Efficient Matrix Element Calculations for the Spectral Domain Method Applied to Symmetrical Multiconductor Transmission Lines

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Abstract

The spectral domain method has proved to be a suitable analytical tool for the characterization of open, symmetrical planar structures. The method requires the repeated calculation of matrix elements, which each involves the time-consuming process of numerical integration over an infinite range. In this paper, suitable basis functions for the expansion of electric surface currents on strips or electric fields in slots, are provided. It is also shown how the use of these basis functions makes possible the efficient and rapid calculation of the matrix elements.

1. Introduction

The spectral domain method has become an important tool in the analysis of microwave and millimetre wave integrated circuits [1]. Of particular interest in this paper is its application on the characterization of open, symmetrical planar structures where substrates are assumed to be infinitely wide [2-6]. The method generally requires a significant amount of analytical preprocessing, but the introduction of the immitance approach [2] has simplified the derivation of the dyadic Green's function elements. However, some practical difficulties are still encountered during the implementation of the method, and in this paper we show how these may be overcome.

Application of the immitance approach yields the spectral dyadic Green's function for the planar structure under consideration. The unknown electric surface currents on strips (or electric fields in slots) are expanded into finite sets of basis functions. For a solution of the dispersion characteristics, the method requires an iterative search for the value of the axial propagation constant, β, which renders the determinant of a square matrix to zero. The matrix elements need to be recalculated during each iteration. The computation of each individual matrix element requires numerical integration over an infinite range, where the integrand contains basis functions that have been used in the expansions. These calculations are the most time-consuming steps in the implementation of the method. Due to the slow rate of convergence of certain integrals, difficulties are encountered in attempts to attain the required accuracy tolerances during the numerical integration. In this paper, we provide suitable basis functions for the expansion of unknown electric currents or fields. The use of these basis functions facilitates the efficient calculation of the matrix elements.

2. Suitable basis functions

Consider the general multilayered planar structure shown in Figure 1. It consists of a number of dielectric layers, with strips and/or slots spaced symmetrically about the y-axis between the different layers. The structure has infinitely thin metallized surfaces with infinitely wide dielectric substrates, and is homogeneous in the z-direction. It may be analyzed in the spectral domain, with the Fourier transform defined as

$$\tilde{\Phi}(\alpha) = \int_{-\infty}^{\infty} \Phi(x)\, e^{j\alpha x}\, dx$$
$$\Phi(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{\Phi}(\alpha)\, e^{-j\alpha x}\, d\alpha \tag{1}$$

The electric surface currents on all finite width strips and the transverse (with respect to the y direction) electric fields in finite width slots are expanded into finite sets of basis functions. Using the immitance approach [2], equivalent circuits may be constructed, where the expanded quantities act as current and voltage sources respectively. From the equivalent circuits, the elements of the spectral dyadic Green's function are obtained in closed form.

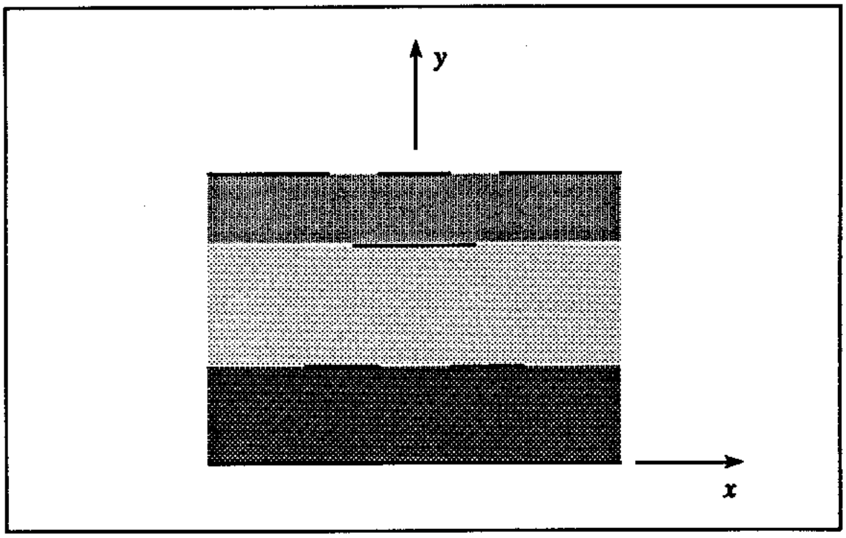

Figure 1 Symmetrical multilayered planar structure.

In symmetrical planar structures, the strips or slots in a specific plane $y = y_1$ belong to one of two categories, namely

1. A *single* strip or slot centred at $x = 0$, as shown in Figure 2(a).
2. A *pair* of strips or slots centred at $x =$ b and $x =$ -b respectively, as depicted in Figure 2(b).

