Efficient Numerical Analysis of Arrays of Identical Elements with Complex Shapes

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***Abstract*─** A fast method-of-moments approach is proposed for the solution of finite arrays of complex identical elements, involving both metal and finite dielectric parts. The method is based on the use of Macro Basis Functions (MBF), also named "Characteristic" Basis Functions, among which interactions are computed very fast with the help of a Multipole approach. Fast evaluation of array patterns or embedded element patterns is obtained through decomposition into a finite series of pattern multiplication problems. Examples are provided for finite arrays of bowtie antennas embedded in dielectric boxes. For periodic arrays, results are compared with infinite-array solutions. The method is also extended to non-periodic structures, for which the Multipole approach appears very useful for interactions outside the near-field region. We show that interactions in the near-field region can benefit from an interpolation procedure.

***Index Terms*─** antenna arrays, method of moments, multipoles, macro basis functions.

I. INTRODUCTION

The numerical simulation of large finite arrays of antennas remains a challenge when the elements are of complex shapes and involve finite dielectric components and metallic parts. Examples of such radiators are ultra-wideband antennas embedded in finite dielectric boxes, which serve for instance as supporting material or as material allowing to reduce the size of the radiating elements. When dealing with piecewise homogeneous media, as is generally the case in antenna technology, integral-equation approaches, like the Method of Moments make sense, since unknowns can be limited to the interfaces between media. However, for the geometries referred to above, the description of currents or equivalent currents may anyway require several hundreds, and sometimes thousands, of coefficients per antenna, even if the latter is smaller than the wavelength.

Several efficient methods have already been developed for periodic structures; however, for the configurations of interest, they suffer from several drawbacks. When the total number of unknowns is larger than a few thousands, iterative methods that rely on fast matrix-vector multiplications can be used, they are based either on multipole decompositions [1], on Fast Fourier Transforms [2] or on QR decompositions [3]. In those cases, when based on Krylov subspace methods, convergence is theoretically guaranteed; however, the number of iterations is difficult to predict and relatively fast convergence generally requires preconditioners, whose performances are also difficult to predict. Besides this, some cases of divergence, generally attributed to effects of numerical round-off, are sometimes observed.

This is mainly why several authors have been trying to circumvent these difficulties by devising non-iterative methods in which elementary decomposition functions are aggregated into *Macro Basis Functions* (MBF's), which would make sense from a physical point of view (they are then named *Characteristic*) [4]-[7]. The determination of the MBFs themselves is a research sub-topic on its own and will not be studied here. Besides the references above, a method is proposed in [8], where aggregations similar to the MBF approach are exploited in an iterative scheme based on multipole decompositions. For the challenging case of electrically connected antennas, specific schemes are presented in [9], [10] and [11]. In the latter, the Array Scanning Method [12], based on infinitearray solutions, is exploited to generate theMBFs.

In this paper, we will concentrate on the computation of fast interactions between Macro Basis Functions (MBF), for the case of complex antennas fully embedded in dielectric blocks, assuming that all the elements of the array are identical, and with an extension to non-periodic arrays. The proposed method relies on a technique initially presented in [13] for the case of radiation by arrays made of metallic elements and in [14] for the case of scattering by finite arrays of homogeneous dielectric objects. The method is based on a combination of the Macro Basis Function (MBF) approach and of the Multipole method. In [15], this method has been extended by placing metallic elements inside finite dielectric objects. Examples were shown in [15] for dipoles embedded in dielectric quasi-spheres. In this paper, further results are shown for wideband bowtie elements embedded in dielectric boxes. It will also be recalled that, in the framework of the MBF approach, the array pattern can strictly be written as a series of pattern multiplication problems, for which the FFT can be exploited. The method will then be extended to the case of non-periodic arrays, for which the acceleration of interactions is even more crucial. Finally, we will also show that the FFT still can be exploited for pattern evaluation of irrergular arrays, provided that a specific interpolation procedure be applied.

This paper is organized as follows. In Section 2, the mathematical formulation is provided. In Section 3, simulation results are shown for bowtie antennas embedded in dielectric volumes. First for the case of a small 2×2 array, for comparison with a brute-force solution; then for a 10×10 array, with a comparison with the infinite-array solution. Finally, in Section 4, the methodology is extended to non-regular arrays. Conclusions are drawn in Section 5.

