Equivalent electric circuits approach for the modeling of non-linear electromagnetic fields

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***Abstract*- In this paper a method for the analysis of non-linear three dimensional electromagnetic field is presented.**

The conductive and magnetic regions of the examined system are subdivided in elementary volume elements in which a uniform current density J and magnetization M is assumed. By integrating Ohm's law inside the conductive regions, a set of equations representing the equilibrium equations of an equivalent electric network is obtained.

The knowledge of the currents in the conductive regions allows the evaluation of the electromagnetic fields and the determination of the forces among different bodies.

Applications of the method to the solution of benchmark problems of time varying linear systems, and non-linear static cases are presented.

I. Introduction

The historical concepts of the electromagnetic theory, characterized by the fields as the quantities that are physically significant, have been recently discussed by John Carpenter in a series of papers [1-3], where he investigated the consequences of a change of approach to electromagnetism. In his view the electric potential V and the magnetic vector potential A become the principal quantities respectively defining a measure of the potential and kinetic energy of a system of charges, while the field vectors E and B are no more than symbols denoting derivatives. Therefore, in this charge based approach, the energy density $w = (\rho V + J\cdot A)/2$ represents the kinetic energy and potential energy of the source charges, while in field theory it is considered as a mathematical equivalent. Even though the two approaches to electromagnetism lead to the same equations in terms of the potentials V and A, they differ substantially from the physical viewpoint. The essence of the V, A treatment is that there is no concern about how the actions are conveyed through space, since V and A quantify all possible interactions between any groups of charges. The potentials V and A at the considered frequencies, can be obtained from the source q's and J's :

$$V(r)=\frac{1}{4\pi\varepsilon}\int_{\Omega}\frac{\rho(r')}{|r-r'|}\,d\Omega(r');\quad A(r)=\frac{\mu}{4\pi}\int_{\Omega}\frac{J(r')}{|r-r'|}\,d\Omega(r') \quad (1)$$

where the charge density ρ and current density J in the integrals are due to the actual charge and current density distributions plus the a priori unknown distributions of the equivalent charges and currents due to the presence of dielectric and magnetic materials.

The approach makes no direct use of the concept of flux, although it provides convenient means of introducing it, therefore for several purposes the field-based and charge-based approaches are equivalent. Hence most of the numerical methods in terms of the potentials V and A can use the same basic equations. Nevertheless the integral formulation presented here can be considered as the logical outcome of the charge-based view of electromagnetism, and its inherent logic differs from other differential and integral methods relating to low frequency electromagnetic fields [4-7].

Since the sources are limited to the q's and J's, the proposed scheme makes no use of magnetic poles, thus the concept of magnetic circuit has no part in this model and the field equations are modelled by electric circuits only. Therefore the presented model has the advantage of a natural and easy linkage of circuit and field equations. Furthermore, it is not affected by numerical instabilities when it takes into account the relative motion among conducting bodies. As an integral formulation, it needs the modelling of conductive regions only, and do not require the specification of boundary conditions. Furthermore the proposed procedure has the characteristic of an easy data input for the definition of the arrangement of the ferromagnetic and non-ferromagnetic regions. By utilizing the symmetries of the examined system it is possible to reduce both the computational time and the required memory workspace.

II. Model

The whole volume Ω of conductive and magnetic regions is subdivided in N elementary volume elements, that can have several shapes (tetrahedrons, bricks, cylinder sectors), as shown in the 2-D decomposition of figure 1. Consequently, the vector potential A can be evaluated from the eq. (1), considering instantaneous propagation for the application of interest, adding the integrals relating to every elementary volume.

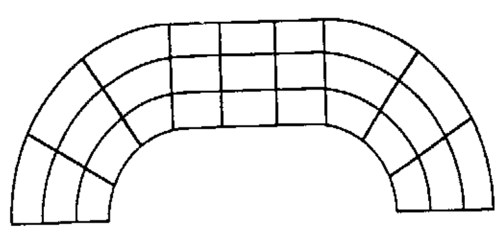

Fig. 1 First decomposition

Connecting the centres of nearby elements by means of segments parallel to the coordinate system unit vectors, we obtain a 3-D grid as shown in the figure 2.