A multiconductor transmission line supports a number of fundamental modes. For symmetrical structures, each mode belongs to one of two mode types, which we denominate as *even* and *odd* type modes. The field distribution of even (odd) type modes is characterized by the fact that a magnetic conductor (electric conductor) may be introduced in the plane of symmetry without disturbing the fields. Depending on the mode type, only basis functions which are either even or odd functions of x need to be included in the expansion of currents on single strips or fields in single slots. The same symmetry considerations hold for the case of a pair of strips or slots. It is however necessary to include basis functions which are both even and odd with respect to the individual axes of a pair of strips or slots [6]. This could be explained by means of the following example. Let Figure 2(b) represent a pair of strips, and we would like to expand the electric surface current $J_x(x, y_1)$ for an even type mode. From the symmetry considerations it then follows that $J_x(x, y_1)$ should be an odd function of x, and that its basis functions should thus all be odd with respect to $x = 0$. However, the current distribution on the right hand strip would in general not be symmetrical about its axis at $x = b$. This fact therefore requires the inclusion of basis functions which are even and basis functions which are odd with respect to $x = b$ in the domain $|x - b| \leq w/2$.

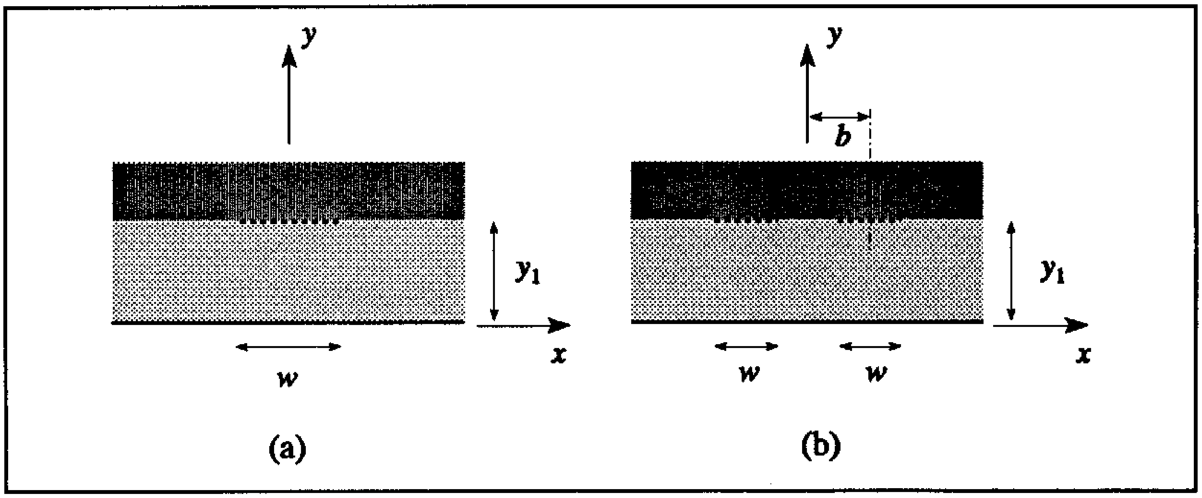

Figure 2 (a) A single strip or slot and (b) a pair of strips or slots.

The surface currents or electric fields in the plane $y = y_1$ may be expanded using any kind of basis functions, as long as they are non-zero only for $|x| \leq w/2$ in the case of Figure 2(a), or only for $|x \pm b| \leq w/2$ in Figure 2(b). The efficiency and accuracy of the method is, however, dependent on the choice of basis functions. The singular behaviour of the electric surface currents parallel to the strip edges or the electric fields normal to the slot edges should therefore be incorporated in the basis functions [7].

In general, the surface currents or electric fields in the plane $y = y_1$ may be expanded as

$$F(x,y_1) = \sum_{n=1}^{N} a_n f_n(x) \qquad G(x,y_1) = \sum_{n=1}^{M} b_n g_n(x) \tag{2}$$

where $F(x, y_1)$ and $G(x, y_1)$ represent $J_x(x, y_1)$ and $J_z(x, y_1)$, or $E_z(x, y_1)$ and $E_x(x, y_1)$. The terms a_n and b_n are unknown coefficients, while $f_n(x)$ and $g_n(x)$ are basis functions which satisfy the edge conditions. In the spectral domain, this becomes

$$\tilde{F}(\alpha,y_1) = \sum_{n=1}^{N} a_n \tilde{f}_n(\alpha) \qquad \tilde{G}(\alpha,y_1) = \sum_{n=1}^{M} b_n \tilde{g}_n(\alpha) \tag{3}$$

We define two sets of functions which may act as building blocks for the appropriate bases. These are given by

$$\left.\begin{aligned} \xi_m(x,w) &= \frac{T_{m-1}(2x/w)}{\sqrt{1-(2x/w)^2}} \\ \zeta_m(x,w) &= \sqrt{1-(2x/w)^2}\; U_{m-1}(2x/w) \end{aligned}\right\} \begin{aligned} &|x| \leq w/2 \\ &m = 1,2,3,.. \end{aligned} \tag{4}$$

$T_m(x)$ and $U_m(x)$ are mth order Chebyshev polynomials of the first and second kind respectively. These functions are shown in Figure 3 for different values of m. $\xi_m(x, w)$ is singular at $|x| = w/2$. Also note that $\xi_m(x, w)$ and $\zeta_m(x, w)$ are even functions of x for m odd, and *vice versa*.

