Plane Wave Scattering by Two Dielectric Coated Conducting Strips

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Abstract **—** A plane electromagnetic wave scattered by two dielectric coated conducting strips is addressed here. Two methods of solutions are introduced. The first is based on solving the Helmholtez wave equation in terms of elliptical coordinates. As a result a Fourier series of radial and angular Mathieu functions of unknown coefficients in each region is obtained. The unknown coefficients can be obtained by enforcing the boundary conditions. The application of the boundary condition requires the use of the addition theorem of Mathieu function. The second method is based on an asymptotic approximate technique introduced by Karp and Russek for solving scattering by wide slit. Numerical examples are calculated using both methods and they are compared with each other. Excellent agreement between both cases is found.

Keywords **—** scattering by cylinders, coated strips, multiple scattering.

I. INTRODUCTION

Scattering from conducting strip and strip grating were the subject of many investigations [1-3]. Also the scattering of an electromagnetic wave from a single strip coated with a dielectric was addressed [4]. The multiple scattering of a plane electromagnetic wave by two strips of parallel edges was also presented in [5]. Recently, the scattering by two dielectric elliptic cylinders [6] and by metamaterial coated elliptic cylinders [7] has been addressed. The scattering of electromagnetic waves by coated strips has not been addressed yet. Therefore this paper presents two methods for solving the scattering of an electromagnetic plane wave by two dielectric coated conducting strips. This geometry can be used to simulate a dielectric coated conducting plane when the strips are close to each other. It could also have an application of diffraction by slit of dielectric coated conducting slits.

II. FORMULATION OF THE PROBLEM

Figure 1 shows two dielectric coated conducting strips of infinite length with their axes parallel to the z axis and widths $2d_1$ and $2d_2$, respectively. The dielectric coating have permittivities ε_1 and ε_2, and focal length is equal to the conducting strip width. The outer surface of the dielectric coating of the first strip has semi-major axis a_1 and semi-minor axis b_1, while a_2 and b_2 are, respectively denoting semi-major and semi-minor axes of the outer surface of the second strip coating. The center of the first coated strip is located at $x = c$ while the center of the second is located at $x = -c$ with respect to the global coordinates (x, y, z). The coated strips are inclined with respect to the x-axis by angle β_1 and β_2, respectively. In addition to the global coordinate system, two coordinate systems (x_1, y_1, z_1) and (x_2, y_2, z_2) are defined at the strip centers such that the plane of the first strip lies in the $x_1 - z_1$ plane while that of the second strip lies in the $x_2 - z_2$ plane.

A plane wave, with $e^{-j\omega t}$ time dependence, is incident with an angle φ_o with respect to the x-axis of the global coordinate system and polarized in z-direction, i.e.,

$$E_z^i = e^{-jk_o(x\cos\varphi_o + y\sin\varphi_o)} \tag{1}$$

where k_o is the wave number in free space. The incident wave can also be expressed in terms of the local coordinates at the coated strip centers. Upon doing the transformation and expanding it in terms of the elliptic wave function, one obtains,

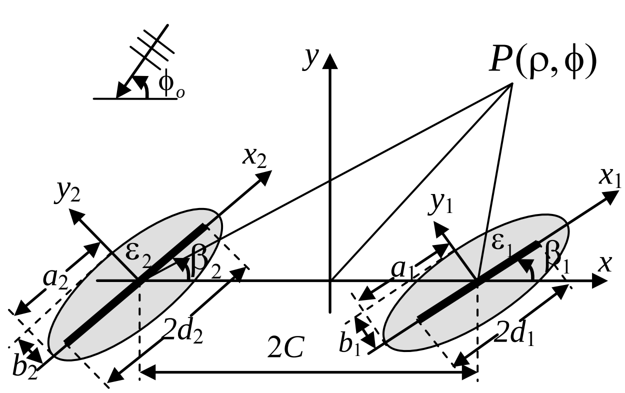

Fig. 1. Geometry of the problem.

$$E^{i}_{z_1} = \sqrt{8\pi}\, e^{-jkc\cos\varphi_o} \sum_{m=0}^{\infty} j^{-m}\left[\frac{1}{N^{(e)}_m(h_1)} Je_m(h_1,\zeta_1) Se_m(h_1,\eta_1)\, Se_m(h_1,\cos\varphi_{01}) + \frac{1}{N^{(o)}_m(h_1)} Jo_m(h_1,\zeta_1)\, So_m(h_1,\eta_1) So_m(h_1,\cos\varphi_{01})\right] \tag{2}$$

and

$$E^{i}_{z_2} = \sqrt{8\pi}\, e^{jkc\cos\varphi_o} \sum_{m=0}^{\infty} j^{-m}\left[\frac{1}{N^{(e)}_m(h_2)} Je_m(h_2,\zeta_2)\, Se_m(h_2,\eta_2)\, Se_m(h_2,\cos\varphi_{02}) + \frac{1}{N^{(o)}_m(h_2)} Jo_m(h_2,\zeta_2)\, So_m(h_2,\eta_2)\, So_m(h_2,\cos\varphi_{02})\right] \tag{3}$$

where

$$\varphi_{01} = \varphi_o - \beta_1 \quad \text{and} \quad \varphi_{02} = \varphi_o - \beta_2 \tag{4}$$

while $Je_m(h,\zeta)$ and $Jo_m(h,\zeta)$ are respectively even and odd modified Mathieu functions of the first kind and order m. Also, $Se_m(h,\eta)$ and $So_m(h,\eta)$ are respectively even and odd angular Mathieu functions of order m. $N^{(e)}_m(h)$ and $N^{(o)}_m(h)$ are even and odd normalized functions, respectively. The Mathieu functions arguments $h_i = k_o d_i$, $\zeta_i = \cosh u_i$ and $\eta_i = \cos v_i$ ($i = 1$ or 2) where u_i and v_i are elliptical cylindrical coordinates defined by,

$$\begin{aligned} x_i &= d_i \cosh u_i \cos v_i \\ y_i &= d_i \sinh u_i \sin v_i \,, \\ z &= z_i \,. \end{aligned} \tag{5}$$

The scattered electric field from the coated strips can be expressed in terms of the local coordinates at the center of each coated strip. Region (I) is inside the dielectric coating and region (II) is outside the dielectric coating. Scattered field from the first strip in region (I) is given by,

$$E^{(I)}_{z_1} = \sqrt{8\pi} \sum_{m=0}^{\infty} A^{(1)}_m \left\{ Je_m(H_1,\zeta_1) - \frac{Je_m(H_1,1)}{Ne_m(H_1,1)} Ne_m(H_1,\zeta_1)\right\} Se_m(H_1,\eta_1) \tag{6}$$

where $Ne_m(h,\zeta)$ is the even modified Mathieu function of the second kind and order $m, H_1 = k_1 d_1$ and $k_1 = k_o\sqrt{\mu_{r_1}\varepsilon_{r_1}}$, $A^{(1)}_m$ are unknown coefficients to be calculated from the boundary conditions. The boundary condition of the vanishing the tangential component of the electric field on the conducting strip surface was satisfied in equation (6).

