Analysis and Design of Planar Waveguide Slot Arrays Using Scattering Matrix Approach

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ABSTRACT. A scattering matrix approach to the analysis and design of planar waveguide slot arrays is described in this paper. The new method employs the scattering matrix for TE_{10} and TE_{20} (to the waveguide axis) modes of isolated slot in the broad wall of a rectangular waveguide, rather than the admittance of the slot derived from approximate shunt or series lumped element model for the slot. The method fully accounts for the external mutual coupling due to the TE_{10} and TE_{20} modes on the slot, and can be easily extended to include higher order waveguide modes. Furthermore, it takes into account the deviation of the slot aperture from the sinusoidal distribution that is normally assumed with a nearly resonant slot. Numerical results demonstrating the viability of the scattering matrix approach, including an example for the design of a 2×3 planar array of longitudinal slots, are presented.

1. INTRODUCTION

In 1978, Elliott and Kurtz devised a method for the design of planar arrays of waveguide-fed longitudinal slots [1], [2, section 8-13]. The method relied on using the artifice of an equivalent array of loaded dipoles to account for the external mutual coupling between the slots. Subsequently, the design procedure was generalized by Elliott and co-workers [3], [4]. This method has been for a long time the only available method for the design of such arrays. It utilizes a set design equations derived under the assumption that the offset longitudinal resonant slot behaves approximately as shunt admittance in a modal representation of the waveguide for the dominant mode. In essence, the design equations relate the lengths and offsets of the slots, which are the unknowns in the design procedure, to the aperture field distributions of the slots, the admittances of the slots, and the reflection coefficients at the waveguide input ports. Initially, experimental data was used for the conductance and susceptance of isolated slot, but more accurate numerical data were later derived through method of moments analyses. Subsequently, the assumption of a shunt admittance model for the longitudinal slot has been lifted by Gulick and Elliott [5], who presented a method which incorporates internal as well as external mutual coupling of the slots rigorously. The method, however, adopts certain approximations for the calculation of incremental lengths and offsets in an iterative design procedure. Furthermore, such a procedure would be costly in terms of computer time and memory for arrays with large number of elements, in particular when the slots are "compound," with no Toeplitz symmetry in the matrix elements.

In this paper, a scattering matrix approach is employed for the analysis and design of planar waveguide-fed slot arrays. The slots are assumed to be equi-spaced along the waveguides' axes, but are otherwise of arbitrary orientation. Furthermore, the slots are assumed to be located in the broad walls of the feed waveguides and radiate in a homogeneous half-space. In the design, the slots are assumed to be all of the same kind, i.e., for instance, all the slots are longitudinal whose locations are described by their offset from the axial centerlines of the waveguides (see Figure 1). The procedure is equally applicable to inclined slots whose centers are located along any given lines in each waveguide by replacing the parameters describing the offsets with those denoting the angular rotations of the slots.

The scattering matrix approach is based on the theory of generalized scattering matrix [6, Section 1-9]. It utilizes Kajfez and Wilton's scattering matrix representation for radiating elements [7], rather than the admittance data derived from approximate shunt or series models of the slots, and, hence, may be applied for the analysis and design of arrays having a variety of modules such as, for instance, slots tuned by metallic posts [8], pairs of narrow slots, etc. Furthermore, it takes into account the deviation in the slot aperture field from the sinusoidal distribution that is normally assumed with a nearly resonant slot [3]. In the analysis, internal mutual coupling is assumed limited to the propagation of TE_{10} and TE_{20} (to the waveguide axis) modes in each waveguide. Such an assumption is necessary

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only for practical considerations. The analysis can be readily generalized to include higher order mode interaction between the slots. External mutual coupling, on the other hand, occurs as a result of the interactions of fields due to all of the slots, and is modeled by outgoing TE_{10} and TE_{20} mode wave sources for each slot. Only numerical verification of the scattering matrix approach was planned, although experimental verification is possible since all quantities involved are directly measurable. Thus, numerical results demonstrating the viability of the scattering matrix approach, including an example for the design of a 2×3 planar array of longitudinal slots, are presented and discussed.

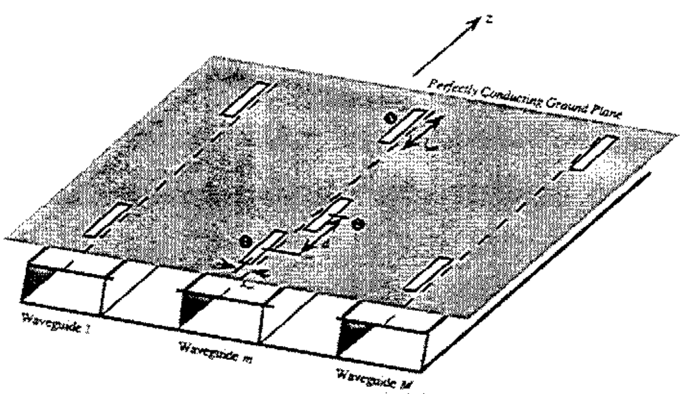

Figure 1. Planar waveguide-fed array of longitudinal slots radiating in a homogeneous half-space

2. ANALYSIS

The planar waveguide-fed slot array considered for analysis is similar to that shown in Figure 1. It consists of N arbitrarily oriented, equi-spaced slots (with spacing "d" between the centers of consecutive slots in the direction of the waveguide) in the broad wall of each of the M feed waveguides. Only a simple change in the orders of the relevant matrices constructed in the course of analysis is needed if different number of slots is assumed in each waveguide. The feed waveguides are all identical, with a wall thickness "t," and of such cross-sectional dimensions that only the dominant (TE_{10} to z) waveguide mod can propagate unattenuated, and the TE_{20} to z mode is the lowest order evanescent mode.