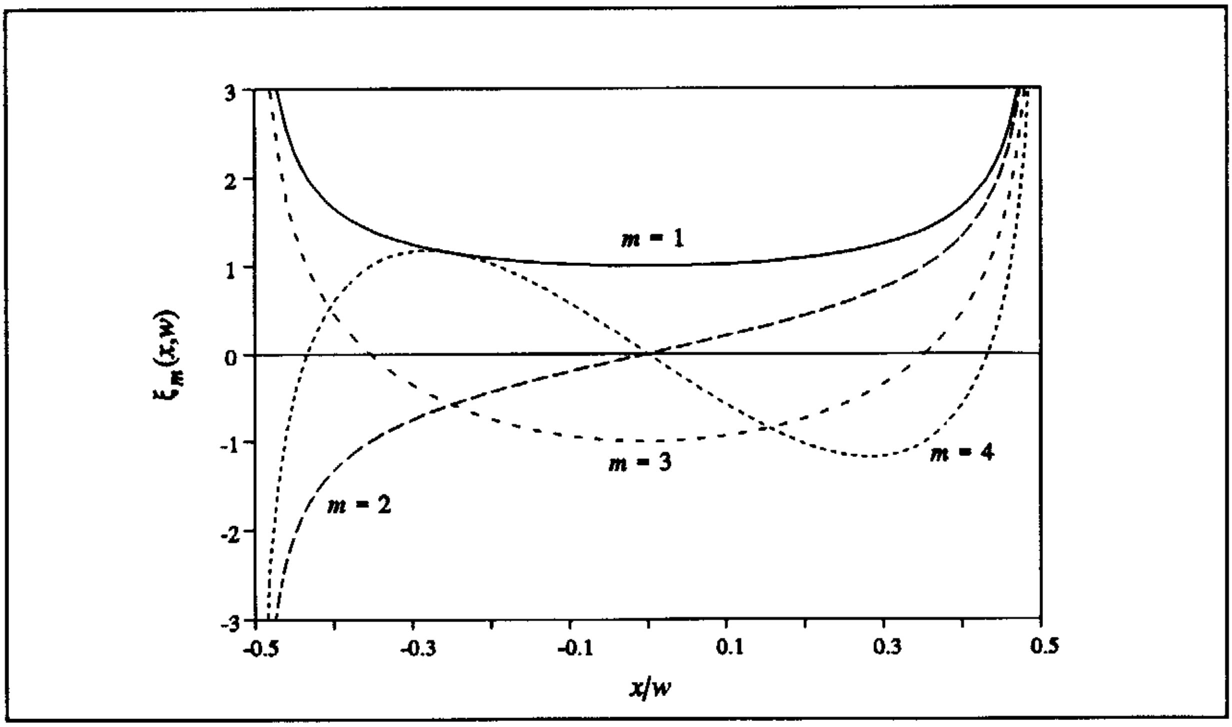

Figure 3(a) The function $\xi_m(x, w)$ for different values of m.