II. FORMULATION

Besides the currents j_m on the metallic part of the antenna, unknown equivalent electric and magnetic currents, j_s and m_s, are considered on the surface S_d of the dielectric objects. Hence, in this Method-of-Moments (MoM) formulation, the metallic part of the antenna couples with the exterior medium and with other antennas only through the equivalent currents on the surface S_d. Following the MBF (also called *Characteristic Basis Function*) methodology, the current distribution on a given cell is obtained as the linear superposition of distributions obtained while solving small problems:

$$\{\mathbf{j_m}, \mathbf{j_s}, \mathbf{m_s}\} = \sum_{p=1}^{P} C_p \; \{\mathbf{j^\circ_{m,p}}, \mathbf{j^\circ_{s,p}}, \mathbf{m^\circ_{s,p}}\}, \quad (1)$$

where $j^{o}_{x,p}$ and $m^{o}_{x,p}$ are the vectors of coefficients describing macro basis function *p*, and C_p is a constant to be determined. Here, these small problems are made either of an isolated transmitting cell (*primary* MBF), or of a receiving cell (*secondary* MBF, [6]), illuminated by the fields radiated by the transmitting cell. The interactions between MBFs are computed as the discretized approximation of:

$$I = \iint \left(\vec{J}_t^* . \vec{E}_s + \vec{M}_t^* . \vec{H}_s \right) dS \,, \quad (2)$$

where $\vec{J}_t$ and $\vec{M}_t$ currents can be either on the metallic or dielectric surfaces, while $\vec{E}_s$ and $\vec{H}_s$ are the fields radiated by a given macro basis function in the region of interest. The (*) superscript stands for complex conjugation, while the *t* index refers to testing functions. For MBFs associated with the same antenna or with neighboring antennas, interaction *I* can be computed with the help of the block of the MoM impedance matrix standing for interaction between those antennas [6]. When pairs of antennas located further away are considered, the computation of interaction I can be carried out much more efficiently, with the help of a multipole formulation. Using the plane-wave expansion of the free-space scalar Green's function, which can be found in [16], we obtain:

$$I = \frac{k}{(4\pi)^2} \iint \left(P_e + P_h \right) T\left(k, \vec{r}_{mn}, \hat{u}\right) dU \,, \quad (3)$$

where *T* is the Multipoles translation function [16], $\hat{u}$ is a unit vector in the direction of integration, and P_e and P_h are specific products of patterns of macro basis functions and of their divergences:

$$\begin{aligned} P_e &= \vec{\mathcal{F}}_t \,.\, \left(-\omega\,\mu\,\vec{\mathcal{F}}_b^\star - k\,\hat{u}\times\vec{\mathcal{G}}_b^\star \right) \\ &\quad + \frac{1}{\omega\,\epsilon}\,\mathcal{DF}_t\,\mathcal{DF}_b^\star \qquad , \end{aligned} \quad (4)$$

$$\begin{aligned} P_h &= \vec{\mathcal{G}}_t \,.\, \left(-\omega\,\epsilon\,\vec{\mathcal{G}}_b^\star + k\,\hat{u}\times\vec{\mathcal{F}}_b^\star \right) \\ &\quad + \frac{1}{\omega\,\mu}\,\mathcal{DG}_t\,\mathcal{DG}_b^\star \qquad , \end{aligned} \quad (5)$$

where the b index denotes the basis functions. In the following, to avoid too heavy notations, the b and t indices will be omitted.

$$\vec{\mathcal{F}} = \sum_{n=1}^{N} j_s^{\circ,\star}(n)\,\vec{F}_n;\ \mathcal{DF} \tag{6}$$

$$= \sum_{n=1}^{N} j_s^{\circ,\star}(n)\,DF_n \tag{7}$$

$$\vec{G} = \sum_{n=1}^{N} m_s^{\circ,\star}(n)\,\vec{F}_n;\ \mathcal{DG} \tag{8}$$

$$= \sum_{n=1}^{N} m_s^{\circ,\star}(n)\,DF_n \tag{9}$$

where $\vec{F}_n$ is the pattern of the n^{th} subsectional basis function $\vec{f}_n$:

$$\vec{F}_n = \iint \vec{f}_n(\vec{r})\,e^{jk\hat{u}.\vec{r}}\,dS\ , \tag{10}$$

and DF_n is the pattern of its divergence. In the formulas above, k is the free-space wavenumber, ω is the radian frequency and ε and μ are the permittivity and permeability of free space. The formulas above can be delineated from [13] and from the well-known expressions of MoM-matrix entries for dielectric materials [17]. Detailed developments, along with slight variations on the formulation, will be given in [18].

