Review of Advanced Electromagnetic Modeling Techniques in the Computer Code FEKO based on the Method of Moments with Hybrid Extensions

Ulrich Jakobus

EM Software & Systems GmbH
Otto-Lilienthal-Str. 36, D-71034 Böblingen, Germany
E-Mail: u.jakobus@emss.de

Abstract — The aim of the present tutorial is to introduce some advanced electromagnetic modeling techniques based on the Method of Moments (MoM) with various hybrid extensions. We are using the computer code FEKO [1] as a reference, and illustrate in the following several extensions that have been made in FEKO to the classical MoM in order to allow an efficient and fast analysis of a variety of complex electromagnetic radiation and scattering problems. The aim is not to go too much into the technical details (for the interested readers suitable references will be given), but rather to present an overview only, with a few selected application examples.

1 Introduction

Today it is no longer possible to imagine being engaged in antenna design or solving EMC problems without the help of computer modeling. These tools have evolved to an indispensable aid for engineers by not only complementing measurements, but at the same time reducing the number of such measurements, and as a consequence resulting in faster design cycles and reduced costs (just to give one example, a single RCS or antenna radiation pattern measurement of a ship in the open sea is quite expensive).

A variety of such computational methods exist for the numerical solution of Maxwell's equations. These include for instance FDTD (finite difference time domain), FEM (finite element method), MoM (method of moments), TLM (transmission line matrix method), PEEC (partial element equivalent circuit) and many more. It is beyond the scope of this tutorial to give a comprehensive overview of these techniques and to discuss the advantages or disadvantages of each. Some material can be found in Refs. [2–4] and in many other books and review papers.

Depending on the specific problem under consideration (such as time versus frequency domain, metallic object or highly heterogeneous material, closed structure or open radiation problem, low or high frequency etc.) some of the available numerical techniques might be more suitable than others. There is no method which can claim to be the best. Thus we believe that the future belongs to hybrid methods, where several different numerical techniques are combined in order to use the most suitable solver for a specific part of a problem.

In the following we focus on the frequency domain MoM with various extensions. The MoM has been developed originally by Harrington [5], and has since been extended by numerous research groups around to world to a mature and powerful technique.

This paper is organized as follows: The MoM is briefly introduced in Section 2, both for metallic and dielectric scattering problems. Then several options to solve high frequency problems within the MoM framework are discussed in Section 3, namely current- and ray-based hybrid methods, but also fast integral equation techniques.

2 Classical Method of Moments

2.1 Metallic objects

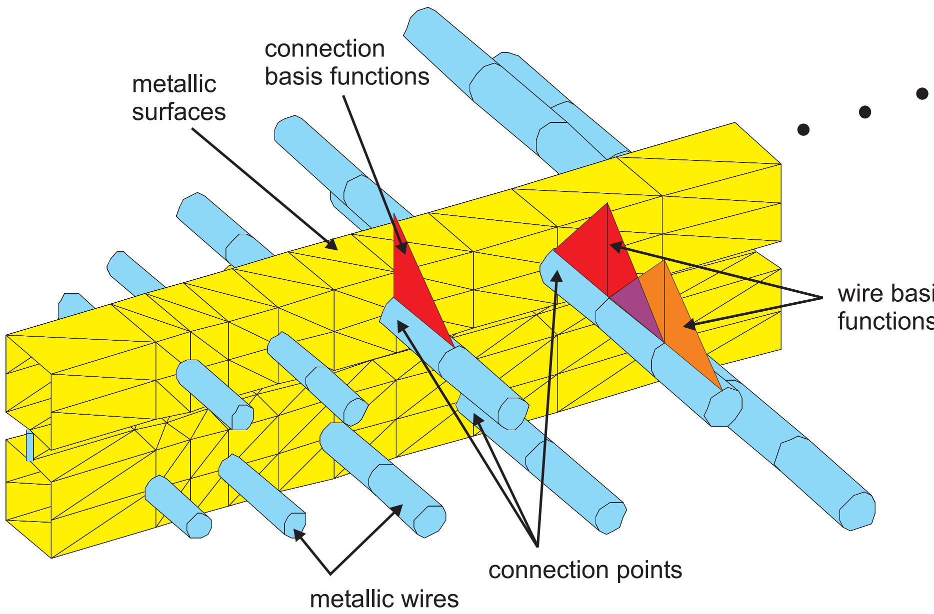

Fig. 1: General representation of a meshed region for the MoM consisting of surfaces and wires.

A general metallic object consisting of metallic wires and surfaces is shown in Fig. 1. This specific structure is part of a logarithmic periodic antenna, but the following discussions are very general and independent of the actual shape.

Metallic wires are discretized into electrically short (i.e. short as compared to the wavelength) wire elements, so-called segments. As indicated in Fig. 1, we are using overlapping triangular basis functions g_n to model the line current I along wires:

$$I = \sum_{n=1}^{N_I} \beta_n \cdot g_n \quad . \tag{1}$$

Likewise, on the metallic surfaces we are using a triangular mesh, and to represent the electric surface current density $\vec{J}$, the well known Rao Wilton Glisson rooftop basis functions $\vec{f}_n$ are used [6], also known as CN/LT (constant normal, linear tangent) [4]. One obtains

$$\vec{J} = \sum_{n=1}^{N_J} \alpha_n \cdot \vec{f}_n \tag{2}$$

with unknown expansion coefficients α_n.

In general, one must also make provision for a current flow from wires to surfaces, so called wire / plate junctions. Special basis functions need to be considered there, which are more or less one half basis function g_n on the wire (maximum current at the connection point) and a singular (with respect to the surface current density $\vec{J}$) behavior on the plate. Further details shall be omitted here, the interested reader is referred to [7–11]. Our implementation is similar to [11] and [12], with some modifications.