In the analysis, internal mutual coupling is assumed limited to the propagation of TE_{10} and TE_{20} modes in each waveguide, whereas external mutual coupling between the slots occurs through the interaction of the fields due to all of the slots. An appropriate model for the slot to work with is therefore Kajfez and Wilton's network representation for radiating elements [7]. In this representation, the ingoing and outgoing wave amplitudes in the mn^{th} module in the array are related according to

$$(1)\quad \mathbf{b}_{mn} = \mathbf{S}_{mn}\mathbf{a}_{mn} + \mathbf{c}_{mn}$$

where the cross-sectional plane at the center of the mn^{th} slot is taken to be the reference plane. For convenience, the ports of each module are placed midway between the slots, i.e., a distance "d" apart, where ports "1" and "2" are located, respectively, before and after the slot along the waveguide axis. In (1), $\mathbf{a}_{mn}$ is the vector of incident TE_{10} and TE_{20} mode amplitudes, $\mathbf{b}_{mn}$ is the vector of corresponding scattered mode amplitudes, $\mathbf{c}_{mn}$ is the vector of outgoing wave sources due to external mutual coupling between the mn^{th} slot and all other slots in the array, all evaluated at the ports of the module, and $\mathbf{S}_{mn}$ is the scattering matrix for the slot. The elements of the vectors are so arranged that the amplitudes of the TE_{10} and TE_{20} modes at port "1", in such an order, are placed first, followed with those at port "2," with the scattering matrix partitioned accordingly. The scattering matrix representation for the mn^{th} slot can then be written in the partitioned form:

$$(2)\quad \begin{bmatrix} \mathbf{b}_1^{mn} \\ \mathbf{b}_2^{mn} \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{11}^{mn} & \mathbf{S}_{12}^{mn} \\ \mathbf{S}_{21}^{mn} & \mathbf{S}_{22}^{mn} \end{bmatrix} \begin{bmatrix} \mathbf{a}_1^{mn} \\ \mathbf{a}_2^{mn} \end{bmatrix} + \begin{bmatrix} \mathbf{c}_1^{mn} \\ \mathbf{c}_2^{mn} \end{bmatrix}$$

The scattering matrix coefficients and the vector of outgoing wave sources of a typical module are functions of the length an location of each slot. These parameters are available to the designer through a method of moments analysis or by measurement. The methods by which these parameters may be computed are explained as the design procedure progresses. In this section, however, attention is focused on the derivation of the design procedure for the array in terms of the scattering matrix and the vector of outgoing wave sources.

Only the incident wave amplitudes, $\mathbf{a}_1^{m1}$ and $\mathbf{a}_2^{mN}$ in the m^{th} waveguide, are known, whereas all other incident and scattered wave amplitudes need to be determined. As a first step towards formulating the design procedure, a relationship between the scattered wave amplitudes, outgoing wave sources, and incident wave amplitudes is derived by relating the forward and backward traveling modal waves between the various slots; *viz.*:

$$(3)\quad \begin{aligned} \mathbf{b}_1^{mn} &= \mathbf{S}_{11}^{mn}\mathbf{a}_1^{mn}\delta_{n1} + \mathbf{S}_{11}^{mn}\mathbf{D}\mathbf{b}_2^{m(n-1)}(1-\delta_{n1}) \\ &+ \mathbf{S}_{12}^{mn}\mathbf{D}\mathbf{b}_1^{m(n+1)}(1-\delta_{nN}) + \mathbf{S}_{12}^{mn}\mathbf{a}_2^{mn}\delta_{nN} + \mathbf{c}_1^{mn} \end{aligned}$$

$$\text{(4)}\quad \begin{aligned}\mathbf{b}_2^{mn} &= \mathbf{S}_{21}^{mn}\mathbf{a}_1^{mn}\delta_{n1} + \mathbf{S}_{21}^{mn}\mathbf{D}\mathbf{b}_2^{m(n-1)}(1-\delta_{n1}) \\ &+ \mathbf{S}_{22}^{mn}\mathbf{D}\mathbf{b}_1^{m(n+1)}(1-\delta_{nN}) + \mathbf{S}_{22}^{mn}\mathbf{a}_2^{mn}\delta_{nN} + \mathbf{c}_2^{mn}\end{aligned}$$

In (3) and (4), δ_{ij} is the Kronecker delta function ($\delta_{ij} = 1$ if $i = j$ and is zero otherwise), and $\mathbf{D}$ is the diagonal matrix

$$\text{(5)}\quad \mathbf{D} = \begin{bmatrix} e^{-j\beta_{10}d} & \\ & e^{-\alpha_{20}d} \end{bmatrix}$$

where $j\beta_{10}$ is the propagation constant for the TE_{10} mode, and α_{20} is the attenuation constant for the TE_{20} mode.