The radiation patterns of the finite structure can be obtained very fast by decomposing the problem into a finite series of pattern multiplication problems. In this series, the contribution from a given MBF can be written as $F_{p,x}(\hat{u})A_p(\hat{u})$, where $F_{p,x}(\hat{u})$ is the element pattern of the current distribution described by the p^{th} macro basis function, computed with the help of the equivalent currents $\mathrm{j}^{\mathrm{o}}_{\mathrm{s,p}}$ and $\mathrm{m}^{\mathrm{o}}_{\mathrm{s,p}}$ on S_d, and which have already been obtained in the course of the MBF+Multipole computations. The index x stands for the polarization of the computed fields. The factor A_p corresponds to the array factor resulting from the array excited with the coefficients associated with the p^{th} macro basis function. Hence, we have:

$$F_x(\hat{u}) = \sum_{p=1}^{p=P} A_p(\hat{u})F_{p,x}(\hat{u})\ , \tag{11}$$

with

$$A_p(\hat{u}) = \sum_{mn} C_{mnp}e^{jk\left(u_x m\, a + u_y\, n b\right)} \tag{12}$$

$$= -j\omega\mu M\ N\,\mathrm{IFFT2}\left(C_{mnp}\left(-1\right)^{m+n}\right) \tag{13}$$

$$= A_p\left(r,\, s\right)\ , \tag{14}$$

where u_x and u_y are the horizontal projections of the unit vector $\hat{u}$ which indicates the direction of observation:

$$u_x\left(r\right) = \lambda / a\left(-1/2 + r/M\right), \tag{15}$$

$$u_y\left(r\right) = \lambda / b\left(-1/2 + s/N\right), \tag{16}$$

with (M, N) the dimension of the two-dimensional inverse FFT, $0 \le r \le N$ and $0 \le s \le M$, while a and b are the array spacings along x and y.

In [15], the method has been validated by comparison with a brute-force solution for a 5×5 array of broadband dipoles embedded in dielectric quasispheres and by comparison with infinite-array results for very large arrays. In the following, results will be shown for bowtie antennas inside dielectric blocks and an extension to non-regular arrays will be provided.

III. NUMERICAL EXAMPLES

The unit cell is sketched in Fig. 1. It is made of a bowtie antenna embedded in a parallelipipedic volume with relative permittivity equal to 4, represented here with 268 RWG-type [19] basis functions. The bowtie antenna is meshed with the help of 112 RWG and one rooftop basis functions. The antennas are terminated with a 100 Ohm load. The size of the box is 1.2 cm and the spacing between elements is 1.25 cm. The array configuration is shown in Fig. 2.

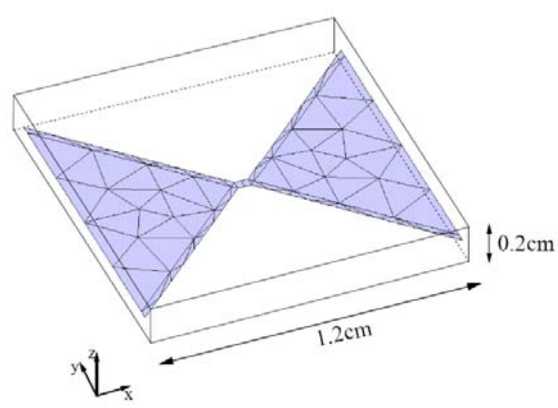

Fig. 1. Mesh of bowtie antenna embedded in dielectric volume.

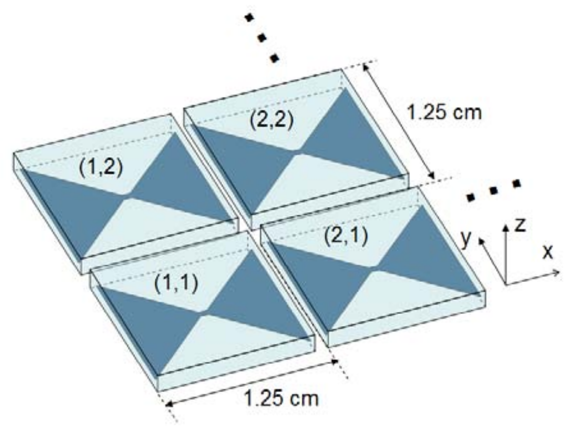

Fig. 2. Array geometry and element numbering.

