Deformation Effect on Transmission Properties of the One Dimensional Photonic Crystal

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***Abstract* —** We considered the influence of multilayer structure parameters which are the index contrast, the period's number and the reference wavelength on the transmission spectrum of a deformed structure. Deformation was introduced by applying the power law $y = x^{k+1}$.We revealed that the higher optical index contrast enhance the deformation effect on transmission properties of the structure at normal incidence. This work is a detailed study of the effect of the deformation introduced in the multilayer stack according to the mentioned law.

***Index Terms* —** Deformation, index contrast, multilayer structure, period's number, reference wavelength, transmission properties.

I. INTRODUCTION

The photonic crystals (PCs) are beginning to have a profound effect on the development of nanoscale devices because they can significantly enhance the interactions between light and matter [1-3]. The properties of photonic crystals are not based on absorption or emission transitions. Instead they are determined by the index of refraction periodicity which can be scaled from submicron dimensions (to control UV/VUV light) to the centimetre scale (to control microwaves) [4,5]. The idea of photonic crystals is to introduce periodicity comparable to the optical wavelength in such a way that a photonic band gap (PBG) is formed. Different users need PCs with different PBG widths. So, flexibility and tunability of the PBG of PCs is crucial for flexible and dynamic nanophotonic circuits in future [6-8]. Chirped structure can be introduced in the photonic crystal to change the PBG. So, not only the quarter wave periodic structures but also the deformed ones have become significant structures of photonic crystals. In this work, the deformation was introduced by applying the power law, so that the coordinates y of the deformed object was determined through the coordinates x of the initial (periodic structure) object in accordance with the following rule: $y = x^{k+1}$. Here k is the coefficient defining the asymmetry degree [9-12]. For example, the periodic structure is projected into itself without any changes of dimensions if $k = 0$. Deviation of the k value from 0 leads to a deformed multilayer structure. This deformation occurs when its interest by the optimization of the deformation degree; but this optimization is not simple, it depends on the structure parameters such as the optical contrast ratio (the ratio of high refractive to low refractive index), the number of periods, and the reference wavelength. Within this, the present work considers the study of the deformed 1D-PCs behaviour when varying these parameters. So, we treat the interaction between the deformation degree k and the other parameters of the structure in aim to optimize the structure by widening the PBG at normal incidence. Through this study, we become able to control the PBG properties of the deformed system by controlling the structure parameters. The numerical method employed to obtain the transmission response of the structure is the transfer matrix method.

II. METHOD OF MODELING

For the calculation of system reflection and transmission, we employed the transfer matrix method (TMM). This technique is a finite difference method particularly well suited to the study of PBG materials and it can solve the standard problem of the photonic band structures and the scattering (transmission, reflection, and absorption) spectrum [13].

It is based on the Abeles method in terms of forward and backward propagating electric field, that is, E^+ and E^- which were introduced to calculate the reflection and transmission. Abeles showed that the relation between the amplitudes [14] of the electric fields of the incident wave E_0^+, reflected wave E_0^-, and the transmitted wave after m layers, E_{m+1}^+, is expressed as the following matrix for stratified films within m layers:

$$\begin{pmatrix} E_0^+ \\ E_0^- \end{pmatrix} = \frac{C_1 C_2 C_3 \ldots C_{m+1}}{t_1 t_2 t_3 \ldots t_{m+1}} \begin{pmatrix} E_{m+1}^+ \\ E_{m+1}^- \end{pmatrix}. \quad (1)$$

Here, C_j is the propagation matrix with the matrix elements.