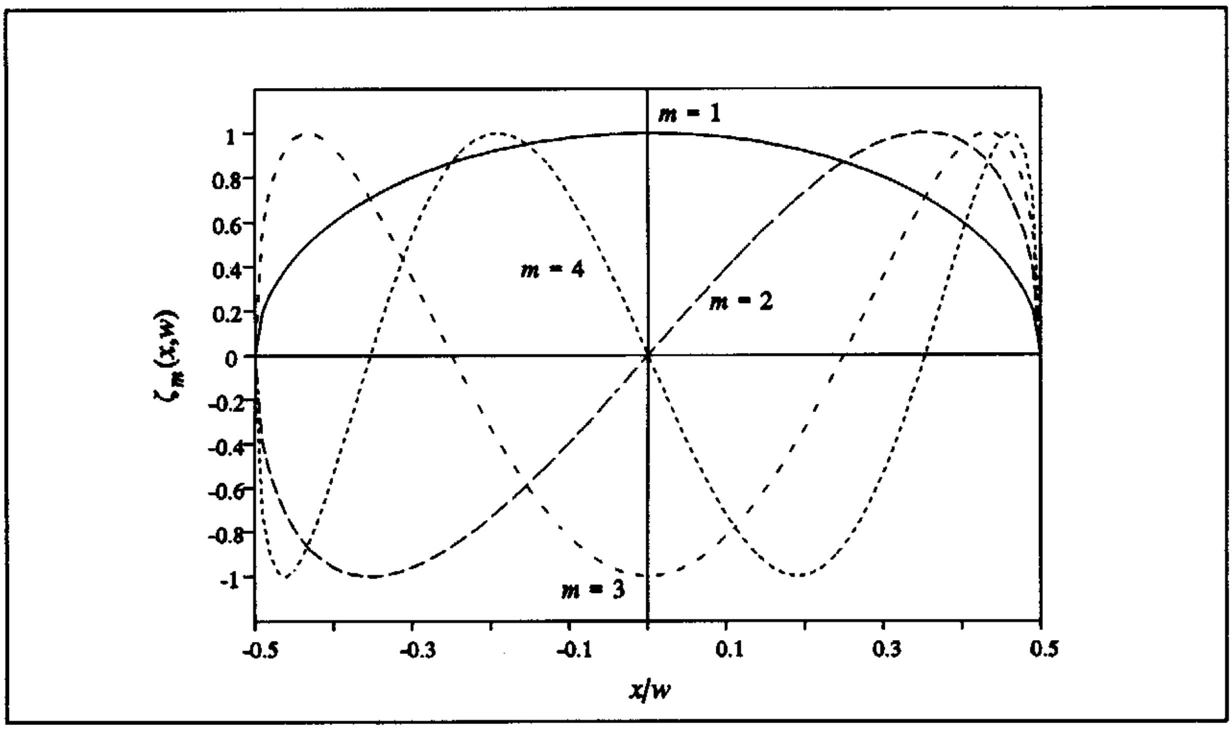

Figure 3(b) The function $\zeta_m(x, w)$ for different values of m.

Their Fourier transforms are given by

$$\tilde{\xi}_m(\alpha,w) = j^{m-1}\,\frac{\pi w}{2}\,J_{m-1}(\alpha w/2) \qquad \tilde{\zeta}_m(\alpha,w) = j^{m-1}\,\frac{\pi m}{\alpha}\,J_m(\alpha w/2) \tag{5}$$

where $J_m(x)$ is the Bessel function of the first kind of order m. The basis functions $f_n(x)$ and $g_n(x)$ may thus be expressed as combinations and permutations of $\zeta_m(x,w)$ and $\xi_m(x,w)$ respectively. Table 1 provides suitable basis functions for the expansion of the different unknown quantities.

Single / Pair	**Strip / Slot**	**Expanded quantity**	**Mode type**	**Basis function**	**Fourier transform of basis function**	
Single	Strip	$J_x(x,y_1)$	Even	$\zeta_{2n}(x,w)$	$\tilde{\zeta}_{2n}(\alpha,w)$	
			Odd	$\zeta_{2n-1}(x,w)$	$\tilde{\zeta}_{2n-1}(\alpha,w)$	
		$J_z(x,y_1)$	Even	$\xi_{2n-1}(x,w)$	$\tilde{\xi}_{2n-1}(\alpha,w)$	
			Odd	$\xi_{2n}(x,w)$	$\tilde{\xi}_{2n}(\alpha,w)$	
	Slot	$E_x(x,y_1)$	Even	$\xi_{2n}(x,w)$	$\tilde{\xi}_{2n}(\alpha,w)$	
			Odd	$\xi_{2n-1}(x,w)$	$\tilde{\xi}_{2n-1}(\alpha,w)$	
		$E_z(x,y_1)$	Even	$\zeta_{2n-1}(x,w)$	$\tilde{\zeta}_{2n-1}(\alpha,w)$	
			Odd	$\zeta_{2n}(x,w)$	$\tilde{\zeta}_{2n}(\alpha,w)$	
Pair	Strip	$J_x(x,y_1)$	Even	$\zeta_n(x+b,w)$ − $\delta\ \zeta_n(x\text{-}b,w)$	$-2j\sin(\alpha b)\ \tilde{\zeta}_n(\alpha,w)$ $2\cos(\alpha b)\ \tilde{\zeta}_n(\alpha,w)$	n odd n even
			Odd	$\zeta_n(x+b,w)$ + $\delta\ \zeta_n(x\text{-}b,w)$	$2\cos(\alpha b)\ \tilde{\zeta}_n(\alpha,w)$ $-2j\sin(\alpha b)\ \tilde{\zeta}_n(\alpha,w)$	n odd n even
		$J_z(x,y_1)$	Even	$\xi_n(x+b,w)$ + $\delta\ \xi_n(x\text{-}b,w)$	$2\cos(\alpha b)\ \tilde{\xi}_n(\alpha,w)$ $-2j\sin(\alpha b)\ \tilde{\xi}_n(\alpha,w)$	n odd n even
			Odd	$\xi_n(x+b,w)$ − $\delta\ \xi_n(x\text{-}b,w)$	$-2j\sin(\alpha b)\ \tilde{\xi}_n(\alpha,w)$ $2\cos(\alpha b)\ \tilde{\xi}_n(\alpha,w)$	n odd n even
	Slot	$E_x(x,y_1)$	Even	$\xi_n(x+b,w)$ − $\delta\ \xi_n(x\text{-}b,w)$	$-2j\sin(\alpha b)\ \tilde{\xi}_n(\alpha,w)$ $2\cos(\alpha b)\ \tilde{\xi}_n(\alpha,w)$	n odd n even
			Odd	$\xi_n(x+b,w)$ + $\delta\ \xi_n(x\text{-}b,w)$	$2\cos(\alpha b)\ \tilde{\xi}_n(\alpha,w)$ $-2j\sin(\alpha b)\ \tilde{\xi}_n(\alpha,w)$	n odd n even
		$E_z(x,y_1)$	Even	$\zeta_n(x+b,w)$ + $\delta\ \zeta_n(x\text{-}b,w)$	$2\cos(\alpha b)\ \tilde{\zeta}_n(\alpha,w)$ $-2j\sin(\alpha b)\ \tilde{\zeta}_n(\alpha,w)$	n odd n even
			Odd	$\zeta_n(x+b,w)$ − $\delta\ \zeta_n(x\text{-}b,w)$	$-2j\sin(\alpha b)\ \tilde{\zeta}_n(\alpha,w)$ $2\cos(\alpha b)\ \tilde{\zeta}_n(\alpha,w)$	n odd n even