Within the MoM framework the unknown expansion coefficients α_n in eqn. (2) and β_n in eqn. (1) are obtained by means of solving a system of linear equations with $N = N_J + N_I$ unknowns. For electrically large structures, N will be large, and thus both memory requirement for the matrix of the system of linear equations (scaling with N^2) as well as the run-time for the solution (scaling typically with N^3 for direct solvers) might be prohibitive. Ways to overcome this problem will be discussed in Section 3.

The system of linear equations results from applying certain boundary conditions. For perfectly conducting objects we are using the electric field integral equation (EFIE) based on

$$\vec{E}_{tan} = \vec{E}_{s,tan} + \vec{E}_{i,tan} = 0 \tag{3}$$

with the scattered (s) and impressed (i) contributions. Explicit expressions for the scattered fields as a function of the sources I and $\vec{J}$ shall be omitted here, they can be found elsewhere (e.g. [13–15]).

We are using the EFIE due to the generality of its application also to open bodies. The alternative formulations MFIE (magnetic) and CFIE (combined) offer advantages: The MFIE is more robust and stable since it is based on a Fredholm integral equation of the second kind, and thus for instance iterative techniques converge faster, and the CFIE in addition to this lower condition number is also more robust with respect to internal resonances. However, both MFIE and CFIE require the bodies to be closed, which is not the case for many practical problems.

Thus we have to use the EFIE in these cases. This is not really a problem, since numerous studies have shown an excellent stability also at resonances, and the relatively high condition number can be dealt with by using either robust direct solvers based on an LU-decomposition with double precision accuracy, or when iterative techniques are required like for the MLFMM (multilevel fast multipole method) then good preconditioners have to be developed (for instance an incomplete LU-decomposition with a small level of fill-in).

In order to show the application of the MoM to metallic bodies, just one simple example shall be presented here: Scattering from a brass strip with dimensions 63.6 mm tall, 6.3 mm wide, and

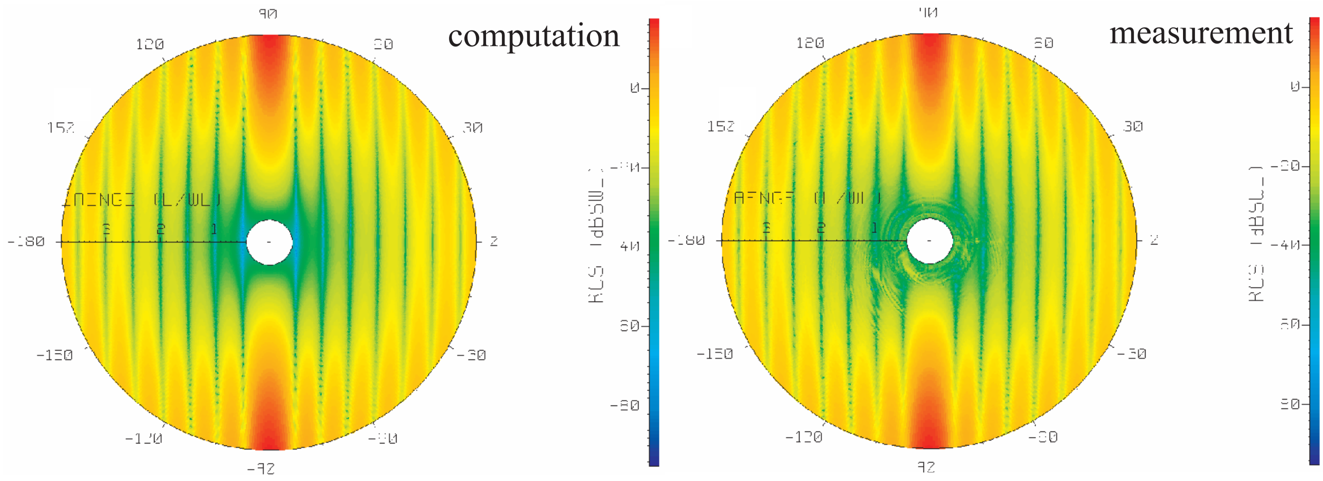

Fig. 2: Computed (left) and measured (right) RCS of a brass strip as a function of the observation angle in the far-field and the frequency (radial direction).

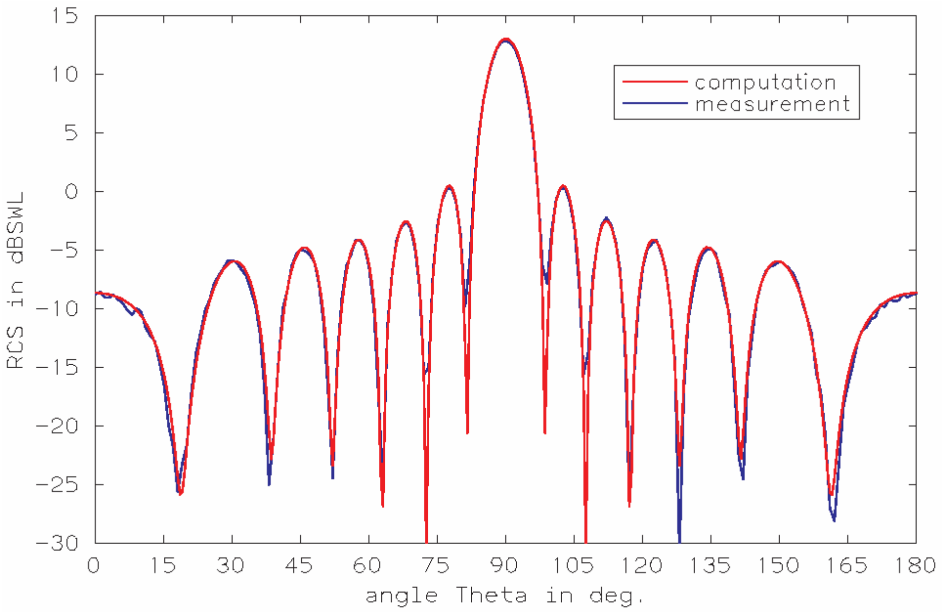

Fig. 3: Comparison of measured and computed RCS for one specific frequency of 15 GHz.