The quantities specified to the designer of the array are basically the complex amplitudes of the scattered modal waves, or the reflection coefficients, at the input ports of the waveguides and the aperture field coefficients (to be defined later) in all the slots. Hence, the task now is to relate the vectors of the scattered wave amplitudes at the inner ports of the array modules to those at the array outer ports. Using (3) and (4), it readily follows that

$$\text{(6)}\quad \mathbf{b}_1^{m1} = \mathbf{S}_{11}^{m1}\mathbf{a}_1^{m1} + \mathbf{G}_{1,m}\mathbf{b}_m + \mathbf{H}_{1,m}\mathbf{c}_m$$

$$\text{(7)}\quad \mathbf{b}_2^{mN} = \mathbf{S}_{12}^{mN}\mathbf{a}_2^{mN} + \mathbf{G}_{N,m}\mathbf{b}_m + \mathbf{H}_{N,m}\mathbf{c}_m$$

where $\mathbf{b}_m$ and $\mathbf{c}_m$ are, respectively, the $2(N-1)$-segment and $2N$-segment vectors

$$\text{(8)}\quad \mathbf{b}_m = \begin{bmatrix} \mathbf{b}_2^{m1} \\ \mathbf{b}_1^{m2} \\ \mathbf{b}_2^{m2} \\ \vdots \\ \mathbf{b}_1^{mN} \end{bmatrix}$$

$$\text{(9)}\quad \mathbf{c}_m = \begin{bmatrix} \mathbf{c}_1^{m1} \\ \mathbf{c}_2^{m1} \\ \vdots \\ \mathbf{c}_1^{mN} \\ \mathbf{c}_2^{mN} \end{bmatrix}$$

$\mathbf{G}_{1,m}$ is the 1 by $2(N-1)$ block matrix

$$\text{(10)}\quad \mathbf{G}_{1,m} = \begin{bmatrix} \mathbf{O} & \mathbf{S}_{12}^{m1}\mathbf{D} & \mathbf{O} & .. & \mathbf{O} \end{bmatrix}$$

and $\mathbf{H}_{1,m}$ is the 1 by $2N$ block connection matrix

$$\text{(11)}\quad \mathbf{H}_{1,m} = \begin{bmatrix} \mathbf{I} & \mathbf{O} & \mathbf{O} & .. & \mathbf{O} \end{bmatrix}$$

where $\mathbf{O}$ is the null matrix and $\mathbf{I}$ is the identity matrix, both of order 2. Similarly, $\mathbf{G}_{N,m}$ and $\mathbf{H}_{N,m}$ are, respectively, the 1 by $2(N-1)$ and 1 by $2N$ block matrices

$$\text{(12)}\quad \mathbf{G}_{N,m} = \begin{bmatrix} \mathbf{O} & .. & \mathbf{O} & \mathbf{S}_{21}^{mN}\mathbf{D} & \mathbf{O} \end{bmatrix}$$

$$\text{(13)}\quad \mathbf{H}_{N,m} = \begin{bmatrix} \mathbf{O} & .. & \mathbf{O} & \mathbf{O} & \mathbf{I} \end{bmatrix}$$

In passing, it is worth noting that in the special case where the M feed waveguides are terminated with known load impedances, $\mathbf{a}_2^{mN}$ and $\mathbf{b}_2^{mN}$ are related according to

$$\text{(14)}\quad \mathbf{a}_2^{mN} = \mathbf{S}_L^m\mathbf{b}_2^{mN}$$

where $\mathbf{S}_L^m$ is the scattering matrix for the load terminating the m^{th} waveguide.

There now remains to relate $\mathbf{b}_m$ to $\mathbf{a}_1^{m1}$, $\mathbf{a}_2^{mN}$, and $\mathbf{c}_m$, thereby obtaining a direct relationship between the vectors of amplitudes of the scattered and incident modal waves at the outer ports of the array and the vector of outgoing wave sources. The vector of outgoing wave sources is later related to the aperture field coefficients. Again, using (3) and (4), there finally results

$$\text{(15)}\quad \mathbf{b}_m = (\mathbf{I} - \mathbf{X}_{mm})^{-1}\left[\mathbf{F}_{1,m}\mathbf{a}_1^{m1} + \mathbf{F}_{N,m}\mathbf{a}_2^{mN} + \mathbf{T}_{mm}\mathbf{c}_m\right]$$

where $\mathbf{I}$ is the identity matrix of order $2(N-1)$, $\mathbf{X}_{mm}$ is the $2(N-1)$ by $2(N-1)$ block banded matrix

$$\text{(16)}\quad \mathbf{X}_{mm} = \begin{bmatrix} \mathbf{O} & \mathbf{S}_{22}^{m1}\mathbf{D} & \mathbf{O} & \mathbf{O} & \mathbf{O} & \mathbf{O} & .. & \mathbf{O} & \mathbf{O} \\ \mathbf{S}_{11}^{m2}\mathbf{D} & \mathbf{O} & \mathbf{O} & \mathbf{S}_{12}^{m2}\mathbf{D} & \mathbf{O} & \mathbf{O} & .. & \mathbf{O} & \mathbf{O} \\ \mathbf{S}_{21}^{m2}\mathbf{D} & \mathbf{O} & \mathbf{O} & \mathbf{S}_{22}^{m2}\mathbf{D} & \mathbf{O} & \mathbf{O} & .. & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & \mathbf{S}_{11}^{m3}\mathbf{D} & \mathbf{O} & \mathbf{O} & \mathbf{S}_{12}^{m3}\mathbf{D} & .. & \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{O} & \mathbf{S}_{21}^{m3}\mathbf{D} & \mathbf{O} & \mathbf{O} & \mathbf{S}_{22}^{m3}\mathbf{D} & .. & \mathbf{O} & \mathbf{O} \\ .. & .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. & .. \\ \mathbf{O} & \mathbf{O} & \mathbf{O} & \mathbf{O} & \mathbf{O} & \mathbf{O} & .. & \mathbf{S}_{11}^{mN}\mathbf{D} & \mathbf{O} \end{bmatrix}$$