Table 1 The nth basis function for the expansion of currents or fields, where $n = 1, 2, 3, \ldots$ and $\delta = \pm 1$ for n odd and n even respectively.

3. Calculation of the matrix elements

Application of Galerkin's method together with Parseval's theorem then results in the eigenvalue equation which needs to be solved numerically [2-7]. The matrix elements that need to be calculated during the process, are all of the form [7]

$$
\begin{aligned}
P(\beta) &= \int_{-\infty}^{\infty} \tilde{G}(\alpha,\beta)\, \tilde{h}_a(\alpha)\, \tilde{h}_b(\alpha)\, d\alpha \\
&= 2\int_{0}^{\infty} \tilde{G}(\alpha,\beta)\, \tilde{h}_a(\alpha)\, \tilde{h}_b(\alpha)\, d\alpha
\end{aligned}
\tag{6}
$$

where $\tilde{G}(\alpha,\beta)$ is an element of the spectral dyadic Green's function. $\tilde{h}_a(\alpha)$ and $\tilde{h}_b(\alpha)$ are Fourier transformed basis functions pertaining to a domain with relevant dimensions denoted by subscripts a and b respectively.

For closed structures, the integrand of equation (6) might have poles located on the axis of integration. This necessitates the application of residue calculus techniques for the evaluation of the integrals [1,8]. These processes are considered to be beyond the scope of this paper.

In the case of open structures, no poles are located on the axis of integration, and therefore no special pole extraction techniques are required for the calculation of the integrals. However, the matrix elements are computed by performing numerical integration over an semi-infinite range, and often the rate of convergence is slow. This presents difficulties in attaining the required accuracy tolerances. By applying the following techniques, these difficulties may be overcome.

Using the basis functions defined in (5) and Table 1, we see that the product $\tilde{h}_a(\alpha)\, \tilde{h}_b(\alpha)$ consists of :

1. A complex constant term.
2. A term of the form α^{-r} with $r = 0$, 1, or 2. For example, if either $\tilde{h}_a(\alpha)$ is proportionate to $\tilde{\zeta}_m(\alpha\,, w_a)$ or $\tilde{h}_b(\alpha)$ is proportionate to $\tilde{\zeta}_m(\alpha\,, w_b)$, then $r = 1$. If neither $\tilde{h}_a(\alpha)$ nor $\tilde{h}_b(\alpha)$ is proportionate to $\tilde{\zeta}_m$, then $r = 0$.
3. A product of two Bessel functions.
4. Zero, one or two $\sin(\alpha b)$ and/or $\cos(\alpha b)$ terms, where $b = b_a$ or $b = b_b$. If both $\tilde{h}_a(\alpha)$ and $\tilde{h}_b(\alpha)$ pertain to single strips or slots, no sines or cosines would appear in the product. If either of the basis functions pertains to a pair of strips or slots, then the product of the bases contains one sine or cosine term. Finally, if both $\tilde{h}_a(\alpha)$ and $\tilde{h}_b(\alpha)$ belongs to pairs of strips/slots, each would contribute a $\sin(\alpha b)$ or $\cos(\alpha b)$ term to the product of the bases.