0.32 mm thick. In Fig. 2 we see on the left hand side the computed RCS, as a function of the far-field angle (also angle in the graph) and as a function of the frequency (in radial direction we have length of the strip normalized to the wavelength, l/λ). The graph to the right shows the measured RCS, these highly accurate measurements have been taken from [16, 17].

One sees some small irregularities for the measured data around the center due to measurement errors, but otherwise measured and computed results are in very close agreement. This is confirmed by a quantitative 2-D plot for one single frequency: Fig. 3 shows the RCS for a fixed frequency of 15 GHz (i.e. $\lambda = 20\,\text{mm}$, then circle in Fig. 2 with a fixed radius $l/\lambda = 63.6/20 = 3.18$). Also this result of RCS versus angle shows a very good agreement.

2.2 Dielectric bodies

Though not the main focus of this paper, for the sake of completeness it shall be mentioned that the MoM is also very well suited for the treatment of dielectric / magnetic bodies (also lossy). The author has compiled another review paper on this topic, see [18], and thus here the main techniques shall be mentioned only:

- Surface equivalence principle for partly homogeneous regions: Here equivalent electric and magnetic surface current densities $\vec{J}$ and $\vec{M}$, respectively, are introduced at the interfaces between different media. Different integral formulations are possible, we focus on the PMCHW technique [13, 19].

- Volume equivalence principle for inhomogeneous bodies. This is based on a 3-D volumetric discretization of the material (as opposed to FEM or FDTD not of the surrounding air). But due to the dense nature of the MoM matrices this technique typically results in big matrices, and a FEM/MoM hybrid method might be the preferred approach for solving highly inhomogeneous scattering problems.

- Special Green's function techniques for selected geometries only. We support this in FEKO for layered spherical bodies [20] and planar multilayer substrates [21]. The latter has efficient far-field approximations, and sophisticated interpolation strategies for a fast evaluation of the occuring Sommerfeld integrals. In principle, this Green's function method can be used for any geometry where Maxwell's equations can be solved analytically, also cylindrical or ellipsoidal structures.

- Special formulations applicable to selected configurations only, but highly accurate and in particular very efficient, see e.g. the treatment of dielectrically coated wires as described in [18], or also the thin dielectric sheet formulations which the MoM offers (e.g. for glass windows of a car).

Further details and in particular many more references to original papers can be found in [18]. Similar to metallic bodies, also one example shall be presented here for the dielectric bodies. Fig. 4 shows the analysis of a planar inverted F antenna (PIFA) in the 1.8 GHz mobile phone range. The solid line without symbols indicates the effective gain in the horizontal plane, if the mobile phone radiates in free space. The two lines with the symbols show the reduction in effective gain due to power loss in the hand, if the hand is included in the model at two different positions partly covering the antenna.

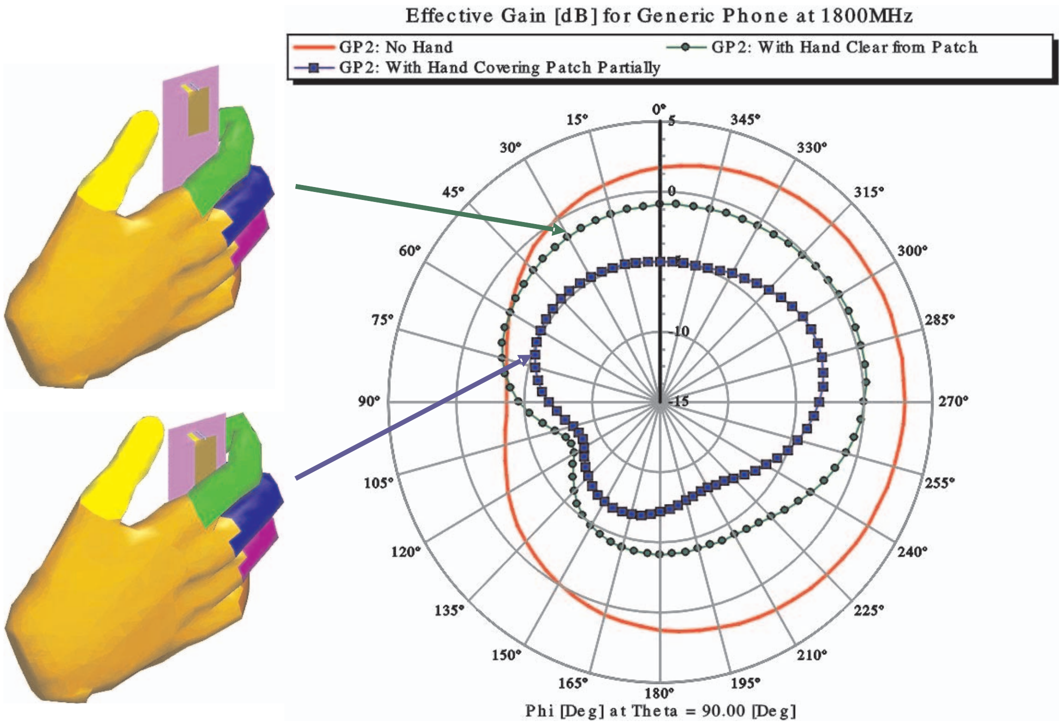

Fig. 4: Influence of the hand on the radiation characteristics of a mobile phone.

3 Solution of high frequency problems

3.1 General considerations

We have seen in Section 2 that for the MoM we need to discretize into electrically small elements (triangular patches, wire segments), and that we end up with N unknowns that need to be determined from the solution of a dense complex system of linear equations.

If we consider the aircraft example in Fig. 5, then for a frequency of 100 MHz we require a mesh of 20 337 metallic triangles, resulting in $N = 30319$ basis functions (each basis function is associated with an edge between two triangular patches). Using double precision accuracy with 16 Bytes for one complex number, the memory requirement for the MoM matrix is $16\,\text{Bytes} \cdot N^2 = 13.7\,\text{GByte}$.

