# *SOME SOURCES OF ERROR IN CEM MODELING AND SIMULATION*


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## Abstract

The modeling and solution of large-scale problems in computational electromagnetics (CEM) requires the application of the *right tool for the right job* in order to minimize the potential for error generation and propagation during each step of the process. The subtleties of this issue are associated with knowing where sources of error can arise, how to quantify them, and what methods can be used to control errors. Sources of error can be categorized as *procedural*, *model-limited*, *technique-limited*, *problem dependent*, *numerical*, and *interpretive*. These by no means represent a complete taxonomy of error sources in CEM, but provide a means of better understanding error budgets and how these may be controlled. This article provides a brief overview of some of the sources of error to be mindful of and the potential pitfalls that may lend to computational uncertainty.


## INTRODUCTION

Generally, CEM techniques can be subdivided into two categories: frequency domain and time-domain. These can be further expressed as either integral or differential formulations of Maxwell's equations. Solutions to these equations fundamentally involve a series of partial differential equations that are subject to boundary constraints, except for some variations that are particular to the physics of a given problem. For instance, a CEM technique can be used to solve the Laplace equation that describes the potential distribution of a closed boundary. Also, a CEM technique can be applied to solving the Helmholtz wave equation, which arises in many electromagnetic radiation problems in open space. Clearly, there are different techniques and formulations for different problem solving applications.

Integral equation methods traditionally involve a dense matrix system, in which tens of millions of unknowns can now be solved with today's high performance computers [1]. Differential equation methods involve a sparse matrix system, in which problems with billions of unknowns have been solved [1]. In some problems, the unknowns are volumetrically distributed, whereas in others, they are distributed over a surface.

In volumetric methods, grid dispersion error has been shown to be a significant issue [1]. As the authors in [1] have shown, in the case of a differential equation solver, the field is propagated from point to point via a numerical grid, giving rise to errors. In volume integral equations, the Green's function in an inhomogeneous region usually can have the incorrect phase velocity to propagate the field from point to point, which is also a source of grid dispersion error. In surface integral equations, an exact closed-form Green's function is used to propagate the field through space. Hence, as reference [1] reports, grid dispersion error is greatly mitigated except for surface waves that propagate on the numerical grid on the surface of the scatterer.

The authors in [1] also cite the effects of numerical noise due to round off errors in ultra large-scale computational electromagnetics. For CEM problems involving hundreds of wavelengths, the solution is particularly sensitive to the phase velocity error. As the authors point out, an error in the phase velocity can generate numerical noise that is intolerable if the goal is to achieve high accuracy computations. It becomes incumbent upon the analyst to find ways of validating computed results using high-quality measurements (in which the measurement errors and uncertainty are also reasonably well quantified), or sometimes even comparing computed results to theoretical closed-form solutions such as the Mie series solution in the case of electromagnetic scattering from a sphere, to check the integrity of the calculations [1]. Once again, the goal is to eliminate as much uncertainty in the computational process as is possible. In other words, it's all about the accuracy.

Indeed, there have been many such studies performed in recent years to identify and quantify the sources of error in CEM modeling in an attempt to find effective ways of countering their effect. Clearly, the main goal of research into controllable error methods is to increase the confidence factor in CEM modeling and simulation since we are relying more and more on simulation-based acquisition to procure new systems. This involves the development of new standards and recommended practices for CEM [2] which are in process, as well as the application of novel mathematical algorithms and numerical methods to assure accuracy as well as computational efficiency. Although much progress has been made in implementing new computational methods such as the multi-level fast multipole algorithm (MLFMA), there is still much work ahead of us in terms of further stabilizing error budgets for state-of-the-art CEM techniques and codes.

Nonetheless, the CEM community remains steadfast in its pursuit of developing new, highly accurate fast solvers. Today's methods exploit variations on the Gaussian elimination method, matrix partitioning and pruning schemes, parallel computing, and other potentially effective methods. This remains an evolving branch of electromagnetics that continues to deal with the dichotomy of ensuring efficient computing without forsaking accuracy. Unfortunately, as the problem size becomes larger, numerical instabilities and computational errors begin to emerge and cannot be ignored—even with today's sophisticated fast solvers. As mentioned above, this is the numerical *noise* due to inherent approximations, limited precision, and round-off error. This numerical noise is proportional to the number of floating point operations that is performed. This noise can be particularly devastating in ultra large scale computing as well as problems that are ill conditioned [1].

In addition to numerical noise error due to precision and round off, other sources of error can be attributed to the geometry, applied physics and mathematical algorithms utilized in the solvers. These and other sources of error are covered next within the context of the taxonomy mentioned earlier.