Similarly, the scattered field from the second strip inside its dielectric is,

$$E^{(I)}_{z_2} = \sqrt{8\pi} \sum_{m=0}^{\infty} A^{(2)}_m \left\{ Je_m(H_2,\zeta_2) - \frac{Je_m(H_2,1)}{Ne_m(H_2,1)} Ne_m(H_2,\zeta_2)\right\} Se_m(H_2,\eta_2) \,. \tag{7}$$

The scattered field in region (II) from the first and the second strips are given by,

$$E^{(II)}_{z_1} = \sqrt{8\pi} \sum_{m=0}^{\infty} B^{(1)}_m\, He^{(1)}_m(h_1,\zeta_1)\, Se_m(h_1,\eta_1) \tag{8}$$

and

$$E^{(II)}_{z_2} = \sqrt{8\pi} \sum_{m=0}^{\infty} B^{(2)}_m\, He^{(1)}_m(h_2,\zeta_2)\, Se_m(h_2,\eta_2) \tag{9}$$

where

$$He^{(1)}_m(h,\zeta) = Je_m(h,\zeta) + j\, Ne_m(h,\zeta) \,.$$

In addition $B^{(1)}_m$ and $B^{(2)}_m$ are unknown coefficients to be calculated from the boundary conditions of homogenous tangential components of electric and magnetic fields at the surface of the dielectric coatings, i.e,

$$E^{inc}_{Z_1} + E^{(II)}_{z_1} + E^{(II)}_{z_2} = E^{(I)}_{z_1} \text{ on dielectric coating (I)} \tag{10}$$

$$E^{inc}_{Z_2} + E^{(II)}_{z_1} + E^{(II)}_{z_2} = E^{(I)}_{z_2} \text{ on dielectric coating (II),} \tag{11}$$

$$H^{inc}_{v_1} + H^{(II)}_{v_1} + H^{(II)}_{v_2} = H^{(I)}_{v_1} \text{ on dielectric coating (I),} \tag{12}$$

$$H^{inc}_{v_2} + H^{(II)}_{v_1} + H^{(II)}_{v_2} = H^{(I)}_{v_2} \text{ on dielectric coating (II).} \tag{13}$$

In order to apply the above boundary conditions, one has to transfer the electric and magnetic field expressions from one coordinate system to the other. This can be done using the addition theorem of the Mathieu functions, namely

- From coated strip (2) to coated strip (1),

$$He_m(a_2,\zeta_2)\,Se_m(a_2,\eta_2)=\sum_{q=0}^{\infty}Ke_{q,m}Je_q(a_1,\zeta_1)\,Se_q(a_1,\eta_1)+\sum_{q=1}^{\infty}We_{q,m}Jo_q(a_1,\zeta_1)\,So_q(a_1,\eta_1) \tag{14}$$

- From coated strip (1) to coated strip (2),

$$He_m(a_1,\zeta_1)\,Se_m(a_1,\eta_1)=\sum_{q=0}^{\infty}Ge_{q,m}\,Je_q(a_2,\zeta_2)\,Se_q(a_2,\eta_2)+\sum_{q=0}^{\infty}Fe_{q,m}Jo_q(a_2,\zeta_2)\,So_q(a_2,\eta_2) \tag{15}$$

where

$$Ke_{q,m}=\frac{\pi(j)^{q-m}}{N_q^{(e)}(a_1)}\sum_{s=0}^{\infty}\sum_{p=0}^{\infty}(-j)^{s-p}\,De_p^q(a_1)\,De_s^m(a_2)\,Be_{s,p} \tag{16}$$

$$We_{q,m}=\frac{\pi(j)^{q-m}}{N_q^{(o)}(a_1)}\sum_{s=0}^{\infty}\sum_{p=0}^{\infty}(-j)^{s-p}\,De_p^q(a_1)\;Do_s^m(a_2)\;Be_{s,p}, \tag{17}$$

$$Ge_{q,m}=\frac{\pi(j)^{q-m}}{N_q^{(e)}(a_2)}\sum_{s=0}^{\infty}\sum_{p=0}^{\infty}(-j)^{p-s}\,De_p^q(a_2)\;De_s^m(a_1)\;Ue_{s,p}, \tag{18}$$

$$Fe_{q,m}=\frac{\pi(j)^{q-m}}{N_q^{(o)}(a_2)}\sum_{s=0}^{\infty}\sum_{p=0}^{\infty}(-j)^{p-s}\,De_p^q(a_2)\;Do_s^m(a_1)\;Ue_{s,p}, \tag{19}$$

and

$$Be_{s,p}=H_{p-s}^{(1)}(kc)\;\cos(p\beta_1-s\beta_2)+(-1)^s H_{p+s}^{(1)}(kc)\;\cos(p\beta_1+s\beta_2), \tag{20}$$

$$Ue_{s,p}=H_{p-s}^{(1)}(kc)\cos(p\beta_2-s\beta_1)+(-1)^s H_{p+s}^{(1)}(kc)\cos(p\beta_2+s\beta_1). \tag{21}$$

Applying the boundary condition (11) one obtains,

$$\begin{aligned}&\sqrt{8\pi}\;e^{-jkc\cos\phi_o}\sum_{m=0}^{\infty}j^{-m}\Bigg[\frac{1}{N_m^{(e)}(h_1)}Je_m(h_1,\zeta_{10})\,Se_m(h_1,\eta_1)\,Se_m(h_1,\cos\phi_{o1})+\frac{1}{N_m^{(o)}(h_1)}Jo_m(h_1,\zeta_{10})\,So_m(h_1,\eta_1)\,So_m(h_1,\cos\phi_{o1})\Bigg]+\\&\sqrt{8\pi}\sum_{m=0}^{\infty}B_m^{(1)}\;He_m^{(1)}(h_1,\zeta_{10})\,Se_m(h_1,\eta_1)+\\&\sqrt{8\pi}\sum_{m=0}^{\infty}B_m^{(2)}\left\{\sum_{q=0}^{\infty}Ke_{q,m}\,Je_q(h_1,\zeta_{10})Se_q(h_1,\eta_1)+\sum_{q=1}^{\infty}We_{q,m}Jo_q(h_1,\zeta_{10})\,So_q(h_1,\eta_1)\right\}\\&=\sqrt{8\pi}\sum_{m=0}^{\infty}A_m^{(1)}\left\{Je_m(H_1,\zeta_{10})-\frac{Je_m(H_1,1)}{Ne_m(H_1,1)}Ne_m(H_1,\zeta_{10})\right\}Se_m(H_1,\eta_1).\end{aligned} \tag{22}$$

Multiplying both sides of equation (22) by $Se_l(H_1,\eta_1)$ and integrating over v_1 from 0 to 2π, one obtains,

$$\begin{aligned}&e^{-jkc\cos\phi_o}\sum_{n=0}^{\infty}j^{-n}\frac{1}{N_n^{(e)}(h_1)}Je_n(h_1,\zeta_{10})\,Se_n(h_1,\cos\phi_{o1})M_{n,m}(h_1,H_1)\\&+\sum_{n=0}^{\infty}B_n^{(1)}\;He_n^{(1)}(h_1,\zeta_{10})\,M_{n,m}(h_1,H_1)+\\&\sum_{n=0}^{\infty}B_n^{(2)}\sum_{q=0}^{\infty}Ke_{q,n}Je_q(h_1,\zeta_{10})\,M_{q,m}(h_1,H_1)\\&=A_m^{(1)}\left\{Je_m(H_1,\zeta_{10})-\frac{Je_m(H_1,1)}{Ne_m(H_1,1)}Ne_m(H_1,\zeta_{10})\right\}N_m^{(e)}(H_1)\end{aligned} \tag{23}$$

where

$$M_{n,m}(h_1,H_1)=\int_0^{2\pi}Se_n(h_1,\eta_1)\,Se_m(H_1,\eta_1)\,dv_1. \tag{24}$$