This is quite large, but not a problem for any modern computer system, in particular if one supports (like FEKO does) parallel processing in connection with efficient out-of-core solver techniques. With an out-of-core solver, one can even solve this structure on a simple notebook computer with the traditional MoM (provided there is enough hard disk space of course).

However, one runs into serious trouble when the frequency is higher than the 100 MHz here. If we only double the frequency, then for 2-D surface meshes, N is already four times larger, and the memory requirement scales with N^2, i.e. then 16 times larger (219 GByte). Trying to solve the aircraft with the traditional MoM at 1 GHz would require approximately 134 TByte, which makes any solution impossible.

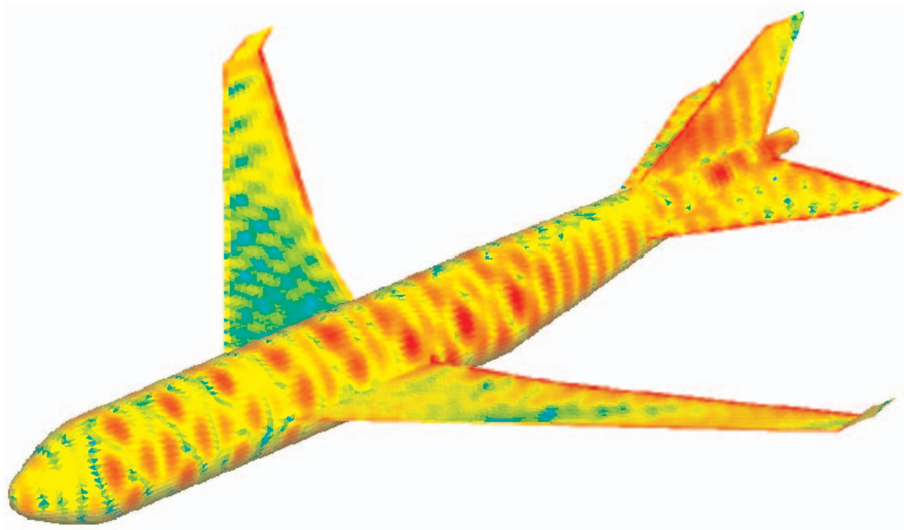

Fig. 5: Model of an aircraft with surface current density when illuminated by an external antenna of the navigation system at 100 MHz.

Alternative techniques have to be used, either by switching to fast integral equation methods, or by using a combination of the MoM with asymptotic high frequency methods. These shall be discussed in the following.

3.2 Current based hybrid method

The MoM is a current based technique: In eqn. (2) we approximate the surface currents $\vec{J}$ by the linear superposition of basis functions with unknown coefficients α_n. The main problem is that within the MoM formulation we need to solve a system of linear equations in order to obtain these N coefficients.

The idea for the current based hybrid method can be illustrated with the help of Fig. 6: Still everywhere currents are introduced, which are unknown in the first place. They are also again expressed as the linear superposition of basis functions with unknown coefficients as in eqn. 2. But the main difference as compared to the MoM is that the expansion coefficients α_n for the currents J^{asym} in the asymptotic region are not determined by solving a system of linear equations, but they are obtained directly by means of high frequency approximations.

The simplest such approximation is Physical Optics (PO), where the asymptotic currents are defined as

$$\vec{J}^{PO} = \begin{cases} 2\,\hat{n} \times \vec{H}_{inc} & \text{illuminated region} \\ 0 & \text{shadowed region} \end{cases} \tag{4}$$

with the outward pointing normal vector $\hat{n}$ and the incident magnetic near-field $\vec{H}_{inc}$. Note that this is not the impressed magnetic near-field of the excitation (which is used in the MoM formulation as the known right-hand side), but this is the incident field, comprised of the impressed field plus the magnetic field radiated by the MoM currents.

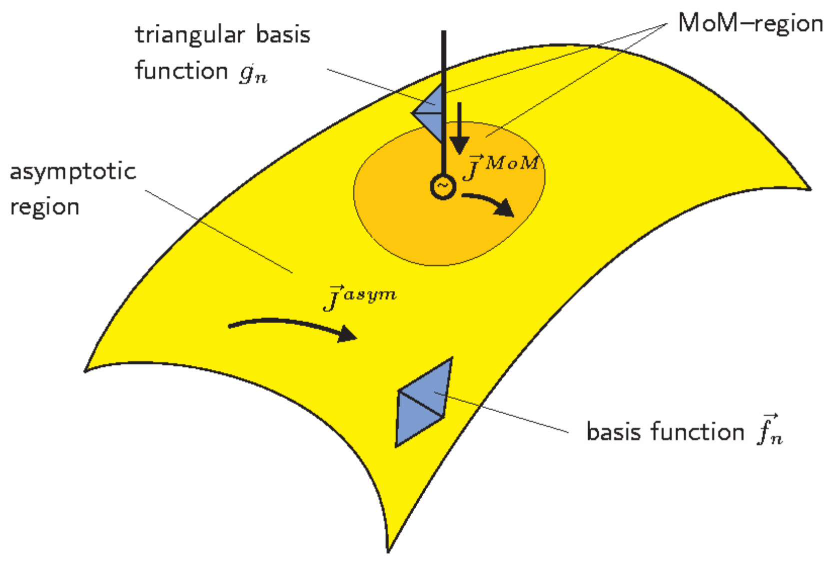

Fig. 6: General idea of splitting the domain into the MoM and an asymptotic region for the current based hybrid method.

More details and also equations for the whole solution procedure of this current based hybrid method can be found in [15, 22].

The PO formulation works reasonably well, but in particular for structures not too big, the accuracy can be improved by either switching to the Physical Theory of Diffraction (PTD) [23, 24], or as we do by adding correction terms to PO (improved PO, IPO) [25, 26] or by switching to Fock currents for curved convex surfaces [27].