Table 1. Coupling coefficients (amplitude and phase) in 2×2 array terminated with 100 Ohm loads, expressed in terms of induced currents, normalized w.r.t. excited element (1,1). Wavelength: 5 cm.

Index	(1,1)	(1,2)	(2,1)	(2,2)
B.F.(dB)	0.0	-14.944	-11.986	-27.370
MBF(dB)	0.0	-14.922	-12.002	-27.477
B.F.(rad)	0.0	1.351	-2.756	0.241
MBF(rad)	0.0	1.357	-2.754	0.255

Simulations have first been carried out at the element level, for a wavelength of 5 *cm*. We verified that, for lossless dielectrics, the power delivered to the antenna corresponds to the radiated power. This verification does not provide ultimate validation but appears as a useful check. Correspondence has been achieved within 0.1 percent of relative error. The array computations have been carried out while considering one primary MBF, corresponding to an isolated excited element and eight secondary MBFs, obtained as the field induced on the eight surrounding elements, due to the incident field radiated by the primary (this way of choosing MBFs was first proposed in [6]). First, still for a 5 cm wavelength, a very small array, made of 2×2 elements has been simulated, in order to see to what extent the MBF approach (so far without the Multipole acceleration) compares with the brute-force solution. In this case, the comparison is provided in terms of coupling coefficients and in terms of embedded element patterns. The coupling coefficients are provided as the currents induced on the 100 Ohm loads of the antennas when one of them is excited with a unit voltage and a 100 Ohm series impedance. Table 1 provides those coupling coefficients in the 2×2 array, expressed in dB with respect to the currents on the series impedance of element (1,1), which is excited. As for the magnitude, the first line stands for the brute-force (B.f.) solution, while the second line stands for the MBF solution. As for the phase, corresponding results are provided in third and fourth lines. It can be seen that the error levels are very low.

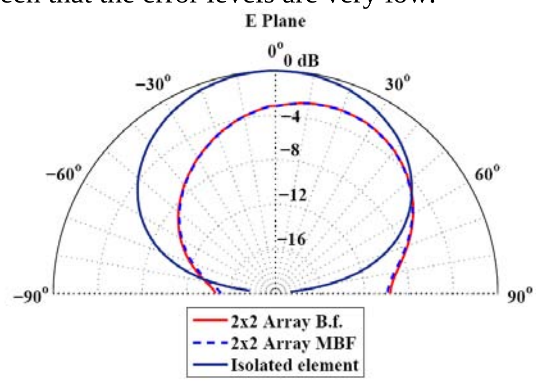

Fig. 3. E-plane cut in embedded element pattern for element (1,1) excited in 2×2 array, compared with pattern of isolated antenna. Wavelength: 5 cm.

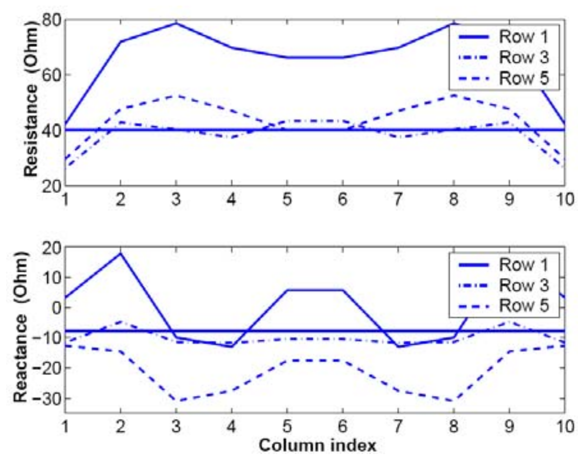

Fig. 4. Active input impedances in 10×10 array and in infinite array (horizontal lines) for broadside scan. Wavelength: 5 cm. Rows are along y in Fig. 2.