$$C_j = \begin{pmatrix} \exp(i\varphi_{j-1}) & r_j \exp(-i\varphi_{j-1}) \\ r_j \exp(i\varphi_{j-1}) & \exp(-i\varphi_{j-1}) \end{pmatrix}, \quad (2)$$

where t_j and r_j are the Fresnel transmission and reflection coefficients, respectively, between the $(j-1)^{th}$ and j^{th} layer. The Fresnel coefficients t_j and r_j can be expressed as follows by using the complex refractive index $\hat{n}_j = n_j + ik_j$ and the complex refractive angle θ_j. For parallel (P) polarization

$$r_{jp} = \frac{\hat{n}_{j-1}\cos\theta_j - \hat{n}_j \cos\theta_{j-1}}{\hat{n}_{j-1}\cos\theta_j + \hat{n}_j \cos\theta_{j-1}}, \quad (3)$$

$$t_{jp} = \frac{2\hat{n}_{j-1}\cos\theta_{j-1}}{\hat{n}_{j-1}\cos\theta_j + \hat{n}_j \cos\theta_{j-1}}. \quad (4)$$

Moreover, for perpendicular (S) polarization:

$$r_{jp} = \frac{\hat{n}_{j-1}\cos\theta_{j-1} - \hat{n}_j \cos\theta_j}{\hat{n}_{j-1}\cos\theta_{j-1} + \hat{n}_j \cos\theta_j}, \quad (5)$$

$$t_{jp} = 2\frac{\hat{n}_{j-1}\cos\theta_{j-1}}{\hat{n}_{j-1}\cos\theta_{j-1} + \hat{n}_j \cos\theta_j}. \quad (6)$$

The complex refractive indices and the complex angles of incidence obviously follow Snell's law: $\hat{n}_{j-1}\sin\theta_{j-1} = \hat{n}_j \sin\theta_j$ (j = 1, 2… m+1). The values ϕ_{j-1} in equation (2) indicate the change in the phase of the wave between $(j-1)th$ and jth boundaries and are expressed by the equation:

$$\varphi_0 = 0, \quad (7)$$

$$\varphi_{j-1} = \frac{2\pi}{\lambda}\hat{n}_{j-1} d_{j-1} \cos\theta_{j-1}. \quad (8)$$

Except for j = 1, λ is the wavelength of the incident light in vacuum and d_{j-1} is the thickness of the $(j-1)th$ layer. By putting $E_{m+1}^- = 1$, because there is no reflection from the final phase, Abeles obtained a convenient formula for the total reflection and transmission coefficients, which correspond to the amplitude reflectance r and transmittance t, respectively, as follows:

$$r = \frac{E_0^-}{E_0^+} = \frac{c}{a}, \quad (9)$$

$$t = \frac{E_{m+1}^+}{E_0^+} = \frac{t_1 t_2 \ldots t_{m+1}}{a}. \quad (10)$$

The quantities a and c are the matrix elements of the all product C_j matrix:

$$C_1 C_2 C_3 \ldots C_{m+1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}. \quad (11)$$

By using equations (9) and (10), we can easily obtain the energy reflectance R as:

$$R = |r|^2. \quad (12)$$

For (S) and (P) polarizations, and the energy transmittance T as:

$$T_S = \mathrm{Re}\left(\frac{\hat{n}_{m+1}\cos\theta_{m+1}}{\hat{n}_0 \cos\theta_0}\right)|t_S|^2, \quad (13)$$

$$T_P = \mathrm{Re}\left(\frac{\cos\theta_{m+1}/\hat{n}_{m+1}}{\cos\theta_0/\hat{n}_0}\right)\left|\frac{\hat{n}_{m+1}}{\hat{n}_0} t_S\right|^2 = \mathrm{Re}\left(\frac{\hat{n}_{m+1}\cos\theta_{m+1}}{\hat{n}_0 \cos\theta_0}\right)|t_P|^2, \quad (14)$$

for S and P polarization, respectively, where Re indicates the real part.

III. MODEL AND FORMALISM

The deformation was introduced by applying a power law, so that the coordinates y which represents the transformed object were determined using the coordinates x of the initial object in accordance with the following rule:

$$y = x^{k+1}, \tag{15}$$

where k is the deformation degree.

The initial optical phase thickness, when we apply the y function, is:

$$\phi = \frac{2\pi}{\lambda} x_0 \cos\theta . \tag{16}$$

The optical phase thickness of the j[th] layer is:

$$\varphi_j = \frac{2\pi}{\lambda} x_0 [j^{k+1} - (j-1)^{k+1}]\cos(\theta_j). \tag{17}$$

Here, $x_0 = \frac{\lambda_0}{4}$ is the optical thickness of each layer of the periodic structure, and λ_0 is the reference wavelength.