As an example, we go back to the aircraft model already considered in Fig. 5. The whole setup is shown in Fig. 7, where this aircraft is standing on the taxiway (150 m parallel to the runway) and

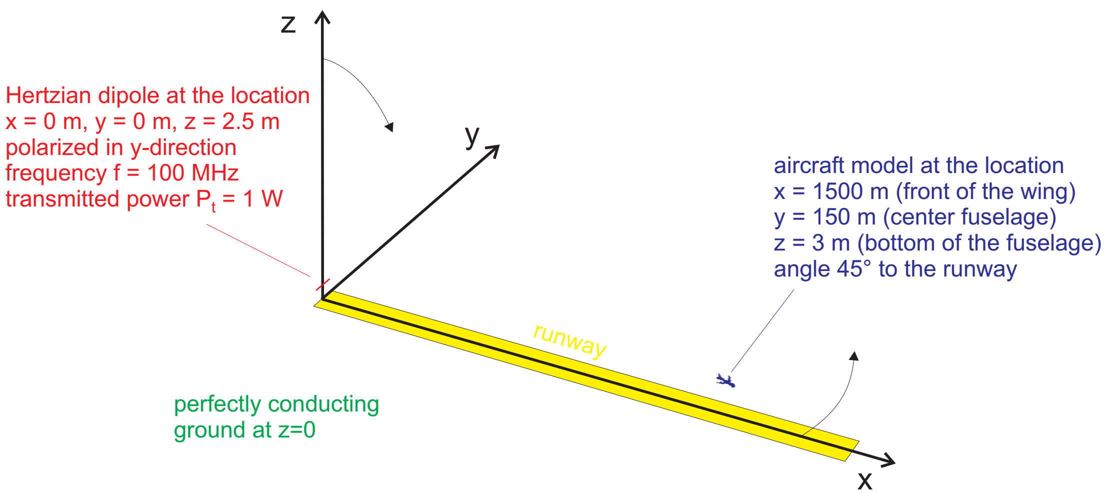

Fig. 7: Runway at an airport with the antenna systems for the instrumental landing system and a parasitic aircraft queuing at the taxiway.

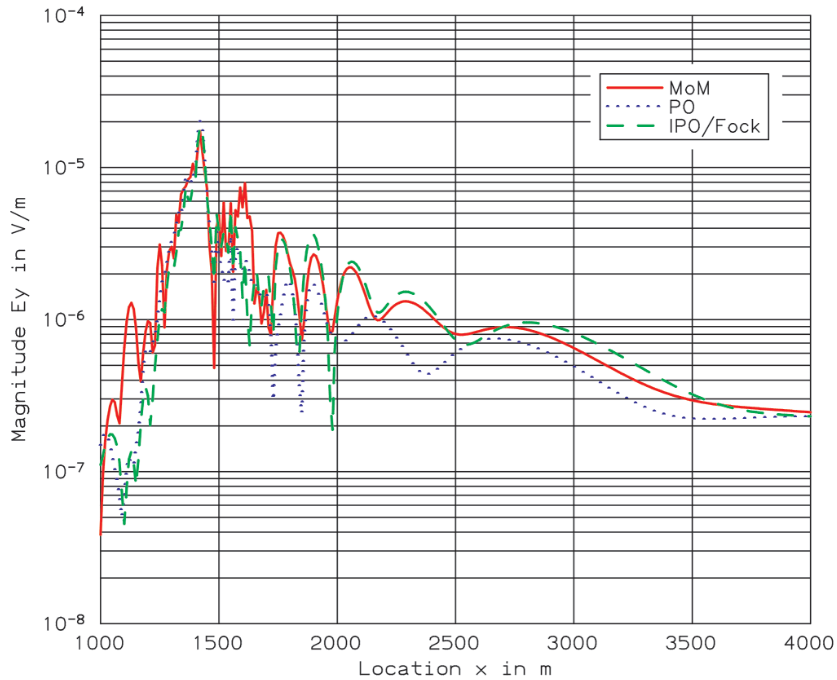

Fig. 8: Disturbance of the landing signal along the runway at an airport due to the presence of an aircraft on the taxiway. Comparison of different numerical techniques.

waiting for take off. This aircraft acts as metallic obstacle which interferes with the instrumental landing system for incoming aircraft.

This disturbance (difference to the signal without the aircraft) is plotted in Fig. 8 along the runway at a height of $z = 3\,\mathrm{m}$, and the results for the traditional MoM are compared to the asymptotic PO and IPO/Fock solutions. For the practical application, the maximum of these curves is the most important quantity, and this is very similar indeed for the three methods.

As already mentioned, for the traditional MoM the memory requirement is 13.7 GByte. Both the PO and IPO/Fock solutions do need only 10.2 MByte, i.e. a factor 1375 less. Also the run-time of the PO solution is by a factor 560 less than for the MoM. The IPO/Fock solution is a bit more time consuming due to the evaluation of special Fock Airy functions and more sophisticated ray tracing following geodesic lines etc., but it is also a factor of 80 faster than the MoM.

3.3 Ray based hybrid method

The current based hybrid method as presented in the previous section clearly reduces the cost of the MoM from N^2 (memory) and N^3 (run-time) to N for both. This dependency proportional to N is a consequence of discretizing the asymptotic domain, and thus the geometrical data (positions, edge length etc.) for N basis functions have to be stored, and when computing scattered near- or far-fields, one has to loop over the N basis functions and sum the contributions.

For very large problems like a co-site interference study on a ship (see Fig. 9) involving also antennas operating in the GHz frequency range, this dependency on N might still be too large.