A major advantage of the MBF approach is that, once the reduced MoM impedance matrix is inverted, solutions are readily obtained for all possible excitations. Characteristics of major interest are the embedded element patterns, obtained with one element excited and all other elements passively terminated. On receive for instance, such a pattern fully describes the sensitivity at a given port of the array. For the 2×2 array referred to above, an E-plane cut of the embedded element pattern of element (1,1) is compared in Fig. 3 with the pattern obtained for an isolated element, as well as with the embedded element pattern obtained in the brute-force case. It can be seen that the MBF approach allows us to estimate with high accuracy the strong effects of mutual coupling on the embedded element patterns. In this case, in view of the very small size of the array, multipoles have not been used to compute the interactions between MBFs.

Other simulations have been carried out for a 10×10 array, with uniform excitations. This leads to (113+ 2×268)×100 = 64.900 surface unknowns on the finite- array structure. 900 unknowns are obtained in the reduced system of equations. The total computation time is of the order of 230 seconds on a 1.6 GHz laptop computer, 100 of which are dedicated to the computation of the MBFs themselves, 90 for computation of interactions in the near-field and 40 for all far-field interactions. The multipole approach has been used for interactions between MBFs as soon as the distance between antennas on which they are residing is larger than a wavelength. Once the reduced impedance matrix is obtained, all coupling coefficients, as well as all embedded element patterns can be computed almost instantly (a few seconds). Examples are provided here for uniform excitation. Fig. 4 shows results obtained for active input impedances on several rows along $\hat{x}$. The horizontal lines stand for the active input impedance obtained with the infinite-array approximation. The latter has been computed with the help of the Method of Moments [20], exploiting a rapidly converging scheme for the periodic Green's function and its gradient (the latter is necessary for the treatment of the dielectric material). It can be seen that, near the middle of the array, active impedances start resembling the infinite-array solution. However, in view of the relatively small size of the array (each array side is 2.5 wavelengths only), and given the oscillatory effects of array truncation on port currents, the infinite-array solution is a relatively poor approximation [21], such that the elements in the middle of the array are not necessarily the closest to the infinite-array solution.

Figure 5 shows the array pattern in the E-plane, obtained with the help of the FFT approach. To emphasize the effects of mutual coupling, we also show the array pattern obtained as a product between the array factor for uniform excitation and the element pattern obtained from the currents corresponding to the infinite-array solution for broadside scan. A 0.5 dB error appears at broadside and the error increases rapidly for successive side-lobes.

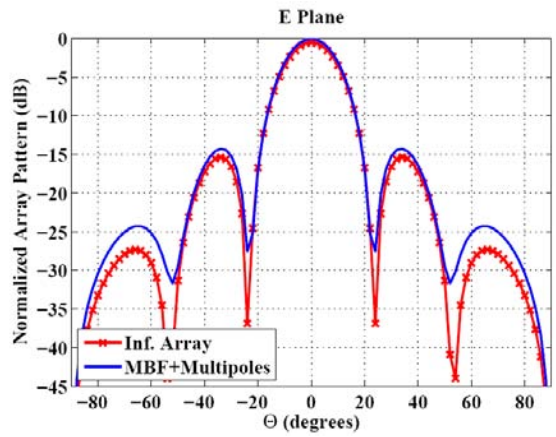

Fig. 5. Array pattern in E-plane, obtained with MBF+Multipole+FFT approach for 10×10 array and broadside scan, compared with solution assuming infinite-array currents. Wavelength: 5 cm.

IV. EXTENSION TO IRREGULAR ARRAYS

The method referred to above has been extended to irregular arrays. If N is the number of surface unknowns on the dielectric-air interface, the multipole approach presented above allows us to compute the interactions between macro basis and testing functions with complexity of the order of N instead of N^2, at least if the spacings between elements is larger than about one wavelength. If M is the number of antennas, for regular arrays, the number of different spacings to be considered for the interaction between a given pair of macro basis and testing functions is of the order of $4M$, thanks to redundance in the periodic structure. However, in general, for the irregular array, this number of interactions is again of the order of M^2, which underscores the importance of computing the interactions very fast. Besides this, for the array pattern (or embedded element pattern) evaluation, the FFT approach presented in Section 2 cannot be used directly. This point will be treated further below.