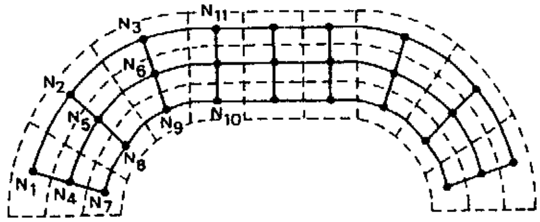

Fig. 2 Grid (solid lines)

Then we associate to every segment of the grid a new elementary volume element having four edges parallel to the segment, and the faces normal to the segment with their centres placed at nodes of the grid, see fig. 3. We assume that, inside every volume element, only the current density and magnetization component parallel to the segment associated to the volume element exists.

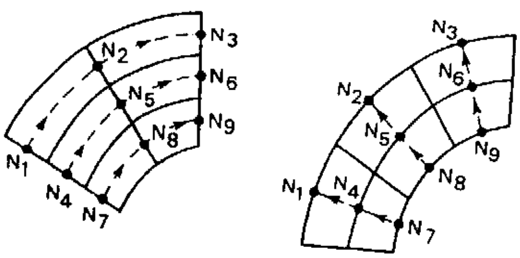

Fig. 3 Association between volume elements (solid lines) and segments (dashed lines)

The vector potential in the generic k-th element is:

$$A_k(t)=\frac{\mu_o}{4\pi}\left[\iiint_{V_s}\frac{J_s(t,x')}{|x_k-x'|}dV_s+\sum_{j=1}^{3N}\iiint_{V_j}\frac{J_{ij}(t,x')}{|x_k-x'|}dV_j\right]+\frac{\mu_o}{4\pi}\left[\sum_{j=1}^{N}\iint_{S_j}\frac{n_j\times M_j(t)}{|x_k-x'|}dS_j+\sum_{j=1}^{3N}\iiint_{V_j}\frac{J_{mj}(t,x')}{|x_k-x'|}dV_j\right] \quad (1a)$$

where V_j is the j-th elementary volume, J_s are the current sources, J_{ij} the induced currents, J_{mj} the volume magnetization currents, M_j the magnetization of the j-th volume element and n_j is the normal to the j-th element surface.

Assuming an uniform distribution of the magnetization M_j inside each volume, we have no volume current densities $J_{mj}=\mathrm{curl}(M_j)=0$. Furthermore assuming an uniform distribution of the current density $J_{ij}(t,x)=J_{ij}(t)$ and $J_s(t,x)=J_s(t)$ we obtain that the magnetic vector potential A, and consequently the flux density B are proportional to the currents I_{ij} and I_s and to the magnetizations M_j. In this way, we can derive the coefficients α, β and λ of eq. (2a), (2b), (2c) by means of analytical expressions [8,9] developed in previous works, thus obtaining a quick and accurate evaluation of the electric parameters of the equivalent circuit. The flux density $B(P,t)=\nabla\times A(P,t)$ or $B_k=\nabla\times A_k$ is equal to:

$$B_k=B_{mk}+B_{sk}+B_{ik}\ \ (2);\quad B_{mk}=\sum_{j=1}^{3N}\alpha_j\cdot M_j\ \ (2a);$$

$$B_{sk}=\sum_{j=1}^{R}\beta_j\cdot I_{sj}\ \ (2b);\quad B_{ik}=\sum_{j=1}^{3N}\lambda_j\cdot I_{ij}\ \ (2c).$$

The B_{sk} field, due to the current sources, is obtained by subdividing the current sources in R elementary elements (slabs or rings). The B_{ik} field ,due to the induced currents, is obtained by adding the contributions of all volume elements, while the B_{mk} field, due to the magnetizing currents, is obtained by taking the curl of the surface integrals, shown in eq. (1a), on the boundaries among every volume element. Considering isotropic materials, we can write the relations between the magnetic field H, the magnetic flux B and the magnetization M inside the material as:

$$H_k=H_{mk}+H_{sk}+H_{ik}\ \ (3);\quad H_{mk}=\frac{B_{mk}}{\mu_o}-M_k\ \ (3a);$$

$$H_{sk}=\frac{B_{sk}}{\mu_o}\ \ (3b);\quad H_{ik}=\frac{B_{ik}}{\mu_o}\ \ (3c).$$