$\mathbf{F}_{1,m}$ and $\mathbf{F}_{N,m}$ are the $2(N-1)$ by 1 block matrix

$$\text{(17)}\quad \mathbf{F}_{1,m} = \begin{bmatrix} \mathbf{S}_{21}^{m1} \\ \mathbf{O} \\ \vdots \\ \mathbf{O} \end{bmatrix}$$

(18) $\mathbf{F}_{N,m} = \begin{bmatrix} \mathbf{O} \\ \mathbf{O} \\ \vdots \\ \mathbf{S}_{12}^{mN} \end{bmatrix}$

and $\mathbf{T}_{mm}$ is a $2(N-1)$ by $2(N-1)$ block matrix whose pq^{th} block is the 2 by 2 matrix $\mathbf{I}\delta_{p+1,q}$.

Knowledge of the vector of outgoing wave sources makes it possible to determine the complex amplitudes of the scattered modal waves at the ports of the modules in the array through (6), (7), and (15). In what follows, the outgoing wave sources are related to appropriately defined aperture field coefficients, and, hence, to the design parameters describing the dimensions and locations of the slots. To this end, the aperture field distribution of the mn^{th} slot is expressed as a weighted sum of the distributions induced due to TE_{10} and TE_{20} modes incident on the slot from both the left and the right, as well as due to mutual external coupling between the slots as

(19) $\mathbf{E}_{mn} = v_{11}^{mn}\mathbf{G}_{11}^{mn} + v_{12}^{mn}\mathbf{G}_{12}^{mn} + v_{21}^{mn}\mathbf{G}_{21}^{mn} + v_{22}^{mn}\mathbf{G}_{22}^{mn} + v_{e}^{mn}\mathbf{G}_{e}^{mn}$

where $\mathbf{G}_{1j}^{mn}$, $j = 1, 2$, are the aperture field patterns excited in the mn^{th} slot, respectively, by TE_{10} and TE_{20} modes of unit amplitude incident on the slot from the left, where $\mathbf{G}_{2j}^{mn}$, $j = 1, 2$, are the aperture field patterns excited in the slot, respectively, by TE_{10} and TE_{20} modes of unit amplitude incident on the slot from the right, and $\mathbf{G}_e^{mn}$ is the aperture field pattern excited in the slot by all other slots by means of external mutual coupling. Thus, v_e^{mn} is a weighted sum of v_{ij}^{pq}, $i, j = 1, 2$, and v_e^{pq}, $p \neq m$, $q \neq n$, with the weights being the strength of $\mathbf{G}_e^{mn}$ in the mn^{th} slot induced by the various field patterns in all other slots. Forming matrices of the weighting coefficients, the following matrix relationship among the aperture field coefficients is obtained

(20) $\mathbf{v}_e = \mathbf{W}\mathbf{v} + \mathbf{W}_e\mathbf{v}_e$

In (20), $\mathbf{v}_e$ and $\mathbf{v}$ are M-segment vectors whose m^{th} segments are the vectors

(21) $\mathbf{v}_{em} = \begin{bmatrix} v_e^{m1} \\ v_e^{m2} \\ \vdots \\ v_e^{mN} \end{bmatrix}$

(22) $\mathbf{v}_m = \begin{bmatrix} \mathbf{v}_1^{m1} \\ \mathbf{v}_2^{m1} \\ \vdots \\ \mathbf{v}_1^{mN} \\ \mathbf{v}_2^{mN} \end{bmatrix}$

where $\mathbf{v}_i^{mn}$ is the vector of aperture field coefficients (v_{ij}^{mn}, i, j, = 1, 2), and $\mathbf{W}$ and $\mathbf{W}_e$ are matrices of the weighting coefficients. Whence

(23) $\mathbf{v}_e = (\mathbf{I} - \mathbf{W}_e)^{-1}\mathbf{W}\mathbf{v}$

It is instructive here to relate the coefficients of the aperture field patterns in the mn^{th} slot to the amplitudes of the incident waveguide modes. Because of the linearity of the electromagnetic field, the relationship is linear of the form

(24) $\begin{bmatrix} \mathbf{v}_1^{mn} \\ \mathbf{v}_2^{mn} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_{11}^{mn} & \mathbf{O} \\ \mathbf{O} & \mathbf{L}_{22}^{mn} \end{bmatrix} \begin{bmatrix} \mathbf{a}_1^{mn} \\ \mathbf{a}_2^{mn} \end{bmatrix}$

where $\mathbf{L}_{11}^{mn}$ and $\mathbf{L}_{22}^{mn}$ are 2 by 2 diagonal matrices. It would be also useful to explicitly relate the aperture field coefficients to the incident and scattered wave amplitudes in a concise manner. So expressed, the relationship reads

(25) $\mathbf{v} = \mathbf{L}_1\mathbf{a}_1 + \mathbf{L}_N\mathbf{a}_N + \mathbf{L}\mathbf{b}$