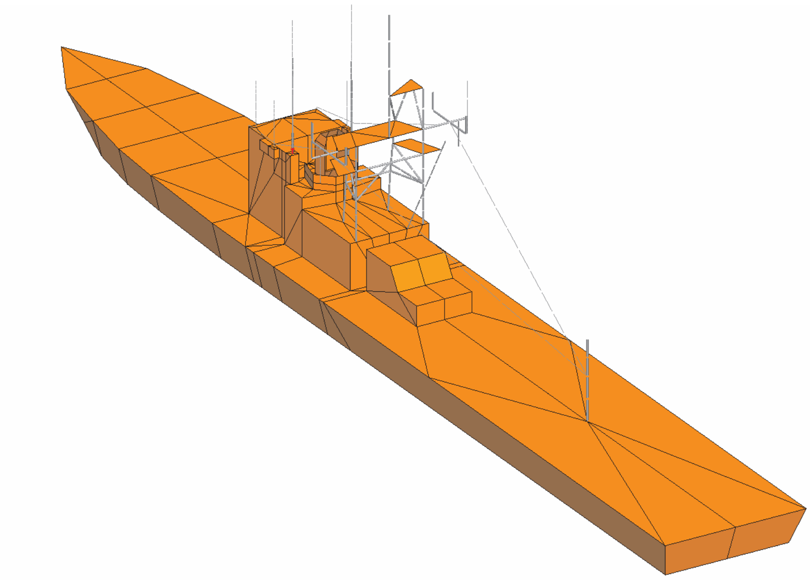

Fig. 9: Antenna coupling study on a ship.

The preferred method in this case is a combination of MoM (for the antennas) with the Uniform Theory of Diffraction (UTD, for the ship body). Then the computational cost with respect to memory and run-time is completely frequency independent. The same resources are required, whether we model antennas at 1 GHz, 10 GHz, or 100 GHz.

Such a combined MoM/UTD hybrid technique has been proposed in a number of early papers, e.g. [28,29]. The formulation regarding our combination of MoM with UTD can be found in [30,31]. There also details are given of how the UTD reflection coefficients have been modified in order to allow MoM sources to be very close (even touching) the UTD region.

3.4 Fast integral equation methods

As mentioned in the introduction, there is no computational technique which can claim to be the best, and which is applicable to all possible scenarios. This is also true for to the current- and ray-based hybrid methods presented in the previous sections. They are extremely useful for a variety of problems, but there are also situations where they cannot be applied, or turn out to be inefficient. We have made several extensions, such as allowing the treatment of dielectric bodies also with PO (see [32]), or including multiple PO reflections, or supporting the treatment of coated surfaces with PO (see [33]). But for instance for convex interior problems, the evaluation of multiple PO reflections is quite time-consuming, and UTD cannot always be used (need for canonical structures with known diffraction coefficients).

Here fast integral equation methods [34–37] can be very useful, they are as general as the MoM regarding the applicability, but have a highly reduced numerical cost. We have focused on a MLFMM (multilevel fast multiple method) implementation in FEKO, where memory scales with $N \log N$, and the CPU-time with $N \log^2 N$.

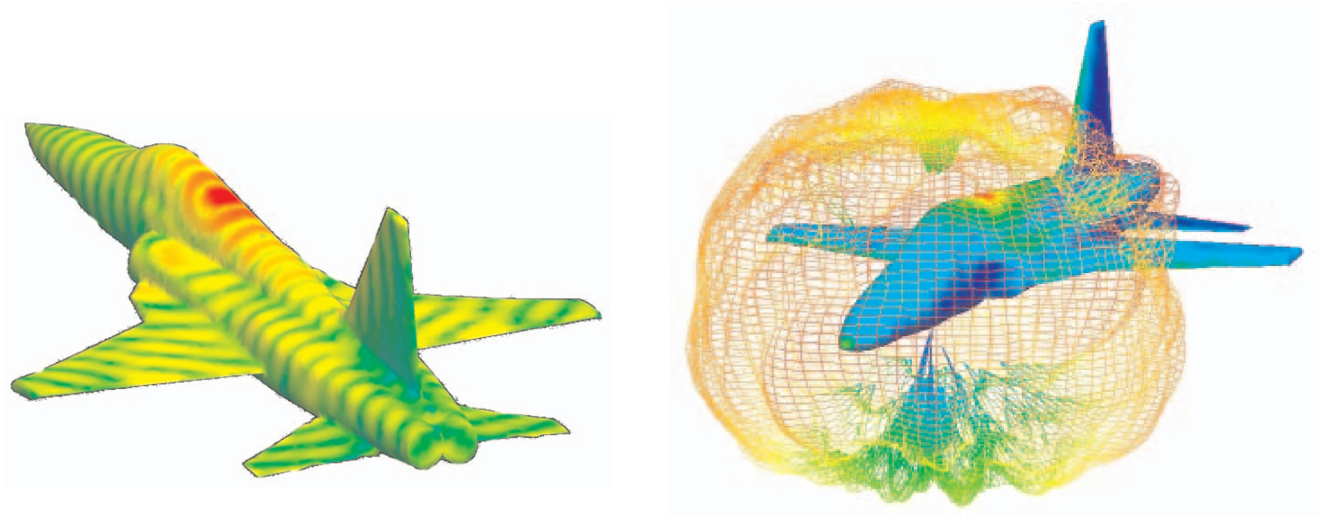

Fig. 10: Surface current density (left) as well as far-field radiation pattern (right) for antenna placement on an aircraft in the 300 MHz UHF frequency band.

In order to show the potential saving as compared to the MoM, two examples shall be considered. The first one is shown in Fig. 10, and deals with antenna placement in the 300 MHz UHF frequency band on an aircraft. The structure consists of 28 634 basis functions, and can still be solved by the traditional MoM, then requiring 12.2 GByte of memory. The MLFMM (here using 6 levels), on the other hand, just requires 437 MByte (this includes everything, also the relatively large memory of 224 MByte for the ILUT preconditioner).

We have also solved a somewhat larger problem, the bistatic RCS computation of a perfectly conducting sphere with a diameter of $10.264\,\lambda$, resulting in $N = 100005$ unknowns. The advantage of this geometry is that an exact Mie series solution exists to compare the result to (the problem is too big to be solved with the traditional MoM as reference, memory requirement 149 GByte). Both this exact and the MLFMM results are plotted in Fig. 11. A good agreement can be observed, also for the larger angles ϑ (the plane wave is incident from $\vartheta = 180°$, thus $\vartheta = 180°$ corresponds to the backscattering).