When the product of two trigonometric terms appear in the integrand of equation (6), the following identities are used to transform it to the sum of two terms.

$$
\begin{aligned}
\sin(\alpha b_a)\sin(\alpha b_b) &= \tfrac{1}{2}\{\cos[\alpha(b_a - b_b)] - \cos[\alpha(b_a + b_b)]\} \\
\cos(\alpha b_a)\cos(\alpha b_b) &= \tfrac{1}{2}\{\cos[\alpha(b_a - b_b)] + \cos[\alpha(b_a + b_b)]\} \\
\sin(\alpha b_a)\cos(\alpha b_b) &= \tfrac{1}{2}\{\sin[\alpha(b_a - b_b)] + \sin[\alpha(b_a + b_b)]\}
\end{aligned}
\tag{7}
$$

Equation (6) may thus in general be rewritten in the form

$$P(\beta) = \sum_{i=0}^{8} A_i \int_0^{\infty} \frac{\tilde{G}(\alpha,\beta)}{\alpha^r} J_k(\alpha w_a/2)\, J_l(\alpha w_b/2)\, p_i(\alpha)\, d\alpha$$

$$= \sum_{i=0}^{8} A_i F_i(\beta) \tag{8}$$

$$\begin{array}{lll} p_0(\alpha) = 1 & p_1(\alpha) = \sin(\alpha b_a) & p_2(\alpha) = \cos(\alpha b_a) \\ p_3(\alpha) = \sin(\alpha b_b) & p_4(\alpha) = \cos(\alpha b_b) & p_5(\alpha) = \sin[\alpha(b_a + b_b)] \\ p_6(\alpha) = \cos[\alpha(b_a + b_b)] & p_7(\alpha) = \sin[\alpha(b_a - b_b)] & p_8(\alpha) = \cos[\alpha(b_a - b_b)] \end{array}$$

where k and l are integers. $A_0, A_1, \ldots A_8$ are complex constants, of which at least seven are zero. Therefore, a maximum of two terms in the series need to be computed. Note that equation (8) is not an expansion of the original integral in (6) - it is merely a general representation of all the possible forms equation (6) might take on after the transformations of (7) had been applied. If, for example, the original integrand in (6) contains the term $\cos(\alpha b_a)\cos(\alpha b_b)$, the only non-zero A_i's in (8) are A_6 and A_8. For the special case where $b_a = b_b$, the non-zero constants are A_0 and A_6.

For $i \neq 0$, the integrands of the $F_i(\beta)$ terms are all oscillatory over the entire range. These integrals have a poor convergence rate when it is evaluated with conventional quadrature routines. They may be computed much more efficiently by treating them as Fourier integrals with the sine and cosine terms as kernels. A special routine such as QDAWF [9] may then be used to evaluate these integrals. QDAWF is an adaptive routine, designed to integrate functions of the form $f(x)\sin(\omega x)$ or $f(x)\cos(\omega x)$ over a semi-infinite range. It integrates the integrand between zeros over a number of subintervals, and invokes an extrapolation scheme in order to estimate the integral.

In general, the $F_0(\beta)$ term may be calculated by using a routine like QDAGI [9]. QDAGI is an integration routine designed to numerically evaluate integrals over an infinite or semi-infinite range. It initially transforms the interval into the finite interval [0, 1], and then uses a 21 point Gauss-Kronrod rule to estimate the integral. The integrand of the $F_0(\beta)$ term is well behaved, provided that the basis functions $\tilde{h}_a(\alpha)$ and $\tilde{h}_b(\alpha)$ are not associated with the same single or pair of strips or slots. Inspection reveals that the bases $\tilde{h}_a(\alpha)$ and $\tilde{h}_b(\alpha)$ then necessarily pertain to strips or slots that are on different vertical planes. (If $\tilde{h}_a(\alpha)$ and $\tilde{h}_b(\alpha)$ are associated with different strips or slots on the same vertical plane, the constant A_0 is zero, and therefore the $F_0(\beta)$ term need not be computed.) The Green's function elements are then of such a form that the integrand decays rapidly for large values of α, and therefore the integral converges quickly.

However, when this provision does not apply, the basis functions $h_a(x)$ and $h_b(x)$ are defined over the same domain (so that $w_a = w_b = w$ and $b_a = b_b = b$). The integrand then decays less rapidly, which causes the rate of convergence to be slow. As reported for the specific case of a single slot in [4], $F_0(\beta)$ may then be converted into a rapidly convergent integral by extracting its asymptotic form and evaluating it in closed form. This is done by stating that

$$F_0(\beta) = \int_0^\infty \tilde{f}(\alpha,\beta)\, d\alpha$$
$$= \int_0^\infty \left[\tilde{f}(\alpha,\beta) - \tilde{f}_\infty(\alpha,\beta)\right] d\alpha + \int_0^\infty \tilde{f}_\infty(\alpha,\beta)\, d\alpha \tag{9}$$

where $\tilde{f}_\infty(\alpha,\beta)$ is the asymptotic form of the integrand. For cases where convergence is slow, the asymptote is usually of the form

$$\tilde{f}_\infty(\alpha,\beta) = C(\beta)\,\frac{\alpha}{\alpha^2+\beta^2}\,J_k(\alpha w/2)\,J_l(\alpha w/2) \tag{10}$$

with $C(\beta)$ a complex function. When $|k - l| = 2s$ with s an integer (as was the case in [4]), the second integral in the final expression of (9) is given in closed form by [10]

$$\int_0^\infty \tilde{f}_\infty(\alpha,\beta)\,d\alpha = \begin{cases} C(\beta)\,(-1)^s\,I_k(\beta w/2)\,K_l(\beta w/2) & k \geq l \\ C(\beta)\,(-1)^s\,I_l(\beta w/2)\,K_k(\beta w/2) & k < l \end{cases} \tag{11}$$

where $I_m(x)$ and $K_m(x)$ are modified Bessel functions of the first and second kind respectively.