For the deformed system, the optical thickness of each layer becomes variable and depends on the j^{th} layer and the deformation degree k. So, the optical thickness of each layer after deformation by the y function takes the following form:

$$x_{0j} = x_0 [j^{k+1} - (j-1)^{k+1}] . \tag{18}$$

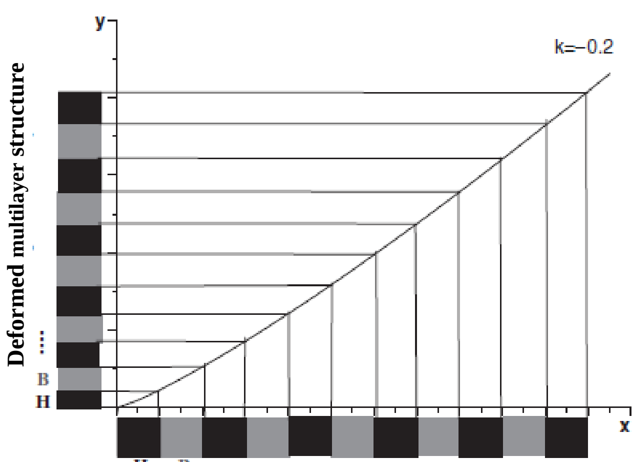

Fig. 1. Principle of introducing a deformation into a periodic multilayer structure, for example for k= -0.2.

It is clear that the optical thickness of each layer increases with k increasing. Figure 1 describes the principle of the transformed system by the y function.

For our study, refractive indices are assumed to be constant in the wavelength region of interest. We define the parameter contrast index x which presents the ratio between the high index and the low index of the layers forming the system $x = \frac{n_H}{n_B}$. The band gap width is defined as the wavelength range when T < 0.01%.

III. STUDY OF DEFORMED STRUCTURE ACCORDING TO THE LAW $y = x^{k+1}$

A. Interaction between the deformation degree and the contrast of indices

The profiles of the optical properties of the system response as a function of k as well as the optimal value of k differ according to the contrast of indices. Figure 2 presents these profiles for some values of contrast.

We perform this study for $k \in [0; 0.4]$. For a given contrast, the forbidden photonic band exists for an interval of k values and it is absent for other k values. The contrast is more important when the interval is large, for a contrast of $1.614 = n_{TiO_2} / n_{SiO_2}$ the band exists only for k lower than 0.2493, for a contrast of 2.41, the band is possible for values lower than 0.3264. Moreover, the curve $\delta\lambda = f(k)$ for a particular contrast have a tendency to increase at the beginning, reach a maximum (the k value corresponding to this maximum increases when increasing the contrast) and then decrease to a value under which, the PBG becomes absent. The PBG middle has the same profile as function of k for different contrasts, the PBG shifts towards the high wavelengths by increasing k. In order to improve the study of the contrast influence on the PBG of the deformed system, we investigate the influence of the index contrast on the deformation effect on the structure transmission spectra i.e. how the increasing of index contrast can improve the significance of deformation. For this, we choose a value of k permitting to have a PBG for the contrast interval studied which is $[1.614; 4]$.