Applying boundary condition (12), one gets,

$$\sqrt{8\pi}\, e^{jkc\cos\phi_o} \sum_{m=0}^{\infty} j^{-m} \Big[\frac{1}{N_m^{(e)}(h_2)} Je_m(h_2,\zeta_{20})$$
$$Se_m(h_2,\eta_2)\, Se_m(h_2,\cos\phi_{o2}) + \frac{1}{N_m^{(o)}(h_2)}$$
$$Jo_m(h_2,\zeta_{20})\, So_m(h_2,\eta_2)\, So_m(h_2,\cos\phi_{o2}) \Big]$$
$$+\sqrt{8\pi}\sum_{m=0}^{\infty} B_m^{(2)}\ He_m^{(1)}(h_2,\zeta_{20})\, Se_m(h_2,\eta_2) +$$
$$\sqrt{8\pi}\sum_{m=0}^{\infty} B_m^{(1)} \Big\{ \sum_{q=0}^{\infty} Ge_{q,m}\, Je_q(h_2,\zeta_{20})\, Se_q(h_2,\eta_2)$$
$$+\sum_{q=1}^{\infty} Fe_{q,m}\, Jo_q(h_{20},\zeta_2)\, So_q(h_2,\eta_2) \Big\} =$$
$$\sqrt{8\pi}\sum_{m=0}^{\infty} A_m^{(2)} \Big\{ Je_m(H_2,\zeta_{20}) - \frac{Je_m(H_2,1)}{Ne_m(H_2,1)}$$
$$Ne_m(H_2,\zeta_{20}) \Big\} Se_m(H_2,\eta_2). \qquad (24)$$

Multiplying both sides of equation (25) by $Se_l(h_2,\eta_2)$ and integrating over v_1 from 0 to 2π, one obtains,

$$e^{jkc\cos\phi_o} \sum_{n=0}^{\infty} j^{-n} \frac{1}{N_n^{(e)}(h_2)} Je_n(h_2,\zeta_{20})$$
$$Se_n(h_2,\cos\phi_{o2}) M_{n,m}(h_2,H_2)$$
$$+\sum_{n=0}^{\infty} B_n^{(2)}\ He_n^{(1)}(h_2,\zeta_{20})\, M_{n,m}(h_2,H_2) +$$
$$\sum_{n=0}^{\infty} B_n^{(1)} \sum_{q=0}^{\infty} Ge_{q,n}\, Je_q(h_2,\zeta_{20})\, M_{q,m}(h_2,H_2) \qquad (25)$$
$$= A_m^{(2)} \Big\{ Je_m(H_2,\zeta_{20}) - \frac{Je_m(H_2,1)}{Ne_m(H_2,1)}$$
$$Ne_m(H_2,\zeta_{20}) \Big\} N_m^{(e)}(H_2)$$

where

$$M_{n,m}(h_2,H_2) = \int_0^{2\pi} Se_n(h_2,\eta_2)$$
$$Se_m(H_2,\eta_2)\, dv_2. \qquad (27)$$

The magnetic field component H_v is given by,

$$H_v = \frac{-jk}{\omega\mu d\sqrt{\zeta^2-\eta^2}} \frac{\partial E_z}{\partial u}. \qquad (28)$$

Employing equation (28) and applying boundary condition (14) lead to,

$$\sqrt{8\pi}\, e^{-jkc\cos\phi_o} \sum_{m=0}^{\infty} j^{-m} \Big[\frac{1}{N_m^{(e)}(h_1)} Je'_m(h_1,\zeta_{10})$$
$$Se_m(h_1,\eta_1)\, Se_m(h_1,\cos\phi_{o1}) + \frac{1}{N_m^{(o)}(h_1)}$$
$$Jo'_m(h_1,\zeta_{10}) So_m(h_1,\eta_1)\, So_m(h_1,\cos\phi_{o1}) \Big]$$
$$+\sqrt{8\pi}\sum_{m=0}^{\infty} B_m^{(1)}\ He_m^{(1)\prime}(h_1,\zeta_{10})\, Se_m(h_1,\eta_1) +$$
$$\sqrt{8\pi}\sum_{m=0}^{\infty} B_m^{(2)} \Big\{ \sum_{q=0}^{\infty} Ke_{q,m}\, Je'_q(h_1,\zeta_{10})\, Se_q(h_1,\eta_1)$$
$$+\sum_{q=1}^{\infty} We_{q,m}\, Jo'_q(h_1,\zeta_{10}) So_q(h_1,\eta_1) \Big\}$$
$$= \sqrt{\frac{8\pi\varepsilon_{r1}}{\mu_{r1}}} \sum_{m=0}^{\infty} A_m^{(1)} \Big\{ Je'_m(H_1,\zeta_{10}) - \frac{Je_m(H_1,1)}{Ne_m(H_1,1)}$$
$$Ne'_m(H_1,\zeta_{10}) \Big\} Se_m(H_1,\eta_1). \qquad (29)$$

Again multiplying both sides of equation (29) by $Se_l(h_1,\eta_1)$ and integrating over v_1 from 0 to 2π, one obtains,

$$e^{-jkc\cos\phi_o} \sum_{n=0}^{\infty} j^{-n} \frac{1}{N_n^{(e)}(h_1)} Je'_n(h_1,\zeta_{10})$$
$$Se_n(h_1,\cos\phi_{o1}) M_{n,m}(h_1,H_1)$$
$$+\sum_{n=0}^{\infty} B_n^{(1)}\ He_n^{(1)\prime}(h_1,\zeta_{10})\, M_{n,m}(h_1,H_1) +$$
$$\sum_{n=0}^{\infty} B_n^{(2)} \sum_{q=0}^{\infty} Ke_{q,n}\, Je'_q(h_1,\zeta_{10})\, M_{q,m}(h_1,H_1) \qquad (30)$$
$$= \sqrt{\varepsilon_r}\, A_m^{(1)} \Big\{ Je'_m(H_1,\zeta_{10}) - \frac{Je_m(H_1,1)}{Ne_m(H_1,1)}$$
$$Ne'_m(H_1,\zeta_{10}) \Big\} N_m^{(e)}(H_1) \quad .$$

Similarly applying boundary condition (15), and employing the orthogonally of the Mathieu functions, results in,

$$e^{jkc\cos\phi_o}\sum_{n=0}^{\infty} j^{-n}\frac{1}{N_n^{(e)}(h_2)}Je'_n(h_2,\zeta_{20}) Se_n(h_2,\cos\phi_{o2})M_{n,m}(h_2,H_2) + \sum_{n=0}^{\infty}B_n^{(2)}\, He_n^{(1)'}(h_2,\zeta_{20})\, M_{n,m}(h_2,H_2) + \sum_{n=0}^{\infty}B_n^{(1)}\sum_{q=0}^{\infty}Ge_{q,n}Je'_q(h_2,\zeta_{20})\, M_{q,m}(h_2,H_2) = \sqrt{\varepsilon_r}A_m^{(2)}\left\{Je'_m(H_2,\zeta_{20})-\frac{Je_m(H_2,1)}{Ne_m(H_2,1)} Ne'_m(H_2,\zeta_{20})\right\}N_m^{(e)}(H_2). \tag{31}$$

From equations (23) and (30), one can obtain,

$$-e^{-jkc\cos\phi_o}\sum_{n=0}^{\infty} j^{-n}\frac{1}{N_n^{(e)}(h_1)}Se_n(h_1,\cos\phi_{o1}) M_{n,m}(h_1,H_1)\left[\frac{Je_n(h_1,\zeta_{10})}{X_m(H_1)}-\frac{Je'_n(h_1,\zeta_{10})}{X'_m(H_1)}\right] =\sum_{n=0}^{\infty}B_n^{(1)}\, M_{n,m}(h_1,H_1)\left[\frac{He_n^{(1)}(h_1,\zeta_{10})}{X_m(H_1)} -\frac{He_n^{(1)'}(h_1,\zeta_{10})}{X'_m(H_1)}\right]+\sum_{n=0}^{\infty}B_n^{(2)}\sum_{q=0}^{\infty}Ke_{q,n}M_{q,m}(h_1,H_1) \left[\frac{Je_q(h_1,\zeta_{10})}{X_m(H_1)}-\frac{Je'_q(h_1,\zeta_{10})}{X'_m(H_1)}\right] \tag{32}$$

where

$$X_m(H_1)=\left\{Je_m(H_1,\zeta_{10})-\frac{Je_m(H_1,1)}{Ne_m(H_1,1)} Ne_m(H_1,\zeta_{10})\right\}N_m^{(e)}(H_1), \tag{33}$$

$$X'_m(H_1)=\sqrt{\varepsilon_r}\left\{Je'_m(H_1,\zeta_{10})-\frac{Je_m(H_1,1)}{Ne_m(H_1,1)} Ne'_m(H_1,\zeta_{10})\right\}N_m^{(e)}(H_1). \tag{34}$$