Then, substituting the equation groups (2) and (3) in the characteristic of the material $H_k=\mathcal{H}(B_k)$ we obtain the equation:

$$M_k=F[B_k]=F[M_1,..,M_{3N},I_{i1},..,I_{i3N},I_{s1},..,I_{sR}] \quad (4)$$

Then we write Ohm's law inside every volume element:

$$\rho_k J_k(t)=-\nabla V_k(t)-\frac{\partial A_k(t)}{\partial t} \quad (5)$$

where A_k is the magnetic vector potential in the k-th elementary parallelepiped, ∇V_k is the irrotational component of the electric field E_k and J_k is the current density in the k-th volume.

We combine equations (1a) and (4) in order to express the derivative of the vector potential with respect to time as a function of the currents inside volume elements.

$$\frac{\partial A_k(t)}{\partial t}=\frac{\mu_o}{4\pi}\left[\frac{\partial J_s(t)}{\partial t}\iiint_{V_s}\frac{1}{|x_k-x'|}dV_s+\sum_{j=1}^{3N}\frac{\partial J_{ij}(t)}{\partial t}\iiint_{V_j}\frac{1}{|x_k-x'|}dV_j+\sum_{j=1}^{3N}\frac{\partial M_j(t)}{\partial t}\iint_{S_j}\frac{1}{|x_k-x'|}dS_j\right] \quad (6)$$

dM/dt can be expressed as a function of dI_{ij}/dt and dI_s/dt in the volume elements by differentiating the constitutive eq. (4) inside every volume element:

$$M_k - F[M_1,..,M_{3N}, I_{i1},..,I_{i3N}, I_{s1},..,I_{sR}] = 0 ,$$

$$\frac{\partial}{\partial t}\left[M_k - F[M_1,..,M_{3N}, I_{i1},..,I_{i3N}, I_{s1},..,I_{sR}]\right] = 0 ,$$

$$\frac{\partial F}{\partial M_1}\frac{\partial M_1}{\partial t} + \ldots + \frac{\partial F}{\partial M_{3N}}\frac{\partial M_{3N}}{\partial t} + \frac{\partial F}{\partial I_{i1}}\frac{\partial I_{i1}}{\partial t} + \ldots$$

$$+ \frac{\partial F}{\partial I_{3N1}}\frac{\partial I_{3N1}}{\partial t} + \frac{\partial F}{\partial I_{s1}}\frac{\partial I_{s1}}{\partial t} + \ldots + \frac{\partial F}{\partial I_{sR}}\frac{\partial I_{sR}}{\partial t} - \frac{\partial M_k}{\partial t} = 0$$

or:

$$\left[\frac{\partial M}{\partial t}\right] = \left[D_M\right]\left[\frac{\partial I}{\partial t}\right] \tag{7}$$

dF/dI and dF/dM constitute the elements of the matrix D_M that gives the relation between dM/dt and dI/dt. These terms are functions of the currents I_{ij} and I_s and of the magnetizations M, that are known at every time instant. Integrating eq. (5) in each volume element Γ_k and meaning the result on the surface S_k we obtain:

$$R_k I_k + \sum_{j=1}^{3N} L_{kj}\frac{\partial I_j}{\partial t} + \sum_{j=1}^{3N}\frac{\partial M_j}{\partial t} + \sum_{j=1}^{R}\overline{L}_{ksj}\frac{\partial I_{sj}}{\partial t} = U_k \tag{8}$$

where U_k is the electric potential difference between the centres of two nearby parallelepipeds, R_k is the electric resistance of the volume Γ_k, I_k is the current in Γ_k , the L_{kj} are the mutual inductances between Γ_k and Γ_j and L_{ksj} are the mutual inductances between the volume Γ_k and the current sources volumes Γ_{sj}. Substituting (7) in (8) we obtain the equation :