However, when $|k - l| = 2s + 1$ (i.e. an odd integer), this integral is not available in closed form. We therefore need to modify the expression for $\tilde{f}_\infty(\alpha,\beta)$ by replacing the Bessel functions with their respective large argument forms [11], so that

$$\tilde{f}_\infty(\alpha,\beta) = C(\beta)\,\frac{\alpha}{\alpha^2+\beta^2}\,\frac{(-1)^{(k+l+1)/2}}{\pi\alpha w/2}\,\cos(\alpha w) \tag{12}$$

and

$$\int_0^\infty \tilde{f}_\infty(\alpha,\beta)\,d\alpha = C(\beta)\,\frac{(-1)^{(k+l+1)/2}}{\beta w}\,e^{-\beta w} \tag{13}$$

The first term in the final expression of equation (9) is then a rapidly convergent integral, and is suitable for efficient numerical computation with a routine such as QDAGI [9].

4. Example

Comparing the basis functions shown in Figure 3 with the conventional trigonometric bases in [3], [6] and [7], shows that the two sets of functions are similar in form. The use of the Chebyshev bases instead of the trigonometric bases, therefore does not result in a reduction (or an increase) in the number of unknowns. For a given accuracy level of the final result, the same number of bases should be included in the expansions, irrespective of the type of basis functions used. When the special calculation techniques described here are *not* used, the CPU time required to compute the integrals is also largely insensitive to the choice of basis functions.

However, if we treat oscillatory integrals as Fourier integrals, and apply asymptotic extraction to slowly converging integrals, the required CPU time is reduced significantly. As an example, we consider certain matrix elements that are calculated during the analysis of the semi re-entrant (SRE) microstrip section [6]. This structure is ideally suited to illustrate these techniques, since it comprises of both a single strip and a pair of strips. Instead of the conventional trigonometric bases used by the authors in [6], we now utilize the appropriate basis functions specified in Table 1 to calculate the following elements :

1. $P^{11}_{11}(\beta)$ for an even type mode:
From the definition of the matrix elements [6, eq. (9)], we see that

$$\begin{aligned} P^{11}_{11}(\beta) &= \int_{-\infty}^{\infty} \tilde{Z}^{11}_{xx}(\alpha,\beta)\, \tilde{J}^{1}_{x1}(\alpha)\, \tilde{J}^{1}_{x1}(\alpha)\; d\alpha \\ &= 2\int_{0}^{\infty} \tilde{Z}^{11}_{xx}(\alpha,\beta)\, \tilde{J}^{1}_{x1}(\alpha)\, \tilde{J}^{1}_{x1}(\alpha)\; d\alpha \end{aligned} \tag{14}$$

where $\tilde{Z}^{11}_{xx}(\alpha,\beta)$ is an element of the spectral dyadic Green's function, while $\tilde{J}^{1}_{x1}(\alpha)$ is a Fourier transformed basis function for the x directed current on the single strip. From Table 1, it follows that $\tilde{J}^{1}_{x1}(\alpha) = \tilde{\xi}_2(\alpha\, , w_1)$, which yields

$$P^{11}_{21}(\beta) = -8\,\pi^2 \int_{0}^{\infty} \frac{\tilde{Z}^{11}_{xx}(\alpha,\beta)}{\alpha^2}\, J_2(\alpha w_1/2)\, J_2(\alpha w_1/2)\; d\alpha \tag{15}$$

If we compare this to equation (8), we see that $A_0 = -8\,\pi^2$ and $A_i = 0$ when $i \neq 0$. The integral $F_0(\beta)$ is evaluated as indicated in equation (9). An expression for $C(\beta)$ is obtained by calculating the Green's function element for $\alpha >> \beta$, so that

$$\tilde{Z}^{11}_{xx}(\alpha,\beta) \rightarrow \frac{\alpha^3}{\alpha^2+\beta^2}\, \frac{j}{\omega\,\epsilon_0(1+\epsilon_{r2})} \tag{16}$$

and

$$C(\beta) = \frac{j}{\omega\,\epsilon_0(1+\epsilon_{r2})} \tag{17}$$

2. $Q^{12}_{32}(\beta)$ for an even type mode:
From [6, eq. (9)], we see that

$$\begin{aligned} Q^{12}_{32}(\beta) &= \int_{-\infty}^{\infty} \tilde{Z}^{12}_{xz}(\alpha,\beta)\, \tilde{J}^{2}_{z2}(\alpha)\, \tilde{J}^{1}_{x3}(\alpha)\; d\alpha \\ &= 2\int_{0}^{\infty} \tilde{Z}^{12}_{xz}(\alpha,\beta)\, \tilde{J}^{2}_{z2}(\alpha)\, \tilde{J}^{1}_{x3}(\alpha)\; d\alpha \end{aligned} \tag{18}$$