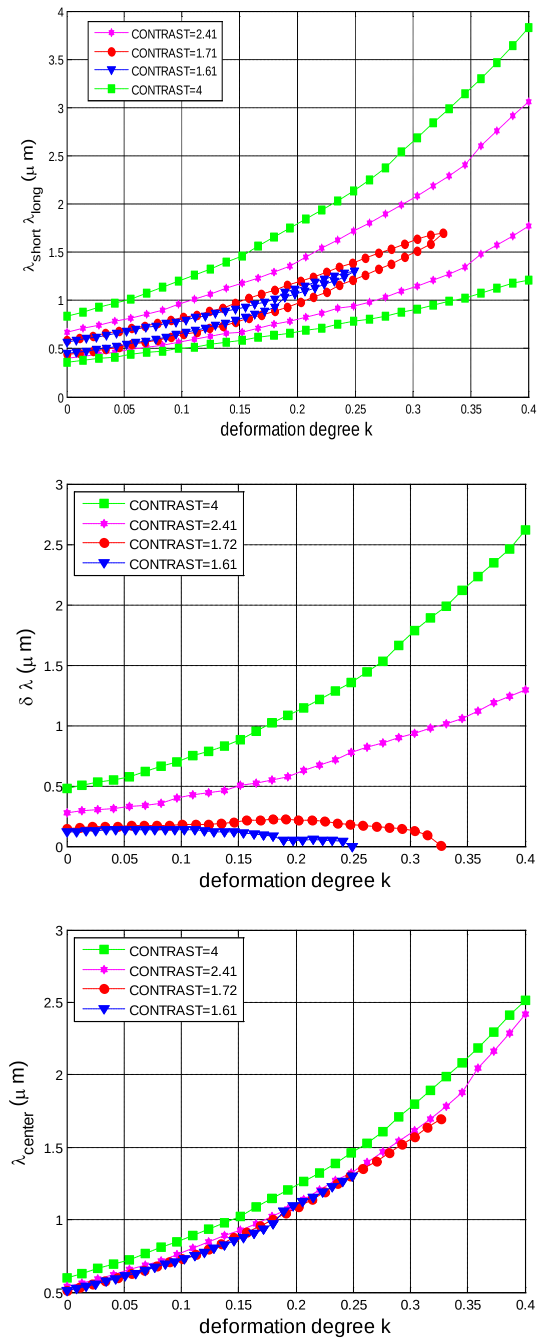

Fig. 2. PBG variation as a function of k for different values of optical index contrast. (a) Variation of the wavelengths of the negligible transmission band extremities as a function of k for different values of optical index contrast. (b) Variation of the bandwidth as a function of k for different values of optical index contrast. (c) Variation of the band center as a function of k for different values of optical index contrast.

We select, for example, the value $k = 0.2$. Then, we display the plot showing the variation of the difference between the PBG width given by a deformed system and that given by the one not deformed according to the index contrast. The widening of the forbidden band according to the contrast is noted for the not deformed system as for the deformed one, but this widening is faster and more significant for the later.

Indeed, Fig. 3 shows that the difference between the bands given by the two systems, deformed and not deformed, increases by increasing the index contrast. We generally conclude that the deformation of the system will have an interest for index contrast values relatively high. It is clear now that the best transmission spectrum is obtained for $x = 4$. Let us choose the best value of k which enables us to have the broadest band belonging to the range $[0.3; 2]$. It is the value $k = 0.2295$.

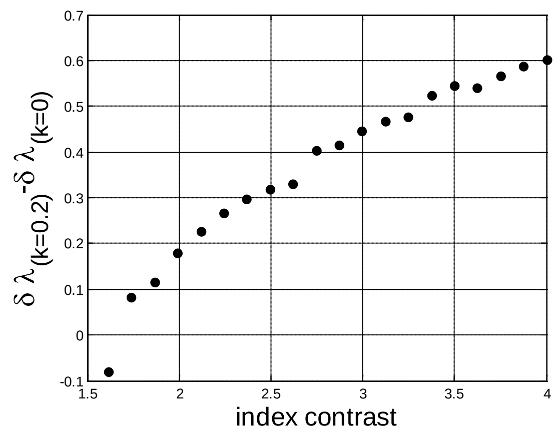

Fig. 3. Difference between the bandwidth of the deformed system and that of the not deformed one as function of the optical index contrast.

Figure 4 shows the transmission spectrum of the system with 31 layers ($j = 15$), contrast=4, and $k = 0.2295$. Its properties are: $\delta\lambda = 1.2676$ µm and $\lambda_{center} = 1.3662$ µm. We note that the PBG covers the three telecommunication wavelengths 0.85 µm, 1.3 µm, and 1.54 µm.

So, these results revealed that the choice of the optical index contrast is very influencing to the quality of the deformation when we consider the normal incidence. That gives us a liberty to choose

x and k according to the needs (spectral range, central wavelength, etc.).