$$R_k I_k + \sum_{j=1}^{3N}(L_{kj} + D_{M_{kj}})\frac{\partial I_j}{\partial t} + \sum_{j=1}^{R}\overline{L}_{ksj}\frac{\partial I_{sj}}{\partial t} = U_k \tag{9}$$

where D_{Mkj} is the element of position k j in the matrix D_M. This equation represents the electric equilibrium equation of a branch of a network where resistive and inductive elements, corresponding to the physical resistances, self and mutual inductances of the elementary volume elements are present.

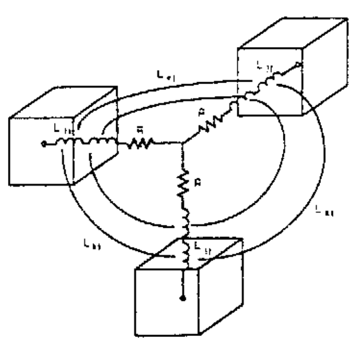

Fig. 4 Branches of the equivalent network

Each of the NO nodes of the network is the center of a star of six branches. Every branch is inductively coupled with all the other branches of the network. Therefore we can consider the segments composing the grid obtained in fig. 1 as branches of an equivalent electric network, and we can write the mesh equations for the loop currents in the network, then obtaining a system of 3N-NO+1 equations. Minimum path meshes are selected in order to have a diagonally dominant matrix. If external voltage generators, capacitor banks, inductances or resistances are electrically connected to the conductive regions, their circuital branches are connected to nodes of the grid. These nodes correspond to the volume elements that are physically in contact with the electrical cables that connect the external elements to the conductive regions. Then their electrical branches are added to the equivalent network of the conductive regions, and the equilibrium of the network is examined.

The solution of the equilibrium equations of the equivalent network, by means of a single-step time-marching method [10], allows the evaluation of the currents in the elementary volumes, and therefore the evaluation of the eddy current distribution in the conductive regions. In order to reduce the computational times the matrix D_M and the magnetizations M are considered constant during every step, being updated only at the beginning of each time interval integration. The complete procedure is constituted by the following steps:

Initialise the currents in the branches of the network; t:=0

for i := 1 to n (last instant) repeat

1• Find the $M(t_i)$ by solving (4)

2• Update the matrix D_M (that express dM/dt as a function of dI/dt at the instant t_i) by means of (7)

3• Make the time marching step by solving (9)

4• t := t + Δt

end for

The computational cost of the method, due to the presence of dense matrixes, is similar to other integral formulations [11]. Nevertheless, the presence of a diagonally dominant matrix and the analytical evaluation of its elements, can significantly reduce the computational times. Furthermore, the impact of parallel processing and the use of iterative algorithms [12] should significantly enhance the numerically efficiency of the method.

III. MOVING ELEMENTS.

We consider a system with a fixed body and a moving one having a velocity v(t). In order to take into account the presence of a moving element, we have to modify eq. (6), that becomes:

$$\frac{\partial A_k(t)}{\partial t} = \frac{\mu_o}{4\pi}\left[\frac{\partial J_s(t)}{\partial t}\iiint_{V_s}\frac{1}{|x_k - x'|}dV_s + J_s(t)\iiint_{V_s}\frac{\partial}{\partial t}\frac{1}{|x_k - x'|}dV_s + \sum_{j=1}^{3N}\frac{\partial J_{ij}(t)}{\partial t}\iiint_{V_j}\frac{1}{|x_k - x'|}dV_j + J_{ij}(t)\iiint_{V_j}\frac{\partial}{\partial t}\frac{1}{|x_k - x'|}dV_j + \sum_{j=1}^{3N}\frac{\partial M_j(t)}{\partial t}\iiint_{S_j}\frac{1}{|x_k - x'|}dS_j + M_j(t)\iiint_{S_j}\frac{\partial}{\partial t}\frac{1}{|x_k - x'|}dS_j\right] \tag{10}$$