Substituting the bases yields

$$Q_{32}^{12}(\beta) = j12\,\pi^2 w_2 \int_0^\infty \frac{\tilde{Z}_{xz}^{12}(\alpha,\beta)}{\alpha}\, J_1(\alpha w_2/2)\, J_6(\alpha w_1/2)\, \sin(\alpha\tau)\, d\alpha \tag{19}$$

where $\tau = d + w_2/2$. The complex constants in (8) are thus given by $A_1 = j\,12\,\pi^2 w_2$ and $A_i = 0$ when $i \neq 1$. $F_1(\beta)$ is calculated as a Fourier integral.

3. $S_{31}^{22}(\beta)$ for an odd type mode:
From the matrix element definition, it follows that

$$S_{31}^{22}(\beta) = 2\int_0^\infty \tilde{Z}_{zz}^{22}(\alpha,\beta)\, \tilde{J}_{z1}^{\,2}(\alpha)\, \tilde{J}_{z3}^{\,2}(\alpha)\, d\alpha \tag{20}$$

which reduces to

$$\begin{aligned} S_{31}^{22}(\beta) &= 2\,\pi^2 w_2^{\,2} \int_0^\infty \tilde{Z}_{zz}^{22}(\alpha,\beta)\, J_0(\alpha w_2/2)\, J_2(\alpha w_2/2)\, \sin^2(\alpha\tau)\, d\alpha \\ &= \pi^2 w_2^{\,2} \int_0^\infty \tilde{Z}_{zz}^{22}(\alpha,\beta)\, J_0(\alpha w_2/2)\, J_2(\alpha w_2/2)\, d\alpha \\ &\quad - \pi^2 w_2^{\,2} \int_0^\infty \tilde{Z}_{zz}^{22}(\alpha,\beta)\, J_0(\alpha w_2/2)\, J_2(\alpha w_2/2)\, \cos(2\alpha\tau)\, d\alpha \\ &= A_0\, F_0(\beta) + A_6\, F_6(\beta) \end{aligned} \tag{21}$$

The $F_6(\beta)$ term is treated as a Fourier integral with the cosine as kernel, while the integral $F_0(\beta)$ is calculated by extracting its asymptotic form. The latter is done by noting that for $\alpha >> \beta$

$$\tilde{Z}_{zz}^{22}(\alpha,\beta) \rightarrow \frac{\alpha}{\alpha^2+\beta^2}\left[\frac{j\beta^2}{\omega\,\epsilon_0(\epsilon_{r2}+\epsilon_{r3})} - \frac{j\omega\,\mu_0}{2}\right] \tag{22}$$

so that

$$C(\beta) = \frac{j\beta^2}{\omega\,\epsilon_0(\epsilon_{r2}+\epsilon_{r3})} - \frac{j\omega\,\mu_0}{2} \tag{23}$$

These matrix elements were calculated on a Persetel PS8/90-3 computer, in the one case by utilizing the special techniques described in section 3, and in the other case by simply integrating the integrands as defined in equations (14), (18) and (20). The calculations were performed for an SRE structure with dimensions $w_1/\lambda_0 = 0.01$, $w_2/\lambda_0 = 0.02$, $d/\lambda_0 = 0.005$, $t/\lambda_0 = 0.01$, $s/\lambda_0 = 0.0025$, $\epsilon_{r2} = \epsilon_{r3} = 2.2$, and with $k_0/\beta = \lambda/\lambda_0 = 0.7$. The integrals were calculated with a relative accuracy criterion of 0.001 (i.e. the error should be smaller than 0.1% of the absolute value of the final result). The CPU times required to calculate the matrix elements using the two approaches are shown in Table 2.

Element	CPU time utilizing special techniques (seconds)	CPU time without special techniques (seconds)
$P_{11}^{11}(\beta)$	0.37	6.72
$Q_{32}^{12}(\beta)$	0.20	0.93
$S_{31}^{22}(\beta)$	0.74	12.15

Table 2 CPU time required to calculate the different matrix elements.

For these examples, the calculation of elements with oscillatory integrands as Fourier integrals, and the application of asymptotic extraction reduce the CPU time by factors of about 4.5 and 18 respectively.

5. Conclusion

Suitable basis functions for the expansion of unknown electric currents or fields as required by the spectral domain method applied to a symmetrical planar transmission line, have been provided. We have shown how through using these basis functions, the required CPU time for the matrix element calculations may be reduced appreciably. Elements with oscillatory integrands are treated as Fourier-type integrals, while asymptotic extraction is performed to enhance the convergence rate of integrals with non-oscillatory integrands. Application of the latter technique has been limited to structures with single strips or slots, but with the additional information furnished in this paper, this procedure may now be utilized during the analysis of any symmetrical multiconductor transmission line.

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