In (25), $\mathbf{a}_1$, $\mathbf{a}_N$, and $\mathbf{b}$ are M-segment vectors whose m^{th} segments are $\mathbf{a}_1^{m1}$, $\mathbf{a}_2^{mN}$, and $\mathbf{b}_m$, accordingly, and $\mathbf{L}_1$ and $\mathbf{L}_N$ are M by M block diagonal matrices whose mm^{th} blocks are the $2N$ by 1 matrices

(26) $\mathbf{L}_{1,mm} = \begin{bmatrix} \mathbf{L}_{11}^{m1} \\ \mathbf{O} \\ \vdots \\ \mathbf{O} \end{bmatrix}$

(27) $\mathbf{L}_{N,mm} = \begin{bmatrix} \mathbf{O} \\ \vdots \\ \mathbf{O} \\ \mathbf{L}_{22}^{mN} \end{bmatrix}$

while $\mathbf{L}$ is an M by M block diagonal matrix whose mm^{th} block is the $2N$ by $2(N-1)$ matrix

(28)

$$\mathbf{L}_{mm} = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & .. & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{L}_{22}^{m1}\mathbf{D} & \mathbf{0} & \mathbf{0} & .. & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{L}_{11}^{m2}\mathbf{D} & \mathbf{0} & \mathbf{0} & \mathbf{0} & .. & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{L}_{22}^{m2}\mathbf{D} & .. & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{L}_{11}^{m3}\mathbf{D} & \mathbf{0} & .. & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & .. & \mathbf{0} & \mathbf{0} & \mathbf{L}_{22}^{m(N-1)}\mathbf{D} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & .. & \mathbf{0} & \mathbf{L}_{11}^{mN}\mathbf{D} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & .. & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

Finally, by way of external mutual coupling, the vector of outgoing wave sources in the waveguide, **c**, is related to **v** and $\mathbf{v}_e$ according to

(29)

$$\begin{aligned} \mathbf{c} = \mathbf{Y}\mathbf{v} + \mathbf{Y}_e\mathbf{v}_e &= \left[\mathbf{Y} + \mathbf{Y}_e(\mathbf{I} - \mathbf{W}_e)^{-1}\mathbf{W}\right]\mathbf{v} \\ &= \left[\mathbf{Y} + \mathbf{Y}_e(\mathbf{I} - \mathbf{W}_e)^{-1}\mathbf{W}\right](\mathbf{L}_1\mathbf{a}_1 + \mathbf{L}_N\mathbf{a}_N + \mathbf{L}\mathbf{b}) \end{aligned}$$

upon using (23) and (25). Here, **c** is an M-segment vector whose m^{th} segment is $\mathbf{c}_m$, $\mathbf{Y}$ and $\mathbf{Y}_e$ are $M \times N$ by $M \times N$ block matrices whose $(mn,pq)^{th}$ blocks are, respectively, the 2 by 2 block matrix, $\mathbf{Y}_{mn,pq}$, and 2-segment vector, $\mathbf{y}_{e,mn,pq}$, where

$$(30)\quad \mathbf{Y}_{mn,pq} = \begin{bmatrix} \mathbf{L}_{11}^{mn,pq} & \mathbf{L}_{12}^{mn,pq} \\ \mathbf{L}_{21}^{mn,pq} & \mathbf{L}_{22}^{mn,pq} \end{bmatrix}$$

$$(31)\quad \mathbf{y}_{e,mn,pq} = \begin{bmatrix} y_{e,1}^{mn,pq} \\ y_{e,2}^{mn,pq} \end{bmatrix}$$

In this notation, $Y_{11,ij}^{mn,pq}$, $Y_{12,ij}^{mn,pq}$, $y_{e,1i}^{mn,pq}$ and are the amplitudes of the left-outgoing TE_{i0} mode source in the mn^{th} slot, respectively, due to the aperture field patterns $\mathbf{G}_{1j}$, $\mathbf{G}_{2j}$, and $\mathbf{G}_e$ in the pq^{th} slot, $i, j = 1, 2$. Similarly, $Y_{21,ij}^{mn,pq}$, $Y_{22,ij}^{mn,pq}$, $y_{e,2i}^{mn,pq}$ and are the amplitudes of the right-outgoing TE_{i0} mode source in the mn^{th} slot, respectively, due to the aperture field patterns $\mathbf{G}_{1j}$, $\mathbf{G}_{2j}$, and $\mathbf{G}_e$ in the pq^{th} slot, i, $j = 1, 2$.

In this analysis, the geometrical parameters describing the slots in the array are given. A method of moments analysis may be carried out to determine the scattering matrices for the various modules, as well as the field patterns in the slots and the corresponding matrices of aperture coefficients, $\mathbf{L}_1$, $\mathbf{L}_N$, and **L**, due to each incident mode. The patterns may be normalized so that v_{ij}^{mn}, $i, j = 1, 2$, and, hence, v_e^{mn}, are the strengths of the field at the centers of the slots. Next, the matrices of weighting coefficients are formed by determining the field due to each slot's field pattern in a homogeneous half-space at the centers of all other slots. The amount of work involved in this step can be substantially reduced by using reciprocity [2, Section 1-14]. This fully determines the field patterns in all of the slots. Consequently, the matrices **Y** and $\mathbf{Y}_e$ relating the aperture field coefficient vectors to the outgoing wave source vectors may be determined using appropriate waveguide Green's functions. Finally, quantities of interest in the waveguide may be determined using some of the formulas given.