When the point x_k belongs to the fixed body eq. (10) is equal to eq. (6), when the point x_k belongs to the moving body, we have that

$$\frac{\partial}{\partial t}\left[\frac{1}{|x_k - x'|}\right] = \frac{1}{|x_k - x'|^3}\frac{\partial x_k}{\partial t} = \frac{v(t)}{|x_k - x'|^3} \tag{11}$$

and

$$J_{ij}(t)\iiint_{V_j}\frac{\partial}{\partial t}\left[\frac{1}{|x_k - x'|}\right]dV_j = J_{ij}(t)v(t)\iiint_{V_j}\frac{1}{|x_k - x'|^3}dV_j$$

$$M_j(t)\iiint_{S_j}\frac{\partial}{\partial t}\left[\frac{1}{|x_k - x'|}\right]dS_j = M_j(t)v(t)\iiint_{S_j}\frac{1}{|x_k - x'|^3}dS_j$$

then we can obtain analytical expressions for the volume and surface integrals. These terms modify eq. (9) which becomes:

$$R_k I_k + \sum_{j=1}^{3N}(L_{kj} + D_{M_{kj}})\frac{\partial I_j}{\partial t} + vK_1 I_j + vK_2 M_j$$

$$+\sum_{j=1}^{R}\overline{L}_{ksj}\frac{\partial I_{sj}}{\partial t} = U_k \tag{12}$$

Therefore we can repeat the procedure described in the previous paragraph, substituting eq. (9) with eq. (12) in the step 4.

IV. Linear systems

When we deal with linear characteristics, eq. (4) becomes:

$$Mk = \frac{1}{\mu_o}\left(1 - \frac{1}{\mu_r}\right)\left[\sum_{j=1}^{R}\beta_j \cdot I_{sj} + \sum_{j=1}^{3N}\lambda_j \cdot I_{ij} + \sum_{j=1}^{3N}\alpha_j \cdot M_j\right] \tag{13}$$

therefore we obtain a linear relation between the magnetizations inside the elementary volumes, and the source and induced currents I_s and I_i.

Consequently dF/dI and dF/dM in the eq.(7) are no more functions of the currents I_i, I_s and of the magnetization M, but are constant values. Therefore the relation between dM/dt and dI/dt is constant in time and the matrix D_M does not have to be updated during the time stepping. Then we can substitute these expressions in the eq. (9) and eliminate the step 3.

The energy of a conductor can be obtained from the equation:

$$W = \frac{1}{2}\iiint_{\Omega} A \cdot J \, d\Omega \tag{14}$$

then the force on the conductor can be obtained by:

$$F = \frac{\partial W}{\partial x} = \frac{\partial}{\partial x}\frac{1}{2}\iiint_{\Omega} A \cdot J \, d\Omega \tag{15}$$

Substituting eq. (2) for A in eq. (15) we have:

$$F = \frac{\partial}{\partial x}\iiint_{\Omega}\frac{\mu_o}{8\pi}\left[\iiint_{V_s}\frac{J_s(t)}{|x_k - x'|}dV_s + \sum_{j=1}^{3N}\iiint_{V_j}\frac{J_{ij}(t)}{|x_k - x'|}dV_j + \sum_{j=1}^{N}\iint_{S_j}\frac{n_j \times M_j(t)}{|x_k - x'|}dS_j\right] \cdot J_{ik}\, d\Omega$$

then carrying out the integral with respect to the volume Ω, summing the contribution of every volume element V_i and taking the derivative with respect to x similarly as in the eq. (10) we have:

$$F = \frac{\mu_o}{8\pi}\sum_{j=1}^{3N}J_{ij}(t)\left[\iiint_{V_s}\frac{J_s(t)}{|x_k - x'|}dV_s + \iiint_{V_j}\frac{J_{ij}(t)}{|x_k - x'|^3}dV_j + \sum_{j=1}^{N}\iint_{S_j}\frac{n_j \times M_j(t)}{|x_k - x'|^3}dS_j\right] \tag{16}$$

that can be evaluated by means of analytical expressions [13]. The evaluation of the electromagnetic force on the moving body allows the determination of its law of motion by means of the mechanical equilibrium equation:

$$\frac{\partial^2 x}{\partial t^2} = \frac{F}{m}$$

V. Results

The method has been tested against benchmarks for linear and non-linear ferromagnetic systems.