From equations (26) and (31) one can find out that,

$$-e^{jkc\cos\phi_o}\sum_{n=0}^{\infty} j^{-n}\frac{1}{N_n^{(e)}(h_2)}Se_n(h_2,\cos\phi_{o2}) M_{n,m}(h_2,H_2)\left[\frac{Je_n(h_2,\zeta_{20})}{X_m(H_2)}-\frac{Je'_n(h_2,\zeta_{20})}{X'_m(H_2)}\right]= \sum_{n=0}^{\infty}B_n^{(1)}\sum_{q=0}^{\infty}Ke_{q,n}\, M_{q,m}(h_2,H_2)\left[\frac{Je_q(h_2,\zeta_{20})}{X_m(H_2)} -\frac{Je'_q(h_2,\zeta_{20})}{X'_m(H_2)}\right]+\sum_{n=0}^{\infty}B_n^{(2)}\, M_{n,m}(h_2,H_2) \left[\frac{He_n^{(1)}(h_2,\zeta_{20})}{X_m(H_2)}-\frac{He_n^{(1)'}(h_2,\zeta_{20})}{X'_m(H_2)}\right] \tag{35}$$

where

$$X_m(H_2)=\left\{Je_m(H_2,\zeta_{20})-\frac{Je_m(H_2,1)}{Ne_m(H_2,1)} Ne_m(H_2,\zeta_{20})\right\}N_m^{(e)}(H_2), \tag{36}$$

$$X'_m(H_2)=\sqrt{\varepsilon_r}\left\{Je'_m(H_2,\zeta_{20})-\frac{Je_m(H_2,1)}{Ne_m(H_2,1)} Ne'_m(H_2,\zeta_{20})\right\}N_m^{(e)}(H_2). \tag{37}$$

Equations (32) and (35) can be written in a matrix form,

$$\begin{bmatrix} C_{m,n}^{(11)} & C_{m,n}^{(12)} \\ C_{m,n}^{(21)} & C_{m,n}^{(22)} \end{bmatrix}\begin{bmatrix} B_n^{(1)} \\ B_n^{(2)}\end{bmatrix}=\begin{bmatrix} Z_m^{(1)} \\ Z_m^{(2)}\end{bmatrix} \tag{38}$$

where

$$Z_m^{(1)}=-e^{-jkc\cos\phi_o}\sum_{n=0}^{\infty} j^{-n}\frac{1}{N_n^{(e)}(h_1)} Se_n(h_1,\cos\phi_{o1})M_{n,m}(h_1,H_1)\left[\frac{Je_n(h_1,\zeta_{10})}{X_m(H_1)} -\frac{Je'_n(h_1,\zeta_{10})}{X'_m(H_1)}\right], \tag{39}$$

$$C_{m,n}^{(11)}=M_{n,m}(h_1,H_1)\left[\frac{He_n^{(1)}(h_1,\zeta_{10})}{X_m(H_1)} -\frac{He_n^{(1)'}(h_1,\zeta_{10})}{X'_m(H_1)}\right], \tag{40}$$

$$C_{m,n}^{(12)} = \sum_{q=0}^{\infty} Ke_{q,n} \; M_{q,m}(h_1, H_1) \left[\frac{Je_q(h_1,\zeta_{10})}{X_m(H_1)} - \frac{Je_q(h_1,\zeta_{10})}{X'_m(H_1)} \right], \tag{41}$$

$$Z_m^{(2)} = -e^{jkc\cos\phi_o} \sum_{n=0}^{\infty} j^{-n} \frac{1}{N_n^{(e)}(h_2)} Se_n(h_2, \cos\phi_{o2}) M_{n,m}(h_2, H_2) \left[\frac{Je_n(h_2,\zeta_{20})}{X_m(H_2)} - \frac{Je'_n(h_2,\zeta_{20})}{X'_m(H_2)} \right], \tag{42}$$

$$C_{m,n}^{(21)} = \sum_{q=0}^{\infty} Ge_{q,n} M_{q,m}(h_2, H_2) \left[\frac{Je_q(h_2,\zeta_{20})}{X_m(H_2)} - \frac{Je'_q(h_2,\zeta_{20})}{X'_m(H_2)} \right], \tag{43}$$

$$C_{m,n}^{(22)} = M_{n,m}(h_2, H_2) \left[\frac{He_n^{(1)}(h_2,\zeta_{20})}{X_m(H_2)} - \frac{{He_n^{(1)}}'(h_2,\zeta_{20})}{X'_m(H_2)} \right]. \tag{44}$$

Once we calculate the unknown coefficients the total scattered field can be calculated from,

$$E_z^{(s)} = \sqrt{8\pi} \left\{ \sum_{m=0}^{\infty} B_m^{(1)} \; He_m^{(1)}(h_1,\zeta_1) \; Se_m(h_1,\eta_1) + \sum_{m=0}^{\infty} B_m^{(2)} \; He_m^{(1)}(h_2,\zeta_2) Se_m(h_2,\eta_2) \right\}. \tag{45}$$

The far field can be evaluated using the asymptotic expansion of $He_m^{(1)}(h,\zeta)$ which is given by,

$$He_m^{(1)}(h,\zeta) = \frac{1}{\sqrt{h\zeta}} e^{j(h\zeta-(2m+1/4)\pi)} \quad . \tag{46}$$

If $h\zeta$ is very large it can be represented in terms of circular cylindrical coordinates, where $h_1\zeta_1 = k_o\rho_1$ and $h_2\zeta_2 = k_o\rho_2$. In this case the total scattered field is given by,

$$E_Z^{(s)} = \sqrt{\frac{2}{\pi k_o \rho}} \quad e^{jk_o\rho} \; P(\varphi) \tag{47}$$

$$E_{total}^{s} = E_{z_1}^{(II)} + E_{z_2}^{(II)} = c(k\rho) P(\varphi),$$

$$c(k\rho) = \sqrt{2/\pi k\rho} \quad e^{jk\rho} e^{-j\pi/4} \quad , \tag{48}$$

$$P(\phi) = 2\pi \sum_{m=0}^{\infty} (-j)^m \left\{ B_m^{(1)} Se_m(h_1, \cos(\phi-\beta_1)) e^{-jkc\cos\phi} + B_m^{(2)} Se_m(h_2, \cos(\phi-\beta_2)) e^{jkc\cos\phi} \right\}. \tag{49}$$

The echo width is,

$$W(\phi) = \frac{4}{k} | P(\phi) |^2 \; . \tag{50}$$

III. APPROXIMATE SOLUTION

The approximate solution is based on a technique that was established by Karp and Russek [8]. Such a technique considers the scattered field from each coated strip as a sum of scattered field from that coated strip due to a plane wave incident plus the scattered field due to a line source of unknown intensity located at the center of the other coated strip. The factious line source accounts in an approximate sense for the multiple scattering between the two coated strips. In order to apply this technique one needs to obtain the far scattered field from one coated strip due to both a plane wave incident and a line source. In such a case, consider a plane wave given by equation (1), is incident on a coated strip located at x_i and $y = 0$, the coated strip is inclined by an angle β_i on the x-axis. The conducting strip has a width $2d_i$ and the coating dielectric constant is ε_i. The scattered field in the region outside the coated cylinder can be written as,

$$E_z^{(s)} = \sqrt{8\pi} \sum_{n=0}^{\infty} A_n^{(i)} He_n^{(1)}(h_i,\zeta_i) Se_n(h_i,\eta_i) \tag{51}$$

while the electric field inside the coating is,

$$E_z^{(I)} = \sqrt{8\pi} \sum_{n=0}^{\infty} B_n^{(i)} \left\{ Je_n(H_i,\zeta_i) - \frac{Je_n(H_i,1)}{Ne_n(H_i,1)} Ne_n(H_i,\zeta_i) \right\} Se_n(H_i,\eta_i). \tag{52}$$