## CEM Error Budgets

Sources of error can be categorized as *procedural*, *model-limited*, *technique-limited*, *problem dependent*, *numerical*, and *interpretive*. These are described below.

First, consider the following fact: all CEM techniques are not necessarily alike even though they are all fundamentally cast from Maxwell's equations. Recall the earlier discussion on the integral and differential formulations of Maxwell's equations, frequency and time domain methods, and Laplace and Helmholtz equations. Specific implementations of electromagnetic theory and CEM techniques are usually aimed at solving certain problems in a certain way for a certain set of boundary conditions and for a certain range of electromagnetic problems. For instance, some techniques are more apt to be used in analyzing exterior radiation and scattering problems, whereas other techniques are better suited to analyzing interior cavity coupling problems. Therefore, even though CEM techniques are based on Maxwell's equations, it is often difficult if not impossible to interchange them for practical problem solving applications or even to compare them in any valid and consistent way.

The "accuracy" of any one CEM technique clearly depends on a number of inter-related factors. These are: (i) the applied physics i.e., how the theoretical equations are cast and what method is used to "map" Maxwell's equations for an infinite space to a finite geometrical space (e.g., boundary element or moment method integral form, finite difference time domain form, finite element method, transmission line model, etc.); (ii) inherent limitations associated with the geometry modeling approach and the procedures followed in building a CEM model; (iii) the numerical solver method used (e.g., full-wave solution, full or banded matrix decomposition, non-matrix solutions, etc.); (iv) the type of problem to be solved (EMI/C, scattering cross section, antenna radiation, printed circuit board trace coupling, etc.); and (v) the methods used to interpret results for computed observables (total and scattered fields, normalized radiation patterns, surface currents, charge densities, etc.).

Hence, the underlying physics formalism, model building blocks (primitives such as canonical surfaces, wires, patches, facets, etc.) and the procedures used to construct models as well as the solution method all conspire to affect the convergence, accuracy, and overall validity of the computed results. The analysis frequency (mesh discretization) and time steps, mathematical basis functions, computer precision, and approximations employed further compound the problem.

***Model-Limited Errors***. This refers to the errors that arise because of limitations associated with the geometrical elements that are used to construct computational models. Sometimes the modeling elements are too gross or simplistic to faithfully represent the geometry at the frequency of interest. For instance, the use of canonical objects in high-frequency ray tracing simulations offers a computationally efficient solution, but does not accurately represent the actual geometry, which in turn leads to approximate solutions. To overcome such difficulties, techniques have been developed to adapt detailed computer-aided design (CAD) models directly in order to derive high-fidelity CEM models. However, this results in a new source of error in that the CAD models themselves may contain subtle flaws that are not readily detected and which can result in errors downstream of the modeling and simulation process.

Research of late has led to the use of curved elemental facets and higher order basis functions, which result in more accurate geometry descriptions and more uniform current distributions on the surfaces of these elements. However, with the exception of certain high-fidelity CEM techniques used for radar cross section simulations, certain limitations still exist with regard to consistently handling the following special cases, which can significantly contribute to the model-limited error budget:

- Multilayer materials, interfaces and discontinuities involving dielectrics
- Open vs. closed boundaries or regions including incomplete geometry definitions, voids, and overlaps (geometrical intersection, union, and subtraction)
- Presence of long, skinny facets
- Modeling doubly-curved surfaces
- Adaptive, non-uniform mesh discretization
- Staircasing at edges and over curved surfaces.

***Procedural Errors***. This refers to the step-by-step approach used in generating and analyzing a CEM model. How one goes about modeling and analyzing a real-world problem is dependent on the type of problem to be solved and what electromagnetic phenomena and observables are of interest among other considerations.

For example, consider the problem of assembling a computational model, and integrating components and their individual electromagnetic contributions to compute a *total budget solution*—not to be confused with *error budget*. This problem is one of resolving a complex system into its parts, analyzing the electromagnetic interactions or relative contributions, and then integrating results in order to arrive at an accurate system analysis—a procedure called *combinatorial* modeling. First, this is an approximate idea. Linear superposition does not work. By solving a problem in components, finding its component radar cross section, for instance, and later adding up the contributions, the total budget solution found this way is a lower bound to the true solution. The difference between the budget solution and the true solution is a function of how strongly the parts interact. The stronger they interact, the larger the difference between the budget solution compared to the true total solution.

For example, five walls of a cavity are not strong scatterers individually, but when the five walls cooperate with each other to form a cavity, they can give rise to resonance scattering, which is much stronger than the scattering from the individual walls. So, a possible approach is to break the system up into weakly interacting components. Then the budget solution is not too different from the true solution. This method can be modified to suit the requirements of subdividing a complex system into weakly interacting components.

