. 8. As seen, it compares well to the static approximation.



(a) Eigenvalues of the original and the preconditioning matrices with NZ = 1 and NZ = 15.



(b) Eigenvalues after preconditioning.

Fig.7. Eigenvalue spectrum distribution.

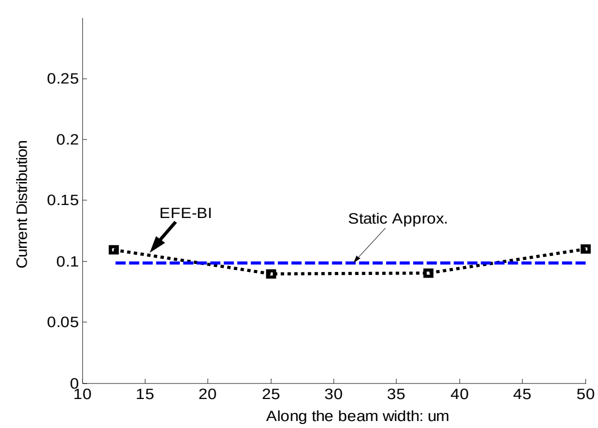


Fig. 8. Current density versus beam width (f = 5 GHz).

IV. CONCLUSION

The extremely small dimensions of RF MEMS switches inevitably lead to highly ill-conditioned matrix systems for RF analysis. Consequently, poor convergence is experienced when the RF MEMS switches are modeled via the conventional FE-BI method. In this paper, we presented a new preconditioner (LESP) to solve the matrix system generated via the extended FE-BI method. This new preconditioner preserves the matrix elements consisting of the largest eigenvalues associated with the original matrix. After preconditioning, the resulting system is almost positive-definite, implying fast and reliable convergence. Using the proposed preconditioner we were able to reliably predict the behavior of RF MEMS switches over a broad range of frequencies (500 MHz – 50 GHz).

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