The first one is a magnetostatic problem proposed by the Institute of Electrical Engineers of Japan [14,15]. The geometry is shown in fig. 5, where the permeability of the iron core is 1000 and the coil was energised with 3000 AT.

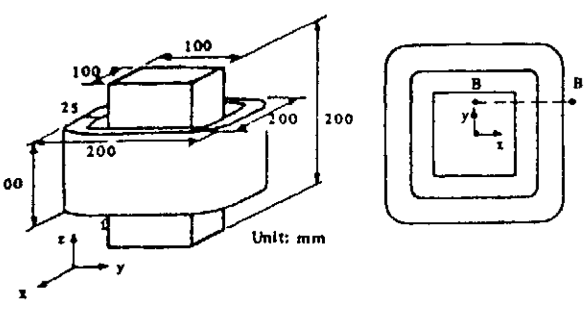

Fig.5. Geometry of the standard IEEJ problem

Fig. 6 displays the z component of the magnetic induction along the x direction at a distance of 10mm from the top of the iron element with y = 45mm.

One eighth of the system was discretized in 250 elementary cubes with good agreement between calculated and experimental results.

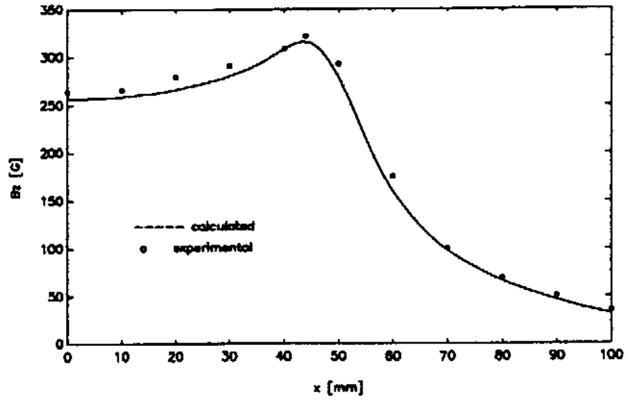

Fig.6. Magnetic induction on the line B-B'

The second problem also proposed by the Institute of Electrical Engineers of Japan [16], featured a time-varying sinusoidal excitation relating to linear ferromagnetic characteristic. The experimental arrangement shown in fig. 7, is composed by a coil energised with 1000 AT, two aluminium plates and a ferrite block, the relative permeability of the ferrite is assumed to be 3000 and the frequency is 50 Hz.

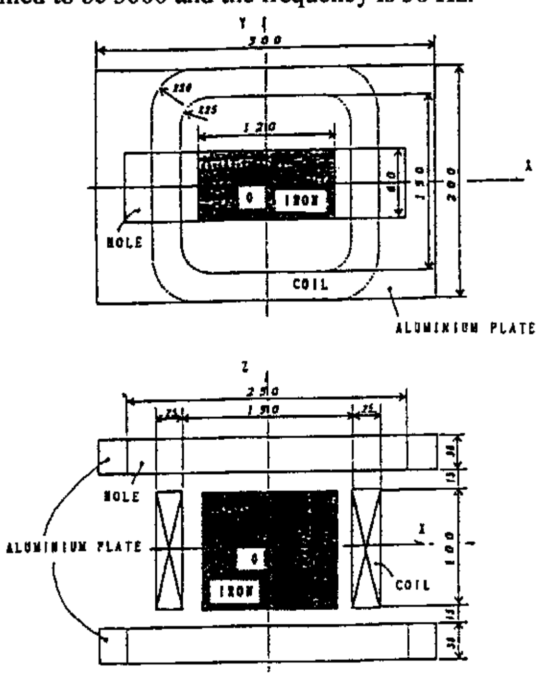

Fig.7. Geometry of the standard IEEJ problem

Figures 8 and 9 show that a good agreement between calculated and experimental results was obtained with 370 elements.