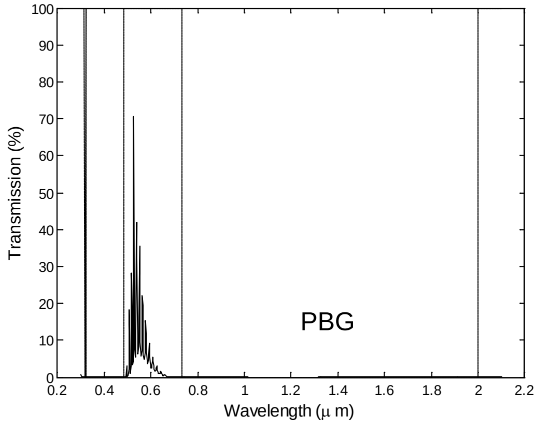

Fig. 4. Transmission at normal incidence of the deformed multilayer structure $TiO2(SiO2/TiO2)^{15}$ as a function of wavelength for a reference wavelength λ_0= 0.5 μm and P-polarized light. k=0.2295, x=4. Condition PBG: T<0.01 %.

B. Interaction between the deformation degree and the number of periods

We consider now the interaction between the degree of deformation and the number of periods forming the structure. The indices of the layers are $n_H = 2.34$ and $n_B = 1.45$, the reference wavelength is 0.5 μm.

Figure 5 shows that for a given number of periods, the PBG exists only for an interval of values of k. For example, for $j = 10$, the band exists only if $k \in [0;0.0643]$. For $j = 15$, k must belong to $[0;0.2493]$. For $j = 18$, k must belong to $[0;0.34]$. If we compare Fig. 5 with Fig. 2 which gives the tendency of the optical properties of the transmission spectrum when varying k for different values of contrast, we notice that the influence of increasing j does not represent a great contribution to ameliorate the deformation effect on the structure response. In this case, the PBG middle shifts quickly towards the higher wavelengths when increasing k. Thus, to increase j doesn't represent a great interest for the studied deformed multi-layer structure.

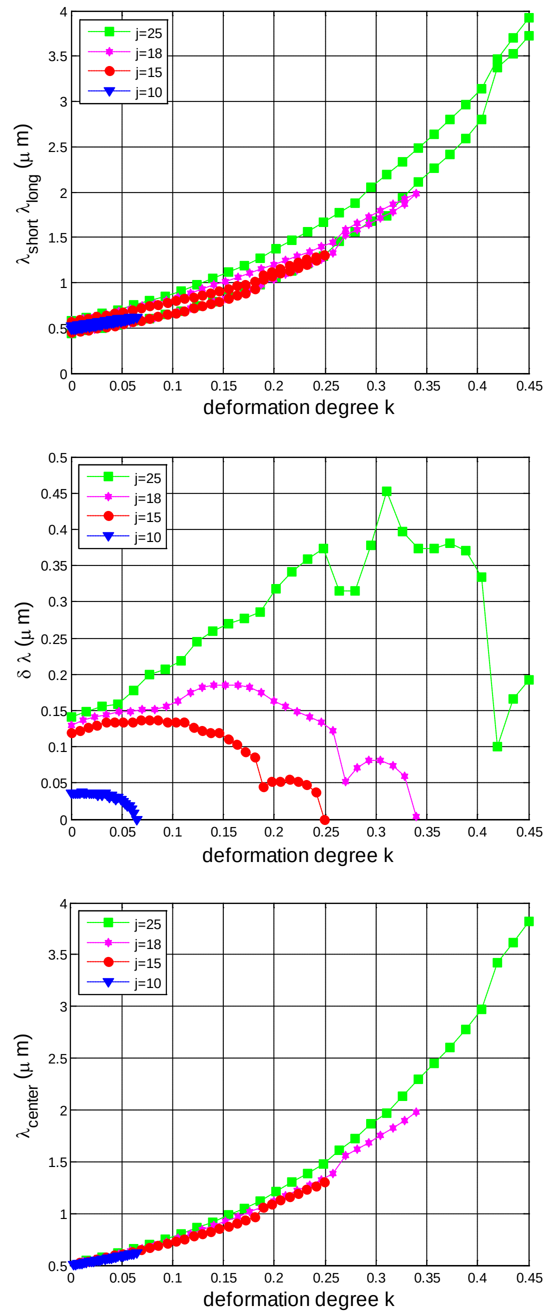

Fig. 5. PBG variation as a function of k for different values of period's number. (a) Variation of the wavelengths of the negligible transmission band extremities as a function of k for different values of period's number. (b) Variation of the bandwidth as a function of k for different values of period's number. (c) Variation of the band center as a function of k for different values of period's number.