Matching the boundary condition gives,

$$\sqrt{8\pi}\; e^{-jkx_i\cos\phi_o}\sum_{n=0}^{\infty} j^{-n}\left[\frac{1}{N_n^{(e)}(h_i)} Je_n(h_i,\zeta_{0i})\, Se_n(h_i,\eta_i)\, Se_n(h_i,\cos\phi_{0i}) + \frac{1}{N_n^{(o)}(h_i)}\, Jo_n(h_i,\zeta_{0i})\, So_n(h_i,\eta_i)\, So_n(h_i,\cos\phi_{0i})\right] \quad (53)$$
$$+\sqrt{8\pi}\sum_{n=0}^{\infty} A_n^{(i)} He_n^{(1)}(h_i,\zeta_{0i})\, Se_n(h_i,\eta_i) = \sqrt{8\pi}\sum_{n=0}^{\infty} B_n^{(i)}\left\{ Je_n(H_i,\zeta_{0i}) - \frac{Je_n(H_i,1)}{Ne_n(H_i,1)} Ne_n(H_i,\zeta_{0i})\right\} Se_n(H_i,\eta_i).$$

Multiplying both sides of equation (53) by $Se_m(H_1,\eta_i)$ and integrating over v_i from 0 to 2π, one obtains,

$$e^{-jkx_i\cos\phi_o}\sum_{n=0}^{\infty} j^{-n}\left\{\frac{1}{N_n^{(e)}(h_i)} Je_n(h_i,\zeta_{0i})\, Se_n(h_i,\cos\phi_{0i}) M_{n,m}(h_i,H_i)\right\} + \sum_{n=0}^{\infty} A_n^{(i)} He_n^{(1)}(h_i,\zeta_{0i})\, M_{n,m}(h_i,H_i) = B_m^{(i)}\left\{ Je_m(H_i,\zeta_{0i}) - \frac{Je_m(H_i,1)}{Ne_m(H_i,1)} Ne_m(H_i,\zeta_{0i})\right\} N_m^{(e)}(H_1). \quad (54)$$

Similarly matching the boundary condition corresponding to H_v, one can get,

$$e^{-jkx_i\cos\phi_o}\sum_{n=0}^{\infty} j^{-n}\left\{\frac{1}{N_n^{(e)}(h_i)} Je'_n(h_i,\zeta_{0i})\, Se_n(h_i,\cos\phi_{0i}) M_{n,m}(h_i,H_i)\right\} + \sum_{n=0}^{\infty} A_n^{(i)} He_n^{(1)\prime}(h_i,\zeta_{0i}) M_{n,m}(h_i,H_i) = \sqrt{\varepsilon_{r_i}}\, B_m^{(i)} \left\{ Je'_m(H_i,\zeta_{0i}) - \frac{Je_m(H_i,1)}{Ne_m(H_i,1)} Ne'_m(H_i,\zeta_{0i})\right\} N_m^{(e)}(H_1). \quad (55)$$

From equations (54) and (55), one obtains,

$$\sum_{n=0}^{\infty} A_n^{(i)}\left\{\frac{He_n^{(1)}(h_i,\zeta_{0i})}{X_m(H_i)} - \frac{He_n'^{(1)}(h_i,\zeta_{0i})}{X'_m(H_i)}\right\} M_{n,m}(h_i,H_i) = -e^{-jkx_i\cos\phi_o}\sum_{n=0}^{\infty} j^{-n}\frac{1}{N_n^{(e)}(h_i)}\left\{\frac{Je_n(h_i,\zeta_{0i})}{X_m(H_i)} - \frac{Je'_n(h_i,\zeta_{0i})}{X'_m(H_i)}\right\} Se_n(h_i,\cos\phi_{0i}) M_{n,m}(h_i,H_i) \quad (56)$$

where

$$X_m(H_i) = \left\{ Je_m(H_i,\zeta_{0i}) - \frac{Je_m(H_i,1)}{Ne_m(H_i,1)} Ne_m(H_i,\zeta_{0i})\right\} N_m^{(e)}(H_i),$$

$$X'_m(H_i) = \sqrt{\varepsilon_r}\,\{\ \}\left\{ Je'_m(H_i,\zeta_{0i}) - \frac{Je_m(H_i,1)}{Ne_m(H_i,1)} Ne'_m(H_i,\zeta_{0i})\right\} N_m^{(e)}(H_i).$$

Equation (56) can be written a matrix form as,

$$\left[F_{m,n}\right]\left[A_m\right] = \left[Y_m\right] \quad (57)$$

where

$$F_{m,n} = \left\{\frac{He_n^{(1)}(h_i,\zeta_{0i})}{X_m(H_i)} - \frac{He_n'^{(1)}(h_i,\zeta_{0i})}{X'_m(H_i)}\right\} M_{n,m}(h_i,H_i), \quad (58)$$

$$Y_m = -e^{-jkx_i\cos\phi_o}\sum_{n=0}^{\infty} j^{-n}\frac{1}{N_n^{(e)}(h_i)}\left\{\frac{Je_n(h_i,\zeta_{0i})}{X_m(H_i)} - \frac{Je'_n(h_i,\zeta_{0i})}{X'_m(H_i)}\right\} Se_n(h_i,\cos\phi_{0i}) M_{n,m}(h_i,H_i). \quad (59)$$

Once the coefficients are calculated the scattered electric field in the outer region is given by equation (51). Since $He_m^{(1)}(h,\zeta) = \frac{1}{\sqrt{h\zeta}} e^{j(h\zeta-(2m+1/4)\pi)}$ and for large $h\zeta$ it can be represented in terms of circular cylindrical coordinates, where $h_i\zeta_i = k_o\rho_i$. In this case the total scattered field is given by,

$$E_z^{(s)} = \sqrt{8\pi}\,\frac{e^{jk_i\rho_o}}{\sqrt{k_o\rho_i}}\sum_{n=0}^{\infty}(-j)^n A_n^{(i)} Se_n(h_i,\eta_i) = c(k_o\rho_i)\, f(h_i,x_i,\phi_i,\phi_{0i}) \tag{60}$$

where

$$f(h_i,x_i,\phi_i,\phi_{0i}) = 2\pi\sum_{m=0}^{\infty}(-j)^m A_m^{(i)} Se_m(h_i,\cos\phi_i). \tag{61}$$

Line source excitation

Consider a line source placed at $(x_{0k},0)$ with respect to the coordinates at the center of strip 1 ($i=1$) or at the center of strip 2 ($i=2$), then the *z*-component of the electric field due to such a line source can be expressed as,

$$E_z^{inc} = 4\left[\sum_{m=0}^{\infty}\frac{Se_m(h_i,\eta_{0k})}{N_m^{(e)}(h_i)}\; Se_m(h_1,\eta_i) \begin{cases} Je_m(h_i,\zeta_{0k})He_m^{(1)}(h_i,\zeta_i) & u>u_{0k} \\ Je_m(h_i,\zeta_i)He_m^{(1)}(h_i,\zeta_{0k}) & u<u_{0k}\end{cases} + \frac{So_m(h_i,\eta_{0k})}{N_m^{(o)}(h_i)}\; So_m(h_i,\eta_i) \begin{cases} Jo_m(h_i,\zeta_{0k})Ho_m^{(1)}(h_i,\zeta_i) & u>u_{0k} \\ Jo_m(h_i,\zeta_i)Ho_m^{(1)}(h_i,\zeta_{0k}) & u<u_{0k}\end{cases}\right] \tag{62}$$

where k takes the values 1 or 2.