3. DESIGN PROCEDURE

All the slots in the design procedure are assumed longitudinal whose locations are specified in terms of the amount of their shift from the axial centerlines of the waveguides. The procedure is equally applicable to inclined slots whose centers are located along any given line in each waveguide by replacing the parameters describing the offsets with those denoting the angular rotations of the slots.

The quantities specified to the designer are the reflection coefficients in the M feed waveguides, $r_1^{m1} = b_1^{m1} \div a_1^{m1}$, the load impedances terminating the feed waveguides, and the aperture field coefficients, $v_{mn} = v_{11}^{mn} + v_{12}^{mn} + v_{21}^{mn} + v_{22}^{mn} + v_e^{mn}$, in all of the slots. The quantities available to the designer to effect the reflection and aperture field coefficients are the complex incident TE_{10} mode amplitudes in the field waveguides, a_1^{m1}, as well as the lengths, l_{mn}, and offsets, x_{mn}, of the slots.

To devise an iterative design procedure for determining the amplitudes of the incident modes and lengths and offsets of the slots, the following elementary relations of differential calculus are utilized:

$$(32)\quad \Delta r_1^{p1} = \sum_{m=1}^{M}\sum_{n=1}^{N}\left[\Delta l_{mn}\frac{\partial}{\partial l_{mn}}r_1^{p1} + \Delta x_{mn}\frac{\partial}{\partial x_{mn}}r_1^{p1}\right]$$

(33)

$$\Delta v_{pq} = \sum_{m=1}^{M}\sum_{n=1}^{N}\left[\Delta l_{mn}\frac{\partial}{\partial l_{mn}}v_{pq} + \Delta x_{mn}\frac{\partial}{\partial x_{mn}}v_{pq}\right] + \sum_{m=1}^{M}\Delta a_1^{m1}\frac{\partial}{\partial a_1^{m1}}v_{pq}$$

for $p = 1, 2, .., M$, and $q = 1, 2, .., N$. Equating the real and imaginary parts in (32) and (33), there then results

(34)

$$\begin{bmatrix} Re(\mathbf{R}_l) & Re(\mathbf{R}_x) & \mathbf{O} & \mathbf{O} \\ Im(\mathbf{R}_l) & Im(\mathbf{R}_x) & \mathbf{O} & \mathbf{O} \\ Re(\mathbf{V}_l) & Re(\mathbf{V}_x) & Re(\mathbf{V}_a) & Im(\mathbf{V}_a) \\ Im(\mathbf{V}_l) & Im(\mathbf{V}_x) & Im(\mathbf{V}_a) & Re(\mathbf{V}_a) \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{l} \\ \Delta \mathbf{x} \\ Re(\Delta \mathbf{a}_1) \\ Im(\Delta \mathbf{a}_1) \end{bmatrix} = \begin{bmatrix} Re(\Delta \mathbf{r}_1) \\ Im(\Delta \mathbf{r}_1) \\ Re(\Delta \tilde{\mathbf{v}}) \\ Im(\Delta \tilde{\mathbf{v}}) \end{bmatrix}$$

where "*Re*" and "*Im*" denote, respectively, the real and imaginary parts of a complex number, $\mathbf{r}_1$ is a vector of length M given by

$$(35)\quad \mathbf{r}_1 = \left[r_1^{p1} \right]$$

$\boldsymbol{l}$, $\mathbf{x}$, and $\tilde{\mathbf{v}}$ are M-segment vectors whose m^{th} segments are the vectors

$$(36)\quad \mathbf{u}_m = \begin{bmatrix} u_{m1} \\ u_{m2} \\ \vdots \\ u_{mN} \end{bmatrix}, \qquad u = l, x, v$$

whereas $\mathbf{R}_l$ and $\mathbf{R}_x$ are 1 by M block matrices whose m^{th} blocks are the M by N matrices

$$(37)\quad \mathbf{R}_u^m = \left[R_{u,pn}^m \right] = \left[\frac{\partial}{\partial u_{mn}} r_1^{p1} \right], \qquad u = l, x$$

$\mathbf{V}_l$ and $\mathbf{V}_x$ are M by M block matrices whose pm^{th} blocks are the N by N matrices

$$(38)\quad \mathbf{V}_u^m = \left[V_{u,qn}^{pm} \right] = \left[\frac{\partial}{\partial u_{mn}} v_{pq} \right], \qquad u = l, x$$

and $\mathbf{V}_a$ is an M by 1 block matrix whose p^{th} block is the N by M matrix

$$(39)\quad \mathbf{V}_a^p = \left[V_{a,qm}^p \right] = \left[\frac{\partial}{\partial a_1^{m1}} v_{pq} \right]$$

The scattering matrix algorithm for array design now can be stated as follows:

ALGORITHM SMATRIX_WG_SLOTS ..

STEP 1: Specify an initial set of vectors $\boldsymbol{l}$, $\mathbf{x}$, and $\mathbf{a}_1$, together with the corresponding set of small increments, $\Delta \boldsymbol{l}$, $\Delta \mathbf{x}$, and $\Delta \mathbf{a}_1$.

STEP 2: Compute $\mathbf{b}$, $\mathbf{r}_1$, and $\tilde{\mathbf{v}}$ using equations (6), (15), (25), and (29).