This is just one of many illustrations of the importance of defining the step-by-step procedures for modeling and analyzing a problem in order to reduce errors and ensure accuracy in the computed results. An ill-posed problem can result in computational instabilities and numerical inaccuracies, for example, when improper sampling is used to try and capture electromagnetic phenomena at resonance or about singularities or at near field caustic points.

Yet other procedural errors again point back to how the basic geometry and CEM model were built—as in the case of canonical modeling objects that are used to approximate a physical system for high-frequency ray tracing computations. Here we can see the relationship between procedural errors and model-limited errors.

The lesson to be learned is this: building and analyzing a CEM model without some a priori understanding of the type of problem to be solved, the basic physics of the problem, and what observables are most appropriate based on the boundary conditions, frequency, and so forth, will likely lead to errors and lend to the uncertainty. In other words—one needs to properly define the problem and the desired "metrics."

***Technique-Limited Errors***. This category pertains to the approximations and potential errors that are introduced when Maxwell's equation are constrained to a particular subset of boundary conditions and modeling problems (also referred to by some as *quadrature* error), expressed either in differential or integral form. As a result, the applied physics can have certain limitations. Examples of the limitations in the physics include:

- Lack of edge and surface traveling wave models
- Approximations to knife-edge, wedge, tip and point diffraction models
- Phase error (loss) over large distances or dimensions at very high frequencies
- High-frequency asymptotic ray tracing approximations (*ansatz* error)
- Lack of a robust set of current expansion or basis functions
- Inability to handle material discontinuities at interfaces (multilayer, anisotropic or inhomogeneous materials, frequency selective surfaces or FSS, etc.)
- Approximated near-to-far-field extrapolation techniques
- Shadow boundaries, creeping wave and related dispersion losses
- Consistent models and techniques for computing rapidly-varying current or field levels in the vicinity of singularities or caustics
- Radiator feed modeling, FSS and mutual coupling for multi-region problems.

Some of the subtle issues here pertain to the applied mathematical algorithms and methods for truncating infinite series and controlling the number of second and higher order electromagnetic interactions (i.e., bounces) to be considered. Government and academic institutions are presently conducting research to find ways of overcoming these and other limitations in the applied physics formalisms and mathematical algorithms.

***Problem-Dependent Errors***. Understanding the physical problem to be solved goes a long way in reducing the potential for errors. For example, one would not necessarily want to use a full-matrix decomposition moment method technique to solve a simple antenna coupling problem at 10 GHz (recall that the effects of numerical noise become more pronounced due to round off and phase velocity errors in ultra large scale computational electromagnetics!). However, for scattering cross section problems at 10 GHz, moment method based techniques in conjunction with the use of fast solvers are desirable in order to obtain highly accurate results. Similarly, a transmission line modeling (TLM) technique may be quite suitable to analyzing an internal cable coupling problem for a closed or bounded cavity, but may not be appropriate for calculating antenna radiation effects for exterior problems involving large, complex structures.

In this case, it is imperative that one defines the problem to be solved. The most suitable physics formalism(s) and solution method(s) can then be determined with a greater degree of confidence. Generally, at a very high level, problems can be classified as one of he following types: EMI/C, scattering cross section, antenna radiation, signal integrity, shielded enclosure problems, and materials problems. These categories can be further subdivided as necessary. EMI/C, for instance, can apply to printed circuit boards or devices as well as to large-scale systems. Remember the rule—*use the right tool for the right job!*

***Numerical Errors***. Solution error is closely tied to the category of technique-limited error in that the physics and the numerical solvers work together to provide a total budget solution. However, in ultra large scale computational electromagnetics, a variety of errors can arise. Solution errors are attributed to the solver method employed e.g., banded matrix iteration, full wave or lower-upper decomposition (LUD) of matrices, exploitation of block Toeplitz matrices, and additional forms of partitioning in conjunction with the application of the Green's function and other methods to arrive at a total solution.

In [1], the authors describe errors arising from an inconsistent Green's function for an MLFMA based technique in which there was a 4$^{th}$-digit error in the wave number as a result of the speed of light constant, c, which was defined in two different ways. In this case, the following equations apply.

$$\mathrm{f}(\mathbf{r}) = \int_{\Omega} \mathrm{g}(\mathbf{r}, \mathbf{r'})_S (\mathbf{r'})\, \mathrm{d}\mathbf{r'}$$

and where

$$\mathbf{y} = \mathrm{A}\mathbf{x}.$$

Here, the MLFMA used the exact value (299,792,458 m/s) whereas the triangular mesh algorithms used the approximate value ($3\mathrm{x}10^8$ m/s). Therefore, extinction theorems will not apply with an inconsistent Green's function. Hence, surface currents may not be correctly calculated in such a way to cancel the internal fields resulting in residual *noise*. This noise and error propagation can be enhanced with large-scale problems and dense matrix systems [1].