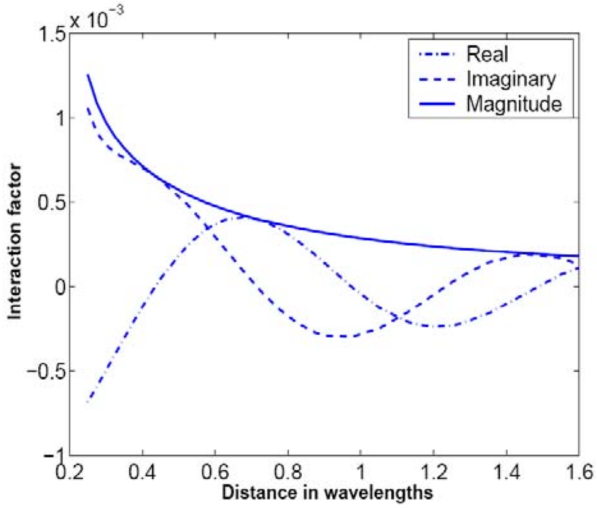

Fig. 6. Interaction factor I between two primary MBFs versus distance along y.

The Multipole approach has been applied for all pairs of elements separated by at least one wavelength, while an approach based on MoM impedance matrix calculations [6] has been used for elements placed closer to each other. For irregular arrays, this still may concern a large fraction of the pairs to be considered. Such interactions concern almost about a fourth of the interactions in the array considered below and take 20 seconds each, which is prohibitive. However, it is interesting to note that the interactions between MBFs on different antennas are a smooth function of the distance between antennas. This is illustrated in Fig. 6 for the interaction between two primaries at increasing distances along $\hat{y}$. It can be seen that this function is particularly smooth, which opens important perspectives for further acceleration of near-field interactions. A method fully exploiting this smooth behavior of near-field interactions versus distance has been initiated in [22] for the case of metallic antennas, and a more advanced version of this technique will be described in a separate publication. Active input impedances for the array configuration depicted in Fig. 7 have been computed for broadside scan at 6 GHz; results are shown in Fig. 8. The horizontal lines again stand for the infinite-array solution with 1.25 cm spacings.

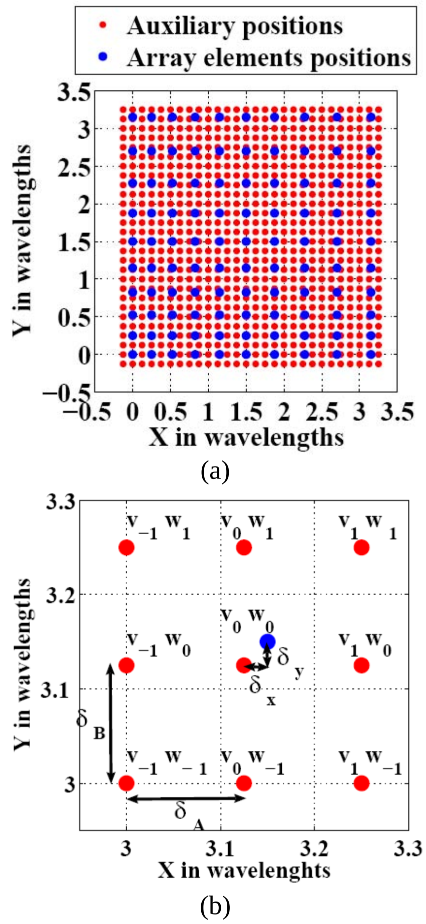

Fig. 7. Positions of elements in irregular array, and underlying finer grid used for NFFT-based pattern computations.

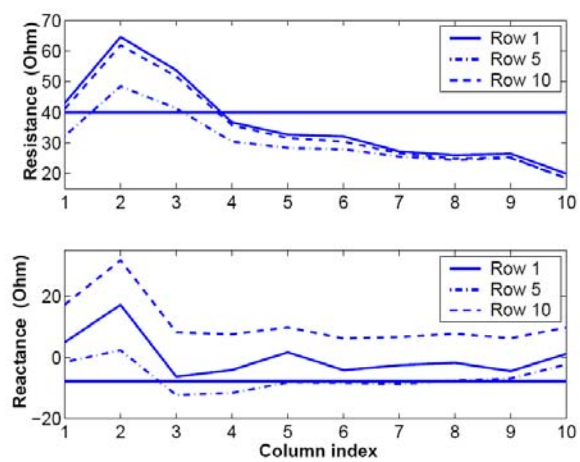

Fig. 8. Active impedances in 10×10 irregular array scanned at broadside. Horizontal lines stand for infinite-array solution. Wavelength: 5 cm. Rows are along y in Fig. 2.