STEP 3: Compute the right-hand side of equation (34) using the vectors $\mathbf{r}_1$ and $\tilde{\mathbf{v}}$ computed in STEP 2, and the specified vectors $\mathbf{r}_1$ and $\tilde{\mathbf{v}}$.
[Initially, if the starting vectors are far away from the final solution, it may be necessary to scale down Δ $\mathbf{r}_1$ and $\Delta\tilde{\mathbf{v}}$ so that iteration may proceed with small increments.]

STEP 4: Increment $\boldsymbol{l}$ by Δ $\boldsymbol{l}$, and compute $\mathbf{R}_l$ and $\mathbf{V}_l$.

STEP 5: Increment $\mathbf{x}$ by Δ $\mathbf{x}$, and compute $\mathbf{R}_x$ and $\mathbf{V}_x$.

STEP 6: Compute $\mathbf{V}_a$.
[Since $\tilde{\mathbf{v}}$ is a linear function of the input vector $\mathbf{a}_1$, this is best done by exciting each waveguide separately with a TE_{10} mode of unit amplitude and computing the resulting $\mathbf{r}_1$ and $\tilde{\mathbf{v}}$ vectors.]

STEP 7: Solve equation (34) for the incremental vectors Δ $\boldsymbol{l}$, Δ $\mathbf{x}$, and Δ $\mathbf{a}_1$. Hence, form the new set of vectors $\boldsymbol{l}$ + Δ $\boldsymbol{l}$, $\mathbf{x}$ + Δ $\mathbf{x}$, and $\mathbf{a}_1$ + Δ $\mathbf{a}_1$.

STEP 8: Repeat STEPS 2 through 7 until $\|\mathbf{r}_1\| \le \varepsilon$ and $\|\Delta\tilde{\mathbf{v}}\| \le \tau$, where ε and τ are arbitrarily small positive numbers.

4. NUMERICAL RESULTS

Two sets of numerical results were carried out to test the scattering matrix approach for linear and planar waveguide-fed narrow longitudinal slots. In the first set, the scattering matrix procedure was validated by computing the reflection coefficients in the feeding waveguides and aperture field coefficients in the slots in an array using the scattering matrix data of individual slot. The second set of numerical experiments was aimed at testing the iterative design procedure. The scattering matrix data for the slots in both the analysis and design experiments were computed using a method of moments procedure with rooftop expansion functions and pulse testing to solve the integral equations for the axial (z-directed) component of the equivalent magnetic current in the slot.

TABLE I
COMPARISON OF REFLECTION COEFFICIENT AND APERTURE FIELD COEFFICIENTS FOR METHOD OF MOMENTS AND S-MATRIX ANALYSES

Parameter	*Method of Moments with all* WG *Modes*	*Method of Moments with* TE_{10} *and* TE_{20} *Inter-Slot Modes*	*S-Matrix Analysis with* TE_{10} *and* TE_{20} *Inter-Slot Modes*
b_1^{11}	0.4519<179.6°	0.4495<178.6°	0.4495<178.6°
b_2^{14}	0.6641<-95.3°	0.6659<-95.8°	0.6671<-95.7°
v_{11}	0.8093<-86.1°	0.7454<-83.6°	0.6767<-87.6°
v_{12}	1.8892<-48.6°	1.9404<-44.3°	2.0547<-43.6°
v_{13}	4.8110<-21.9°	4.7637<-21.0°	4.6161<-21.0°
v_{14}	5.9100<-40.9°	5.9484<-40.0°	5.9385<-41.1°