The enhancement of numerical noise and round off error propagation stems from the application of the Green's function and the process of solving for the large number of current or field unknowns (N) for a dense matrix system. A**x** typically requires $N^2$ operations, whereas MLFMA can perform the action in O(N) or O(NlogN) operations for densely packed sources and sparsely packed sources, respectively [1]. Therefore, one could conclude that a fewer number of operations would result in less error propagation. However, there are actually various numerical noise contributions at play in solving for the unknowns. These are product noise, subtraction noise, Gaussian elimination noise, matrix error noise (quadrature error in evaluating matrix coefficient terms), as well as phase velocity error where the phase velocity is incorrectly defined, which in turn can give rise to errors in the exponential function calculations [1]. This is related to the process of solving an integral equation which formulates a cooperative behavior among the current elements so as to produce a field that exactly cancels the incident field within a metallic scatterer, for instance. The authors in [1] point out that this cooperative behavior requires that all the current elements "talk" to each other on the same "wavelength" or the same phase velocity. Hence, any inconsistency in the phase velocity will not allow the current elements to cooperate effectively with each other.

Next, the sources of matrix error can be traced to the problem of (i) geometrical modeling error; (ii) integral equation discretization (including basis function expansion error and quadrature error); (iii) matrix equation solution error (using iterative solvers, LUD, and banded matrices); (iv) matrix vector product error due to matrix equation factorization error (in the case of fast algorithms) and pre-corrected FFT errors; and (v) associated round off and numerical precision errors [1, 3].

***Interpretive Errors***. The human's own ability to interpret the computed observables can invoke a *Heisenberg uncertainty* principle of sorts. The process of modeling and analyzing problems that reveal singularities, caustics, and harmonic resonance behavior as well as situations where abrupt discontinuities of currents or field point mismatches exist at/between multiple region (multilayer material) interfaces, can call into question the suitability of the technique and/or the solution method let alone the accuracy of the computed results. Oftentimes, there is a balance of objective and subjective reasoning at play at this stage of the modeling and simulation task. The proof comes in validating the results against ground truth or measurement benchmarks.

Research has been conducted to establish a standardized method of interpreting computed data results in a highly objective and consistent way using novel technique comparison and Feature Selective Validation (FSV) methods that are design to reduce uncertainty [4, 5].

## Controlling Error

Some possible ways to enhance accuracy and control error in the CEM modeling and simulation process include:

- Use of high-fidelity geometrical models and automated CAD healing capabilities

- Incorporating additional physics models to more accurately handle special cases such as surface and edge traveling waves
- Using higher-order surface modeling elements
- Applying robust current expansion functions (e.g., RWG type)
- Applying new hybrid techniques to accurately model multiple regions (enforcing current continuity and field point matching at interfaces)
- Novel exploitation of symmetry and bodies of revolution (BOR) techniques
- Using "adaptive" optimization algorithms for accuracy and computational efficiency
- Utilizing novel partitioning and decomposition of submatrices
- New and effective ways of sifting out, ranking, and suppressing "off diagonal" noise error sources
- Applying ensemble parameter reasoning (using AI/expert systems to automatically build valid CEM models)
- Applying novel smoothing functions to control staircasing error
- Using extended precision computing
- Exploiting matrix-free fast solvers and HPCs to handle large problems across a broad frequency range
- Developing component-level techniques that can be integrated and extrapolated to provide accurate system-level (total budget) solutions.

Some errors can be easily removed by extending bit precision. Other errors can only be removed by employing better algorithms and methods.

## SUMMARY

This article highlighted the various sources of error in the overall CEM modeling and simulation process. An overview of some of the sources of error and the potential pitfalls that may lend to computational uncertainty was provided. This is applicable to a broad range of problems ranging from the modeling of printed circuit board radiated and conducted emissions/immunity to analyzing large, complex system electromagnetic effects. Concerns have been raised regarding the lack of well-defined methodologies to achieve CEM technique validations within a consistent level of accuracy. This points to the need to identify and quantify the sources errors and to employ controllable error schemes when and where feasible.

The modeling and solution of large-scale problems in CEM requires the application of the *right tool for the right job* in order to minimize the potential for error generation and propagation. This starts by knowing where sources of error can arise, how to quantify them, and what methods can be used to control errors. Sources of error were generally categorized as *procedural*, *model-limited*, *technique-limited*, *problem dependent*, *numerical*, and *interpretive*. These are by no means complete and inclusive, but these provide insights into better understanding error budgets and how these may be controlled.

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