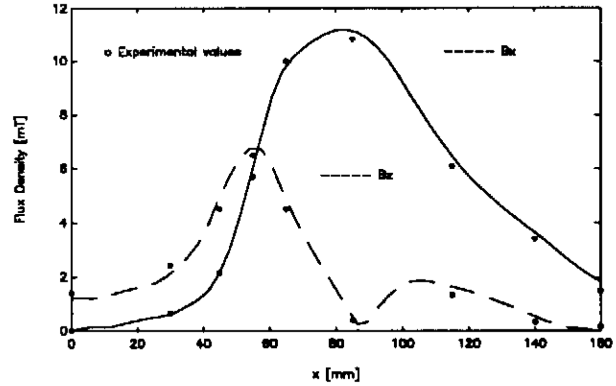

Fig.8. Magnetic flux density at z=57.5mm, y=0.0

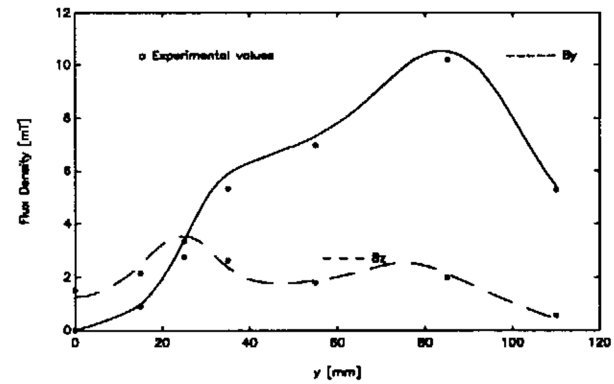

Fig. 9. Magnetic flux density at z=57.5mm, x=0.0

Experimental measurements are given also when a hole in the aluminium block is present. Figures 10 and 11 show a good agreement between calculated and experimental results with the same discretization.

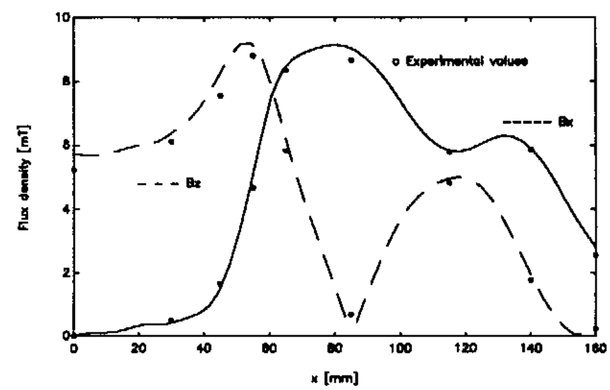

Fig. 10. Magnetic flux density at z=57.5mm, y=0.0

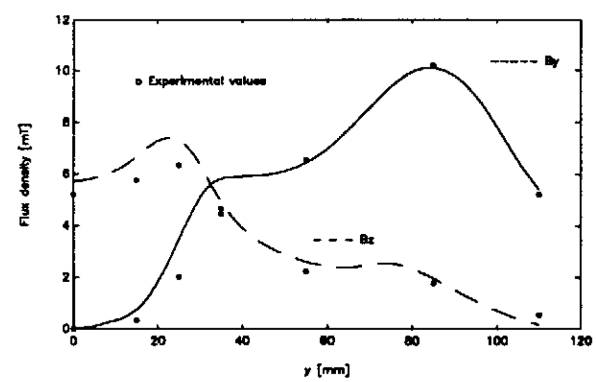

Fig. 11. Magnetic flux density at z=57.5mm, x=0.0

Furthermore, the method was tested by comparing calculated and experimental results relating to the TEAM problem 13 [17] as shown in fig. 12.