TABLE II
ITERATIVE DESIGN OF A 2 × 3 PLANAR ARRAY OF WAVEGUIDE-FED LONGITUDINAL SLOTS USING THE S-MATRIX ANALYSIS

Iteration / Parameter	1	2	3	4	5	6	7	8	9
l_{11}, x_{11}	1.6000, 0.6000	1.5761, 0.4055	1.6241, 0.5399	1.5148, 0.1999	1.5267, 0.2193	1.5495, 0.2493	1.5694, 0.2566	1.5618, 0.2363	1.5755, 0.2566
l_{12}, x_{12}	1.6000, -0.6000	1.6174, -0.6005	1.6287, -0.6299	1.6659, -0.7681	1.6601, -0.6254	1.6258, -0.4626	1.5894, -0.3556	1.5892, -0.3735	1.5888, -0.3639
l_{13}, x_{13}	1.6000, 0.6000	1.5972, 0.5056	1.4984, 0.1696	1.5159, 0.2186	1.5271, 0.2329	1.5482, 0.2529	1.5681, 0.2545	1.5656, 0.2434	1.5681, 0.2415
l_{21}, x_{21}	1.6000, 0.6000	1.6126, 0.5924	1.6147, 0.5374	1.6115, 0.4793	1.6032, 0.4345	1.5912, 0.3772	1.5845, 0.3321	1.5852, 0.3267	1.5849, 0.3345
l_{22}, x_{22}	1.6000, -0.6000	1.5579, -0.3789	1.5522, -0.3386	1.5508, -0.3172	1.5487, -0.2986	1.5463, -0.2644	1.5522, -0.2209	1.5626, -0.2093	1.5529, -0.1970
l_{23}, x_{23}	1.6000, 0.6000	1.6151, 0.6057	1.6196, 0.5493	1.6180, 0.5162	1.6066, 0.4521	1.5923, 0.3803	1.5852, 0.3330	1.5849, 0.3276	1.5865, 0.3381
a_1	j0.25	0.0021+j0.2231	0.0067+j0.1997	0.0107+j0.1694	0.0064+j0.1657	-0.0013+j0.1615	0.0024+j0.1597	0.0021+j0.1571	-0.0004+j0.1613
a_2	j0.25	0.0031+j0.2248	0.0028+j0.2121	0.0027+j0.2024	0.0033+j0.1904	0.0035+j0.1744	0.0018+j0.1617	0.0004+j0.1614	0.0000+j0.1613
sf	0.1	0.1	0.1	0.2	0.4	0.8	1.0	1.0	1.0
r_1	-0.5506-j0.3120	-0.4696+j0.2092	-0.4030-j0.1866	-0.2668-j0.1400	-0.2106-j0.0986	-0.1218-j0.0540	-0.0157-j0.0076	0.0018-j0.0082	-0.0001-j0.0096
r_2	-0.5548-j0.3094	-0.4790+j0.2207	-0.3830-j0.1758	-0.2941-j0.1103	-0.2248+j0.0987	-0.1242-j0.0861	-0.0242-j0.0249	0.0076+j0.0094	-0.0058+j0.0002
v_{11}	1.2942+j0.2724	1.2071+j0.0788	1.4423+j0.1725	0.8277+j0.1767	0.8843+j0.1803	0.9831+j0.1375	1.0295+j0.0236	0.9752+j0.0178	1.0369-j0.0296
v_{12}	1.1460+j0.2557	1.1483+j0.1780	1.1452+j0.0728	1.1963-j0.0311	1.1546-j0.0680	1.0589-j0.0414	0.9779+j0.0236	0.9967-j0.0213	0.9907+j0.0145
v_{13}	1.3207+j0.2355	1.3377+j0.2222	0.6586+j0.1319	0.8839+j0.2045	1.9213+j0.2152	0.9904+j0.1641	1.0223+j0.0286	0.9952+j0.0046	0.9955-j0.0082
v_{21}	1.1761+j0.1564	1.1504+j0.0544	1.2689+j0.2191	1.0427+j0.0596	1.0208+j0.0358	1.0044+j0.0130	0.9957-j0.0069	0.9887+j0.0036	1.0178-j0.0011
v_{22}	1.3863+j0.4900	1.2902+j0.3465	1.2248+j0.2656	1.1718+j0.0954	1.1849+j0.1398	1.1447+j0.1735	1.0508+j0.0907	1.0105-j0.0350	0.9859+j0.0274
v_{23}	1.2088+j0.1110	1.2107+j0.1546	1.0036-j0.0335	1.1284+j0.0946	1.0702+j0.0589	1.0125-j0.0189	0.9903-j0.0126	0.9952+j0.0053	1.0027-j0.0098

A. Validation of the Scattering Matrix Approach for Linear and Planar Arrays:

A linear array in the broad wall of a thin ($t = 0$) standard X-band waveguide whose cross-sectional dimensions are 0.2286 m and 0.01016 m was considered for the analysis. the slots in the array had the length-offset pairs (0.015 m, 0.001 m), (0.016 m, -0.002 m), (0.016 m, 0.003 m), and (0.015 m, -0.004 m) with inter-slot spacing $d = 0.02$ m. The waveguide was terminated with a short circuit placed a quarter of a waveguide wavelength from the center of the last slot. Furthermore, the waveguide was excited by a TE_{10} waveguide mode of unit amplitude at 9.0 GHz. The results obtained using the scattering matrix analysis are compared with those obtained using the method of moments analysis in Table I. Inspection of the table readily shows that the results obtained from the method of moments by limiting internal mutual coupling between the slots to the TE_{10} and TE_{20} modes are in better agreement with the data obtained using the scattering matrix approach, with the results obtained with the contribution of all higher order modes accounted for are only slightly different for the inter-slot distance considered). Similar results were obtained for $M \times N = 2 \times 2$, 2×3, and 2×4 planar arrays.

B. Design of a 2 × 3 Planar Array:

A 2×3 planar array with inter-slot spacing of half waveguide wavelength in standard X-band waveguides terminated with short-circuits placed a quarter of a waveguide wavelength away from the centers f the last slots was considered for design. The design specifications were zero reflection in the waveguides and aperture field coefficients of unity in the six slots. The results of the iterative design procedure at 9.0 GHz are summarized in Table II. In the table, the lengths and offsets are given in centimeters, and "*sf*" refers to the scale factor by which the right-hand side of (34) was multiplied at a particular stage of interaction. At each step of iteration, the scattering matrix data of the individual slots and the outgoing wave sources due to external mutual coupling were calculated using the method of moments procedure assuming the walls of the waveguide are of zero thickness. Iteration was halted when $\|\mathbf{r}_1\| \le 10^{-3}$ and $\|\Delta\tilde{\mathbf{v}}\| \le 10^{-2}$. It is worth noting that the computation time for the design of the longitudinal slot array can be drastically reduced by pre-computing the scattering matrix data for a number of equidistant closely spaced length-offset pairs and using such data in a *look-up* data set.

5. SUMMARY

The scattering matrix approach for the analysis and design of planar waveguide-fed slot arrays has been described in this paper. The new approach utilizes the scattering matrix data of individual slots rather than the admittance data derived from approximate shunt or series models of the slots. The method fully accounts for the external mutual coupling due to the TE_{10} and TE_{20} modes on the slots, and can be easily extended to include higher order waveguide modes.

Numerical experiments demonstrating the viability of the scattering matrix approach, including an example design of 2×3 planar array of longitudinal slots have been presented.

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