$$\zeta_{0k} = \left[\frac{1}{2}\left(\frac{s_{0k}^2}{d_i^2}+1\right) + \left(\frac{1}{4}\left(\frac{s_{0k}^2}{d_i^2}+1\right)^2 - \frac{x_{0k}'^2}{d_i^2}\right)^{1/2}\right]^{1/2}, \tag{63}$$

$$\eta_{0k} = \frac{x'_{0k}}{\zeta_{0k} d_i}, \tag{64}$$

$$s_{0k} = \left((x_i - x_{0k})^2 + (y_i - y_{0k})^2\right)^{1/2}, \tag{65}$$

$$\psi_{0k} = \tan^{-1}\left[\frac{y_{0k}-y_i}{x_{0k}-x_i}\right] - \beta_i, \tag{66}$$

$$x'_{0k} = s_{0k}\cos\psi_{0k} \quad y'_{0k} = s_{0k}\sin\psi_{0k}. \tag{67}$$

The scattered field in the region outside the coated cylinder can be written as,

$$E_z^{(s)} = 4\sum_{n=0}^{\infty} C_n^{(i)} He_n^{(1)}(h_i,\zeta_i)\; Se_n(h_i,\eta_i). \tag{68}$$

While the electric field inside the coating is given by,

$$E_z^{(I)} = 4\sum_{n=0}^{\infty} D_n^{(i)}\left\{ Je_n(H_i,\zeta_i) - \frac{Je_n(H_i,1)}{Ne_n(H_i,1)} Ne_n(H_i,\zeta_i)\right\} Se_n(H_i,\eta_i). \tag{69}$$

Matching the boundary condition gives,

$$\begin{aligned}\sum_{n=0}^{\infty}\frac{Se_n(h_i,\eta_{0k})}{N_n^{(e)}(h_i)}\; & Se_n(h_1,\eta_i) Je_n(h_i,\zeta_{0i}) He_n^{(1)}(h_i,\zeta_{0k}) + \frac{So_n(h_i,\eta_{0k})}{N_n^{(o)}(h_i)} So_n(h_i,\eta_i) Jo_n(h_i,\zeta_{0i}) Ho_n^{(1)}(h_i,\zeta_{0k}) \\ &+ \sum_{n=0}^{\infty} C_n^{(i)} He_n^{(1)}(h_i,\zeta_{0i})\; Se_n(h_i,\eta_i) \\ &= \sum_{n=0}^{\infty} D_n^{(i)}\left\{ Je_n(H_i,\zeta_{0i}) - \frac{Je_n(H_i,1)}{Ne_n(H_i,1)} Ne_n(H_i,\zeta_{0i})\right\} Se_n(H_i,\eta_i).\end{aligned} \tag{70}$$

Multiplying both sides of equation (70) by $Se_m(H_1,\eta_i)$ and integrating over v_i from 0 to 2π, one obtains,

$$\begin{aligned}\sum_{n=0}^{\infty}\frac{He_n^{(1)}(h_i,\zeta_{0k})}{N_n^{(e)}(h_i)} & Je_n(h_i,\zeta_{0i})\; Se_n(h_i,\eta_{0k}) M_{n,m}(h_i,H_i) + \sum_{n=0}^{\infty} C_n^{(i)}\; He_n^{(1)}(h_i,\zeta_{0i}) M_{n,m}(h_i,H_i) \\ &= D_m^{(i)}\left\{ Je_m(H_i,\zeta_{0i}) - \frac{Je_m(H_i,1)}{Ne_m(H_i,1)} Ne_m(H_i,\zeta_{0i})\right\} N_m^{(e)}(H_1).\end{aligned} \tag{71}$$

Similarly matching the boundary condition corresponding to H_v, one can get,

$$\sum_{n=0}^{\infty} \frac{He_n^{(1)}(h_i,\zeta_{0k})}{N_n^{(e)}(h_i)} Je'_n(h_i,\zeta_{0i})\, Se_n(h_i,\eta_{0k})$$
$$M_{n,m}(h_i,H_i) + \sum_{n=0}^{\infty} C_n^{(i)} He_n'^{(1)}(h_i,\zeta_{0i}) \tag{72}$$
$$M_{n,m}(h_i,H_i) = D_m^{(i)} \left\{ Je'_m(H_i,\zeta_{0i}) - \frac{Je_m(H_i,1)}{Ne_m(H_i,1)} Ne'_m(H_i,\zeta_{0i}) \right\} N_m^{(e)}(H_1).$$

Solving equations (71) and (72), one gets,

$$\sum_{n=0}^{\infty} C_n^{(i)} \left\{ \frac{He_n^{(1)}(h_i,\zeta_{0i})}{X_m(H_i)} - \frac{He_n'^{(1)}(h_i,\zeta_{0i})}{X'_m(H_i)} \right\}$$
$$M_{n,m}(h_i,H_i) = -\sum_{n=0}^{\infty} \frac{He_n^{(1)}(h_i,\zeta_{0k})}{N_n^{(e)}(h_i)} \tag{73}$$
$$\left\{ \frac{Je_n(h_i,\zeta_{0i})}{X_m(H_i)} - \frac{Je'_n(h_i,\zeta_{0i})}{X'_m(H_i)} \right\}$$
$$Se_n(h_i,\eta_{0k}) M_{n,m}(h_i,H_i).$$

Equation (73) can be written in a matrix form similar to equation (57), where,

$$Y_m = -\sum_{n=0}^{\infty} \frac{He_n^{(1)}(h_i,\zeta_{0k})}{N_n^{(e)}(h_i)}$$
$$\left\{ \frac{Je_n(h_i,\zeta_{0i})}{X_m(H_i)} - \frac{Je'_n(h_i,\zeta_{0i})}{X'_m(H_i)} \right\} \tag{74}$$
$$Se_n(h_i,\eta_{0k}) M_{n,m}(h_i,H_i).$$

Once the coefficients are calculated the scattered electric field in the outer region is given by equation (51).

$$E_z^{(s)} = \sqrt{8\pi}\, \frac{e^{jk_i\rho_i}}{\sqrt{k_o\rho_i}} \sum_{n=0}^{\infty} (-j)^n C_n^{(i)} Se_n(h_i,\eta_i) \tag{75}$$
$$= c(k_o\rho_i) g(h_i,\phi_i,\zeta_{ok},\eta_{ok}),$$

where

$$g(h_i,\phi_i,\zeta_{0k},\eta_{0k}) = \sqrt{8\pi} \sum_{m=0}^{\infty} (-j)^m C_m^{(i)} \tag{76}$$
$$Se_m(h_i,\cos\phi_i).$$

Now consider the problem of the two strips shown in Fig. 1. Assuming a fictitious line source C_1 at the center of the first strip and another line source C_2 at the center of the second strip the far scattered field from the first strip is given by,

$$E_1^s = c(k_o\rho_1)\big[f(h_1,x_1,\phi_1,\phi_{01}) + C_2 g(h_1,\phi_1,\zeta_{01},\eta_{01}) \big]. \tag{77}$$

Similarly, the far scattered field due to the second strip is given by,

$$E_2^s = c(k_o\rho_2)\big[f(h_2,x_2,\phi_2,\phi_{02}) + C_1 g(h_2,\phi_2,\zeta_{02},\eta_{02}) \big]. \tag{78}$$

The partial scattered field from the first strip due to the scattered field from the second strip can be determined by considering the scattered field from the second strip as the intensity of a line source at $\phi_2 = -\beta_2$ times the well-known response [8], i.e.,

$$E_{12}^s = c(k_o\rho_1)\big[f(h_2,x_2,-\beta_2,\phi_{02}) + C_1 g(h_2,-\beta_2,\zeta_{01},\eta_{01}) \big] g(h_1,\phi_1,\zeta_{02},\eta_{02}). \tag{79}$$