Figure 6 gives for each j the corresponding omnidirectional bandwidth for $x = 4$ and $k = 0.2295$. With these conditions, the minimum number of periods is 7 which permits to have a PBG (width =0.0264 µm).

Figure 7 shows the variation of the difference between the PBG width given by a deformed system and that given by the not deformed system according to j with j varies between 7 and 15 periods. For $j = 7$, the deformation has a negative effect on the system response because it reduces the complete bandwidth compared to the not deformed system. But starting from $j = 8$, the deformation improves the response of the system.

C. Interaction between the deformation degree and the reference wavelength λ_0

By studying the interaction between the reference wavelength variation and that of the deformation degree (Fig. 8), we can say that the reference wavelength variation doesn't have any effect on k optimization. Moreover, the curves corresponding to the various values of λ_0 have almost the same tendency, the widening of the band and its displacement according to λ_0 are already noted for the not deformed system. So, the variation of λ_0 does not influence the effect of the deformation.

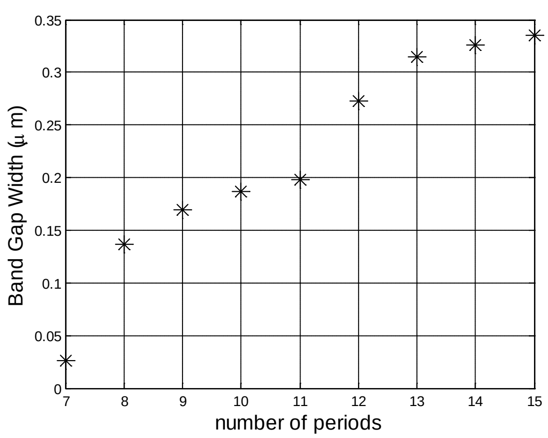

Fig. 6. Variation of the bandwidth as function of the period's number. x=4, k=0.2295.

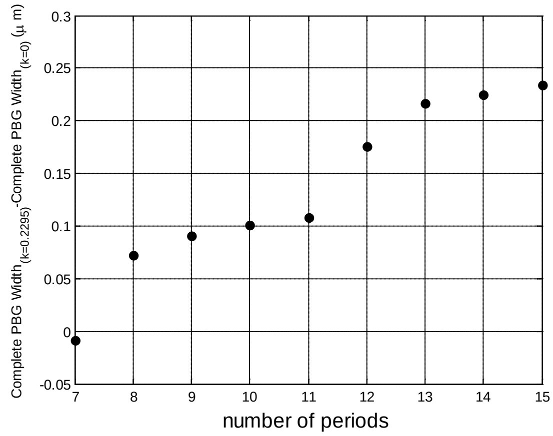

Fig. 7. Difference between the bandwidth of the deformed system and that of the not deformed one as function of the period's number. x =4, k=0.2295.

Moreover, the increase of λ_0 doesn't make it possible to increase the degree of deformation, we are always limited by the value 0.2493 of k. Some is the value of λ_0, the PBG does not exist for values of $k > 0.2493$. The optimal value of k for all the curves is the same one, it is 0.07737.

Figure 9 shows the difference between the PBG width given by a deformed system with $k = 0.07737$ and that given by the not deformed system according to λ_0, it is clear that the difference does not progress much with the wavelength reference. We thus note that the variation of the reference wavelength doesn't represent a good way to improve the effect of the deformation on the system response.