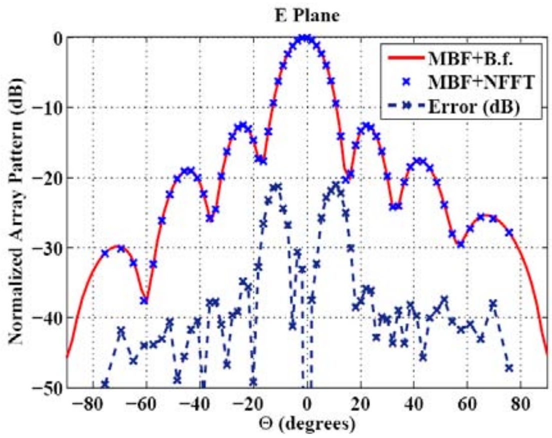

Fig. 9. Pattern of the 10×10 irregular array scanned at broadside, computed with brute-force approach and with the help of NFFT approach. The lower curve shows the errors incurred by use of NFFT with λ/8 grid and second order separable interpolation rule.

As for the pattern computations, we try to describe an equivalent uniform array for the irregular problem, such that the FFT approach still can be used in a straightforward manner. This approach can be connected to NFFT methods also applied to radar processing and medical imaging [24]. Here, a simple interpretation, inspired from interpolation methods, will be used. The phase factor associated with each element in the array factor expression depends on the element's position. In the following, it will be estimated as a linear combination, described with weights u_{ij}, of its value at a few points located around the antenna, on a regular grid. Hence, this problem is similar to a standard interpolation technique. The method amounts to distributing, for each element in the array, the MBFs coefficients onto a set of auxiliary neighbor elements located on a regular grid, as shown in Fig. 7(b), where 9 neighboring elements are considered. The distribution rule simply corresponds to the set of weights u_{ij}. The procedure to obtain the weights on the auxiliary elements with the help of a 1-D quadratic interpolation technique is illustrated in Fig. 7(b). The weight u_{ij} can be expressed as a product $u_{ij} = v_i \, w_j$. The weights $v_i(\delta_x)$ are computed as $v_{-1} = \delta_x^0\left(1-\delta_x^0\right)/2$,

$v_0 = 1-\left(\delta_x^0\right)^2$,

and $v_1 = \delta_x^0\left(1+\delta_x^0\right)/2$; similar expressions are used to define $\omega_j(\delta y)$. In those formulas, $\delta_x^0 = \delta_x/\delta_A$ and $\delta_y^0 = \delta_y/\delta_B$ are incremental positions on the regular grid, normalized w.r.t. the grid spacings. More sophisticated techniques may be employed to perform the aforementioned distribution [23], [25] but their evaluation is outside the scope of this paper.

Once the new coefficients and the regular grid are obtained, expression (14) can be employed again for the pattern computation. Performing the IFFT over for the auxiliary grid implies a higher computational cost, in view of the use of a finer grid: the $N\log_2 N$ complexity now becomes $CN\log_2 CN$, with C the ratio between the number of points in the fine grid and the number of antennas. However, for large arrays, this will remain competitive with the brute-force approach with complexity of the order of N^2. The proposed technique has been applied to the 10×10 array of Fig. 7(a), considering a fine grid with spacing λ/8. Results are shown in Fig. 9, together with the error incurred by the interpolation. For this example, the bruteforce solution applied to the pre-calculated MBF patterns takes 12.44 s, while the NFFT approach takes 2.23 s. Much larger time savings are expected for larger arrays.

V. CONCLUSION

A fast numerical approach, combining Macro Basis Functions and Multipole approaches, has been presented for arrays comprising dielectric elements that contain metallic parts. Once the macro basis functions have been computed, the complexity of the method no longer depends on the number of unknowns in the unit cell. The accuracy of the method has been demonstrated by comparison with a full-wave approach for a very small array. Besides this, we explained how the FFT can be exploited for the very fast estimation of radiation patterns (array patterns or embedded element patterns). This is possible thanks to the fact that the MBF approach allows us to decompose the full array pattern into a finite superposition of pattern multiplication problems. Finally, we showed that the method can be extended to the analysis of irregular arrays. In this case, more interactions need to be computed and we underscored the possibility of further acceleration by illustrating the smoothness of such interactions versus inter-element distance; we also illustrated the effectiveness of the NFFT for the fast pattern computation in the case of irregular arrays.

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