On the other hand this partial scattered field is given by,

$$E_{12}^s = C_2\, c(k_o\rho_1)\, g(h_1,\phi_1,\zeta_{02},\eta_{02}) \,. \tag{80}$$

Using equivalence between equations (79) and (80), one obtains,

$$f(h_2,x_2,-\beta_2,\phi_{02}) + C_1 g(h_2,-\beta_2,\zeta_{01},\eta_{01}) = C_2. \tag{81}$$

In a similar way one can obtain,

$$f(h_1,x_1,\pi-\beta_1,\phi_{01}) + C_2\, g(h_1,\pi-\beta_1,\zeta_{02},\eta_{02}) = C_1. \tag{82}$$

Solving equations (81) and (82), one can get,

$$C_1 = \frac{Nu_1}{Du_1} \tag{83}$$

where

$$Nu_1 = f(h_1,x_1,\pi-\beta_1,\phi_{01}) + f(h_2,x_2,-\beta_2,\phi_{02}) g(h_1,\pi-\beta_1,\zeta_{02},\eta_{02})$$
$$Du_1 = 1 - g(h_1,\pi-\beta_1,\zeta_{02},\eta_{02})\, g(h_2,-\beta_2,\zeta_{01},\eta_{01})$$
$$C_2 = \frac{Nu_2}{Du_2}. \tag{84}$$

where

$$Nu_2 = f(h_2, x_2, -\beta_2, \phi_{02}) + f(h_1, x_1, \pi - \beta_1, \phi_{01}) g(h_2, -\beta_2, \zeta_{01}, \eta_{01})$$

$$Du_2 = 1 - g(h_1, \pi - \beta_1, \zeta_{02}, \eta_{02}) g(h_2, -\beta_2, \zeta_{01}, \eta_{01}).$$

Once C_1 and C_2 are known, one can determine the z-component of the total scattered field from the two strips, i. e.,

$$E_z^s = c(k_o \rho) P(\phi) \tag{85}$$

$$p(\phi) = e^{-jk_o c \cos\phi} \left[f(h_1, x_1, \phi - \beta_1, \phi_{01}) + C_2 g(h_1, \phi - \beta_1, \zeta_{02}, \eta_{02}) \right] + e^{jk_o c \cos\phi} \left[f(h_2, x_2, \phi - \beta_2, \phi_{02}) + C_1 g(h_2, \phi - \beta_2, \zeta_{01}, \eta_{01}) \right] \tag{86}$$

The plane wave scattering properties of a two-dimensional body of infinite length are conveniently described in terms of the echo width equation (50).

IV. RESULTS AND DISCUSSION

In the following results the first method is implemented in all calculations, however every presented result is checked using the approximate method and a good agreement is found. Also in all figures the case of no dielectric coating $\varepsilon_r = 1$ is introduced in order to illustrate the effect of the dielectric coating on the scattering echo width. The first case is of a plane wave normally incident on two dielectric coating conducting strips with $\beta_1 = \beta_2 = 0^o$. As can be seen from Fig. 2 the forward and backscattering echo widths are increasing with thin coating of dielectric and decreases as the dielectric coating increases.

For the second case same coated strips are considered except in the angles $\beta_1 = \beta_2 = 90^o$ is the only change. The scattering echo width was calculated again for different thickness which shows that it is increasing in forward and backward directions for thin dielectric and decreasing for thick one. The third example has different geometrical parameter as shown in Fig. 4. The angle of incidence was taken as $\phi_o = 60^o$. Again this example illustrates the effect of the coating thickness in the back and forward scattering echo width. As one can see from Fig. 4, the scattering echo width is increasing for thin dielectric layer and decreases as it gets thick. The forth example shows the echo width pattern for angles $\beta_1 = 45^o$ and $\beta_2 = -45^o$ while $\phi_o = 90^o$.

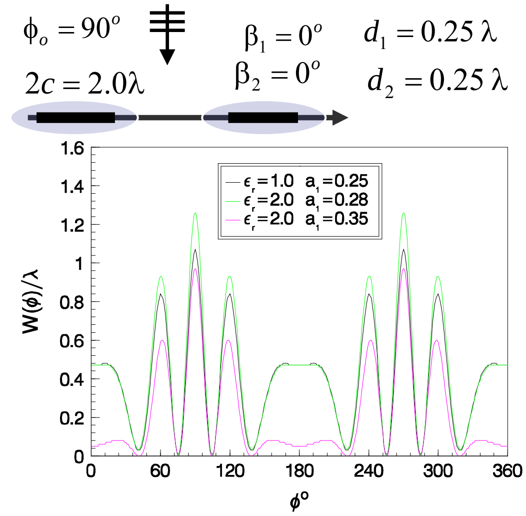

Fig. 2. Echo width pattern for different coating thickness.

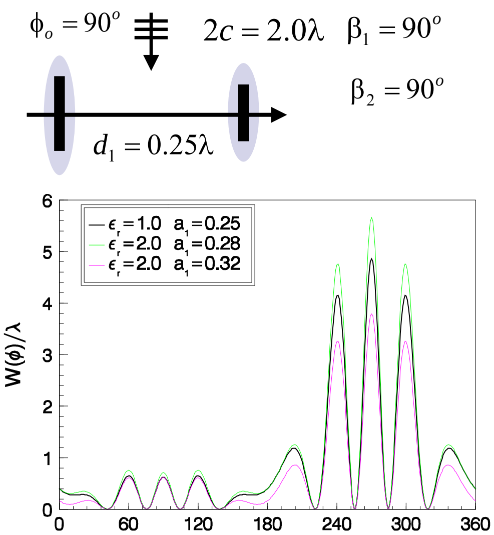

Fig. 3. Echo width pattern for different coating thickness.

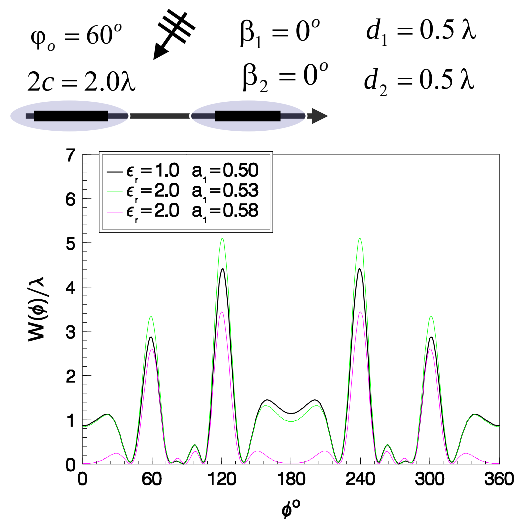

Fig. 4. Echo width pattern for different coating thickness.

As shown in Fig. 5 that same behavior can be concluded for this case. The fifth case is similar to the forth except that $\beta_1 = \beta_2 = 45^o$. Figure 6 illustrates the echo width pattern for fifth example, where the thin dielectric coating produces higher forward and backward echo width while the thick one produces less echo width values. In the sixth case different geometrical parameters were considered where larger strips are considered and $\beta_1 = 0^o$, $\beta_2 = 90^o$ while $\phi_o = 60^o$.The echo width pattern corresponding to this case is plotted in Fig. 7.

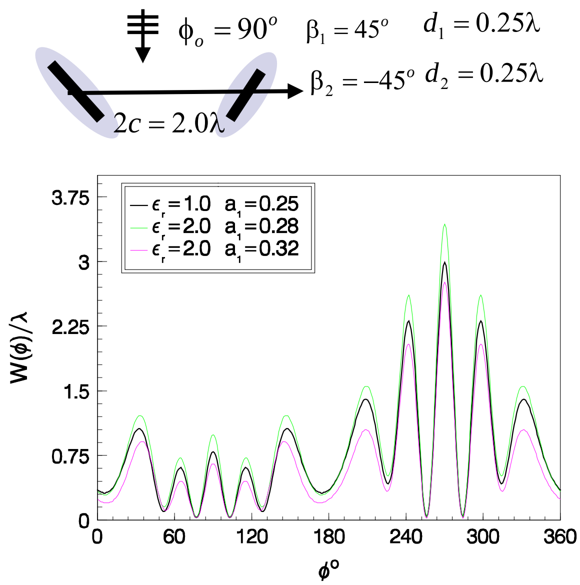

Fig. 5. Echo width pattern for different coating thickness.