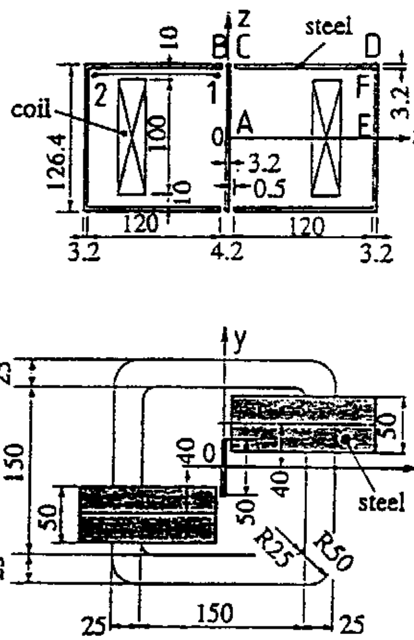

Fig.12. TEAM Problem 13 geometry.

The calculated results shown in the figures 13 and 14 agree with the experimental results and with results obtained with other models.

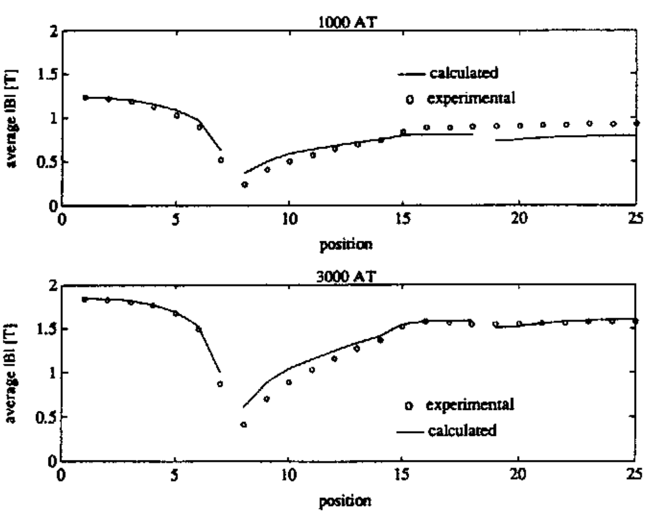

Fig.13. Comparison of average flux densities in iron

Only one fourth of the system was discretized, by using 430 elements. A modified Newton-Raphson algorithm [17] with a relaxation factor r = 0.5 was used for the solution of the non-linear system, the linear solution was taken as starting value for the calculation, and convergence was reached in 10 steps .

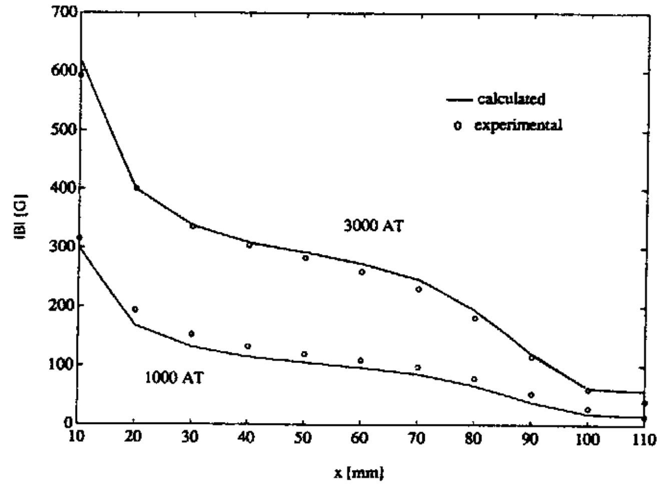

Fig.14. Comparison of flux densities in air

VI. Conclusions

An integral formulation for 3-D non-linear electromagnetic fields analysis has been presented. The method was formulated for the analysis of non-linear systems including the presence of moving bodies.

The method allows an easy modeling of homogenous and inhomogenous materials, can simply take into account the relative motion and the electromagnetic forces among conducting bodies, and allows an easy linkage between circuit and field equations.

The method has been tested on standard problems both for linear time varying systems and non-linear static systems and has given a good agreement with experimental results. Work is in progress for the implementation and validation of the non-linear time-varying case, and for the inclusion of the motion of bodies.

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