The run-time for this specific example is 14 hours on an Intel Pentium 4 PC with 1.8 GHz clock rate. It should be mentioned that this is for the EFIE, although for this specific sphere example the MFIE or CFIE would result in much faster solutions. But see the general discussion in Section 2 of why we prefer to use the EFIE.

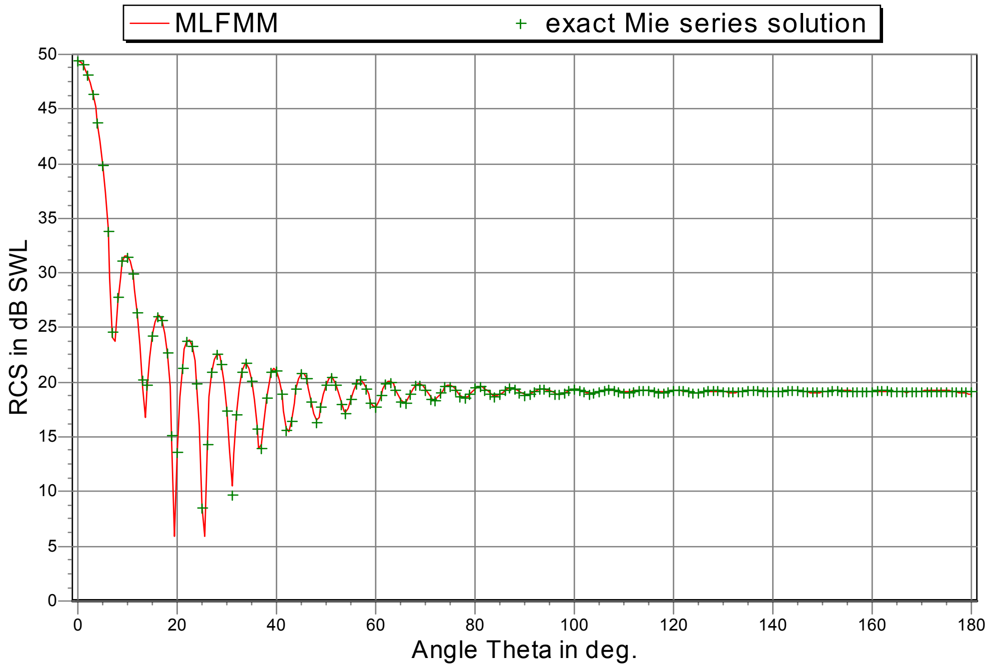

Fig. 11: Bistatic RCS computation for a perfectly conducting sphere of diameter $10.264\,\lambda$ with the MLFMM and comparison to the exact Mie series solution.

4 Conclusions

We have presented an overview on some advanced modeling techniques based on the method of moments, which allow the solution of electromagnetic radiation and scattering problems for a wide frequency range. For the higher frequencies, current- or ray-based hybrid techniques have been proposed (depending on the specific problem at hand), in addition to the popular fast integral equation techniques.

References

[1] EM Software & Systems - S.A. (Pty) Ltd, Stellenbosch, South Africa, *FEKO — Field Computations Involving Bodies of Arbitrary Shape*, 2003. `http://www.feko.info`.

[2] T. Itoh, *Numerical Techniques for Microwave and Millimeter-Wave Passive Structures*. John Wiley and Sons, 1989.

[3] M. N. O. Sadiku, *Numerical Techniques in Electromagnetics*. Boca Raton: CRC Press, 1992.

[4] A. F. Peterson, S. L. Ray, and R. Mittra, *Computational Methods for Electromagnetics*. IEEE Press, New York, 1998.

[5] R. F. Harrington, *Field Computation by Moment Methods*. New York: Macmillan Company, 1968.

[6] S. M. Rao, D. R. Wilton, and A. W. Glisson, "Electromagnetic scattering by surfaces of arbitrary shape," *IEEE Transactions on Antennas and Propagation*, vol. 30, pp. 409–418, May 1982.

[7] N. C. Albertsen, J. E. Hansen, and N. E. Jensen, "Computation of radiation from wire antennas on conducting bodies," *IEEE Transactions on Antennas and Propagation*, vol. 22, pp. 200–206, Mar. 1974.

[8] E. H. Newman and D. M. Pozar, "Electromagnetic modeling of composite wire and surface geometries," *IEEE Transactions on Antennas and Propagation*, vol. 26, pp. 784–789, Nov. 1978.

[9] A. W. C. Chu, S. A. Long, and D. R. Wilton, "The radiation pattern of a monopole antenna attached to a conducting box," *IEEE Transactions on Antennas and Propagation*, vol. 38, pp. 1907–1912, Dec. 1990.

[10] S. M. Rao, *Electromagnetic Scattering and Radiation of Arbitrarily–Shaped Surfaces by Triangular Patch Modeling*. PhD thesis, University of Mississippi, 1980.

[11] D. Zheng and K. A. Michalski, "Analysis of coaxially fed microstrip antennas of arbitrary shape with thick substrates," *Journal of Electromagnetic Waves and Applications*, vol. 5, no. 12, pp. 1303–1327, 1991.

[12] D. R. Wilton and S. U. Hwu, "JUNCTION: a computer code for the computation of radiation and scattering by arbitrary conducting wire/surface configurations," in *Proceedings of the 5th Annual Review of Progress in Applied Computational Electromagnetics, Monterey*, pp. 43–51, ACES, Mar. 1989.

[13] A. J. Poggio and E. K. Miller, "Integral equation solutions of three–dimensional scattering problems," in *Computer Techniques for Electromagnetics* (R. Mittra, ed.), ch. 4, pp. 159–264, Oxford: Pergamon Press, 1973.

[14] N. Morita, N. Kumagai, and J. R. Mautz, *Integral Equation Methods for Electromagnetics*. Boston: Artech House, 1990.