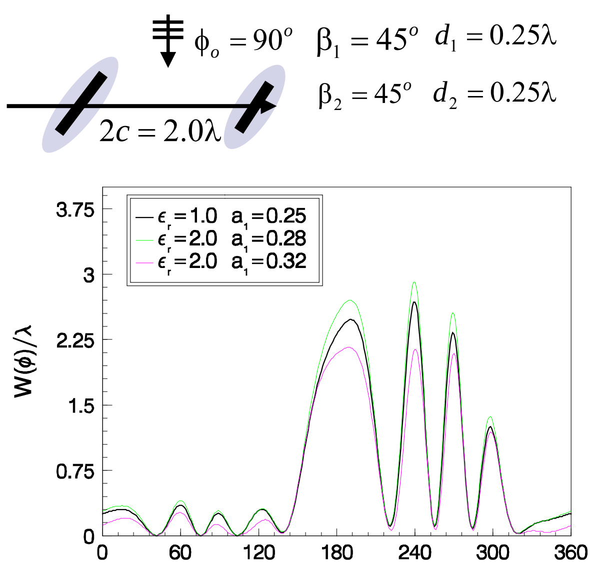

Fig. 6. Echo width pattern for different coating thickness.

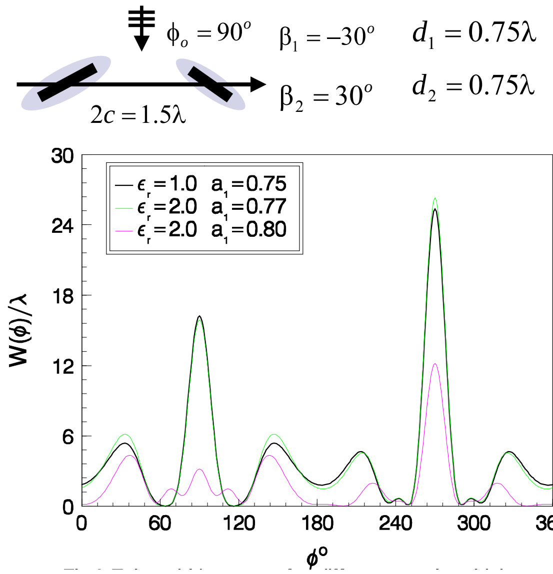

Fig. 7. Echo width pattern for different coating thickness.

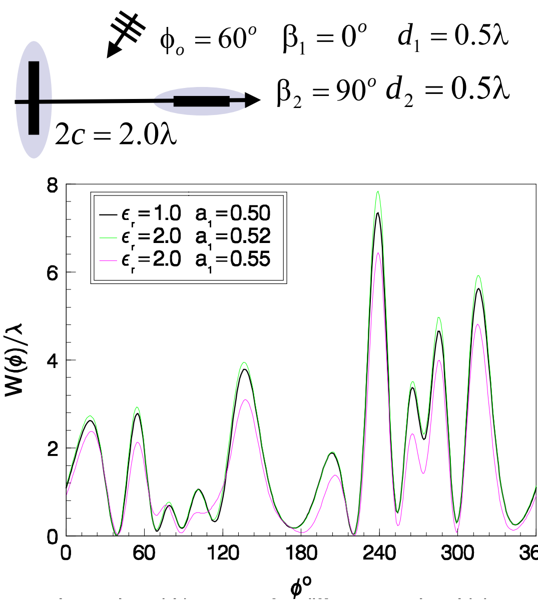

Fig. 8. Echo width pattern for different coating thickness.

Again the same behavior is noticed as previous cases. In the seventh example a relatively large strips are considered and the separation between them is also decreased relative to previous cases. Again the echo width pattern showed a very slight increase for very thin coating and then it gets lower as the coating thickness increases. In the last example new parameters were introduced in order to show the effect of the dielectric permittivity on the scattering echo width. As one can see in Fig. 9, echo width pattern is decreasing with increasing the dielectric permittivity.

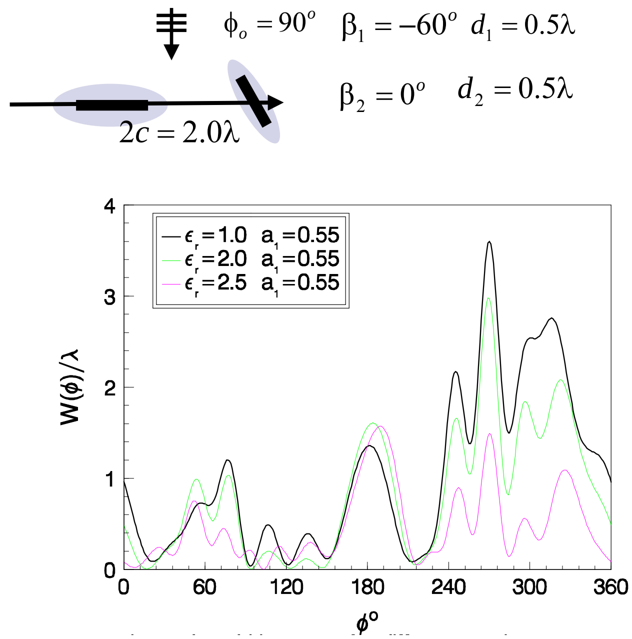

Fig. 9. Echo width pattern for different coating permittivity.

V. CONCLUSIONS

Scattering of an electromagnetic wave by two dielectric coated conducing strips is achieved to study the effect of the coating on the echo width. It is found that very thin dielectric coating increases the scattering echo width in forward and back directions, and as the thickness increases the forward and backscattered echo width decreases. It is also found that for a constant thickness the scattering echo width pattern decreases with the increasing of dielectric permittivity of the coating.

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REFERENCE

[1] J. J. Bowman, T. Senior, and P. Uslenghi, *Electromagnetic and Acoustic Scattering by Simple Shapes.* Amsterdam: North-Holland Publishing Company, 1969.

[2] P. M. Morse and P. J. Rubensten, "The diffraction of waves by ribbons and by slits," *Physics Review*, vol. 54, pp. 895-898, 1938.

[3] R. I. Primich, "Some electromagnetic transmission and reflection properties of a strip grating," *I. R. E. Trans. on Antennas and Propagation*, vol. 5, pp. 176-182, 1957.

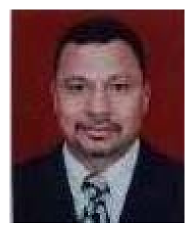
[4] R. Holland and V. Cable, "Mathieu function and their applications to scattering by a coated strip," *IEEE on Electromagnetic Compatibility*, vol. 34, pp. 9-16, Feb. 1992.

[5] H. A. Ragheb and M. Hamid, "Scattering by two conducting strips with parallel edges," *Canadian J. of Phys*, vol. 66, pp. 376-383, 1987.

[6] A. Sebak, "Electromagnetic scattering by two elliptic cylinders," *IEEE antyennasa and Propag.*, vol. 42, no. 11, pp. 1521-1527, 1994.

[7] B. N. Khatir and A. Sebak, "Transverse electric wave scattering by parallel metamaterial coated elliptic cylinders," *EMTS 2007 - International URSI Commission B - Electromagnetic Theory Symposium, Ottawa, ON, Canada*, July 26-28, 2007.

[8] S. N. Karp and A. Russek, "Scattering by s wide slit," *J. of Applied Physics*, vol. 27, pp. 886-889, 1952.



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