VI. CONCLUSION

We can consider the present work very interesting since it presents an optimization multi-parameter of the chirped multi-layer structure. We investigated the behaviour of the optical properties of the system versus the variation of its parameters. The interest of the deformation is not always concluded, it depends on the selected parameters of the system. This study can represent a support which gives for which parameters of the system we can consider the deformation interesting and which are the corresponding values of k optimum. In general, we can say that if we want to improve the performances of the optical

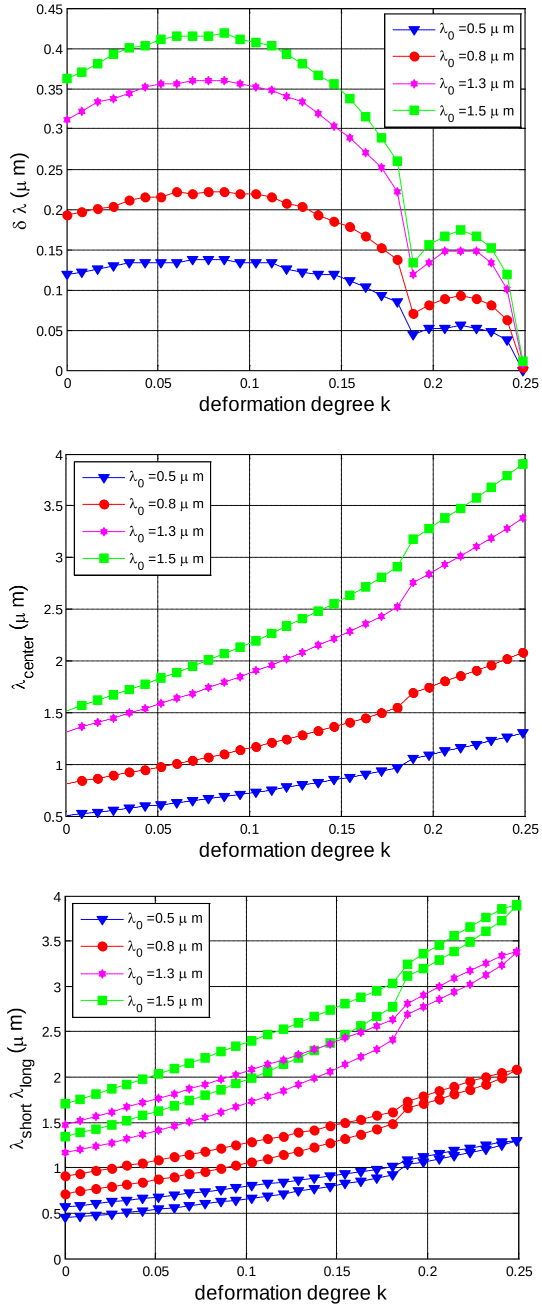

Fig. 8. PBG variation as a function of k for different values of reference wavelength. (a) Variation of the bandwidth as a function of k for different values of reference wavelength. (b) Variation of the band center as a function of k for different values of reference wavelength. (c) Variation of the wavelengths of the negligible transmission band extremities as a function of k for different values of reference wavelength.

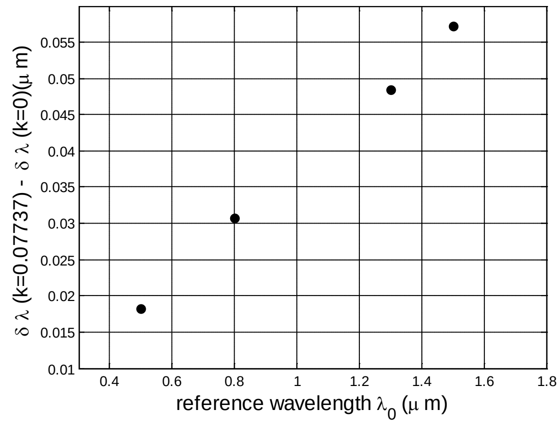

Fig. 9. Difference between the bandwidth of the deformed system and that of the not deformed one as function of the reference wavelength. k=0.07737.

components at normal incidence, it is preferable to choose the maximum index contrast and the highest degree of deformation which gives the best response which meets our needs.

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