[15] U. Jakobus and F. M. Landstorfer, "Improved PO–MM hybrid formulation for scattering from three–dimensional perfectly conducting bodies of arbitrary shape," *IEEE Transactions on Antennas and Propagation*, vol. 43, pp. 162–169, Feb. 1995.

[16] C. W. Trueman and S. R. Mishra, "A WWW–based data base for code validation," in *ACES 12th Annual Review of Progress in Applied Computational Electromagnetics*, vol. II, pp. 1092–1099, Mar. 1996.

[17] C. L. Larose, C. W. Trueman, and S. R. Mishra, "Measured RCS polar contour maps for code validation," *Applied Computational Electromagnetics Society Journal*, vol. 11, pp. 25–43, Nov. 1996.

[18] U. Jakobus, "Comparison of different techniques for the treatment of lossy dielectric/magnetic bodies within the method of moments formulation," *AEÜ International Journal of Electronics and Communications*, vol. 54, no. 3, pp. 163–173, 2000.

[19] B. M. Kolundžija, "Electromagnetic modeling of composite metallic and dielectric structures," *IEEE Transactions on Microwave Theory and Techniques*, vol. 47, pp. 1021–1032, July 1999.

[20] H.-O. Ruoß, U. Jakobus, and F. M. Landstorfer, "Efficient EM analysis of hand–held mobile telephones close to human head using modified method of moments," *Electronics Letters*, vol. 31, pp. 947–948, June 1995.

[21] J. J. van Tonder and U. Jakobus, "Full–wave analysis of arbitrarily shaped geometries in multilayered media," in *14th International Zurich Symposium on Electromagnetic Compatibility*, pp. 459–464, Feb. 2001.

[22] U. Jakobus and F. M. Landstorfer, "Current–based hybrid moment method analysis of electromagnetic radiation and scattering problems," *Applied Computational Electromagnetics Society Journal*, vol. 10, pp. 38–46, Nov. 1995. Special Issue on Advances in the Application of the Method of Moments to Electromagnetic Radiation and Scattering Problems.

[23] P. Y. Ufimtsev, Ме́тод краевых волн в физи́ческой тео́рии дифракции *(The Method of Edge Waves in the Physical Theory of Diffraction).* Москва́: Сове́тское Радио Пресс, 1962.

[24] P. Y. Ufimtsev, "Elementary edge waves and the physical theory of diffraction," *Electromagnetics*, vol. 11, pp. 125–160, 1991.

[25] U. Jakobus and F. M. Landstorfer, "Improved physical optics approximation for flat polygonal scatterers," in *11th International Zurich Symposium on Electromagnetic Compatibility*, pp. 355–360, Mar. 1995.

[26] U. Jakobus and F. M. Landstorfer, "Improvement of the PO–MoM hybrid method by accounting for effects of perfectly conducting wedges," *IEEE Transactions on Antennas and Propagation*, vol. 43, pp. 1123–1129, Oct. 1995.

[27] U. Jakobus and F. M. Landstorfer, "Hybrid MM–PO–Fock analysis of monopole antennas mounted on curved convex bodies," in *Conference Proceedings of the 12th Annual Review of Progress in Applied Computational Electromagnetics*, (Monterey), pp. 101–108, Applied Computational Electromagnetics Society, Mar. 1996.

[28] E. P. Ekelman and G. A. Thiele, "A hybrid technique for combining the moment method treatment of wire antennas with the GTD for curved surfaces," *IEEE Transactions on Antennas and Propagation*, vol. 28, pp. 831–839, Nov. 1980.

[29] G. A. Thiele and T. H. Newhouse, "A hybrid technique for combining moment methods with the geometrical theory of diffraction," *IEEE Transactions on Antennas and Propagation*, vol. 23, pp. 62–69, Jan. 1975.

[30] U. Jakobus and F. M. Landstorfer, "A combination of current– and ray–based techniques for the efficient analysis of electrically large scattering problems," in *Conference Proceedings of the 13th Annual Review of Progress in Applied Computational Electromagnetics*, (Monterey), pp. 748–755, Applied Computational Electromagnetics Society, Mar. 1997.

[31] I. P. Theron, D. B. Davidson, and U. Jakobus, "Extensions to the hybrid method of moments/uniform GTD formulation for sources located close to a smooth convex surface," *IEEE Transactions on Antennas and Propagation*, vol. 48, pp. 940–945, June 2000.

[32] U. Jakobus, "Extension of the MoM/PO hybrid technique to homogeneous dielectric bodies," in *Conference Proceedings of the 14th Annual Review of Progress in Applied Computational Electromagnetics*, vol. II, (Monterey), pp. 920–927, Applied Computational Electromagnetics Society, Mar. 1998.

[33] U. Jakobus and I. P. Theron, "Treatment of coated metallic surfaces with physical optics for the solution of high–frequency EMC problems," in *15th International Zurich Symposium on Electromagnetic Compatibility*, pp. 257–261, Feb. 2003.

[34] E. Bleszynski, M. Bleszynski, and T. Jaroszewicz, "AIM: Adaptive integral method for solving large–scale electromagnetic scattering and radiation problems," *Radio Science*, vol. 31, pp. 1225–1251, Sept. 1996.

[35] W. C. Chew, J.-M. Jin, C.-C. Lu, E. Michielssen, and J. M. Song, "Fast solution methods in electromagnetics," *IEEE Transactions on Antennas and Propagation*, vol. 45, pp. 533–543, Mar. 1997.

[36] C.-C. Lu and W. C. Chew, "A multilevel algorithm for solving a boundary integral equation of wave scattering," *Microwave and Optical Technology Letters*, vol. 7, pp. 466–470, July 1994.

[37] R. Coifman, V. Rokhlin, and S. Wandzura, "The fast multipole method for the wave equation: A pedestrian prescription," *IEEE Antennas and Propagation Magazine*, vol. 35, pp. 7–12, June 1993.