WAVE CONCEPT ITERATIVE PROCESS MERGES WITH MODAL FAST FOURIER TRANSFORMATION TO ANALYZE MICROSTRIP FILTERS.

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ABSTRACT. *A novel iterative method based on the concept of waves is reported for use in the field theory, computer aided design and optimization of high frequency integrated circuits. It consists of a recursive relationship between a given source and reflected waves from the discontinuity plane which is divided into cells. A high computational speed has been achieved by using Modal Fast Fourier Transformation (MFFT). The theory as well as its procedure implementation is described. Numerical results are successfully compared with published data.*

1 INTRODUCTION

With the increasing demand of wireless services, more and more features have to be implemented in smaller devices part. As a matter of fact, the integration for the front end and antenna becomes more and more complex. Therefore, it is not anymore for the designer of MMIC or passive circuits a problem of single design, however a more global approach has to be taken. It is obvious that all the constitutive parts of the systems are interacting with each other and the development of fast and efficient software tools that can accurately predict the electrical behavior of the components being investigated is of primary importance.

A significant amount of research has been devoted to these subjects, and a variety of special purpose methods have been developed [3]. Several numerical methods can be used such as the moment method [4], the Finite Element Method (FEM) [5-6] and the Finite Difference Time Domain Method (FDTD) [2]. However, major limitations for these ones are the necessity of having large computer storage as the problem size increases in terms of wavelength, as well as time consumption for the mathematical treatment

The purpose of this paper is to present an efficient iterative technique which can overcome the limitations of the above methods and is suitable to analyze a general structure. The basic theory and its implementation in a global structure solving procedure will first be explained. To demonstrate the validity of this approach a microstrip filter characterization will then be shown. Based on the treated examples, comparisons in terms of computational performances are given in the last part of the paper.

2 BASIC PRINCIPLE

To demonstrate the application of the general definition of waves, we consider a planar circuit with arbitrary shape printed on a dielectric interface as shown in Fig.1.

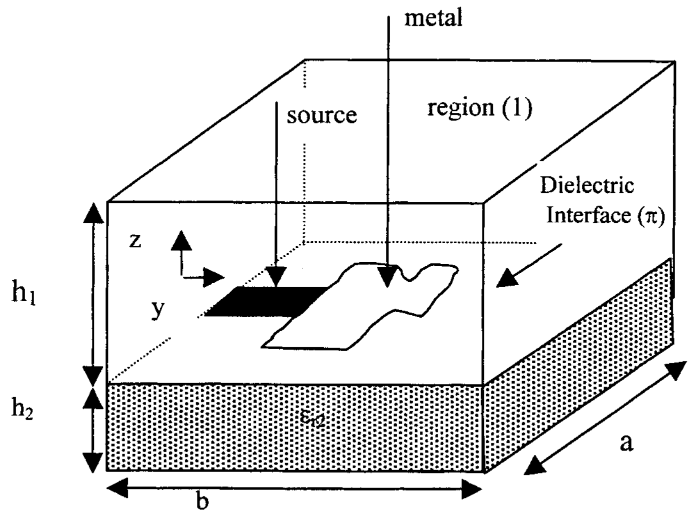


Fig.1: Planar circuit studied

To initialize the iterative process, an electric field source E_0 is defined on the discontinuity plane (π). As a consequence, two spatial waves with two components A_1 (x, y) and A_2 (x, y) are generated by the upper and lower metallic box given two spectral waves. These lasts come back to the dielectric interface then produced the waves for the next iteration. This process is described on Fig.2.

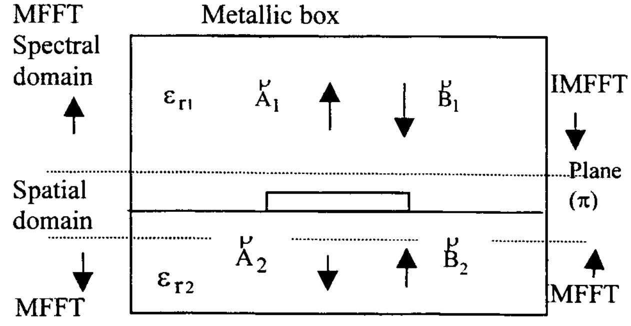


Fig.2 : Iterative process

The wave concept is introduced by the transverse electric E_i and current density J_i in terms of waves[7][8]. It leads to the following set of equations.

$$A_i = \frac{1}{2\sqrt{Z_{oi}}}(E_i + Z_{oi} J_i) \qquad (1)$$

$$B_i = \frac{1}{2\sqrt{Z_{oi}}}(E_i - Z_{oi} J_i) \qquad (2)$$

Z_{oi} is the characteristic impedance of region i (i=1,2) which is equal to 120 * π * ε_r (ε_r: relative permittivity of the medium).
A specific scattering problem may be formulated in analogy to standard scattering expression in antenna or radar theory. In this context a schematic description is illustrated in Fig.3.
The main operation of the iterative procedure with the above given schematic can be summarized for one iteration by the following steps [9].
- Discretisation of the interface circuit plane (π), on which the boundary conditions have to be satisfied (spatial domain).
-Using the Modal Fast Fourier Transformation.
-Application of the reflection operator in its spectral form.
-Inverse Modal Fast Fourier Transformation backs into spatial domain.

2.1 Space domain formulation

The interpretation of waves A_i and B_i in Fig.2 may be viewed as the scattering matrix or more exactly the scattering operator associated with the discontinuity surface (π). Let H_d, H_m and H_s denote the indicator functions of respectively the dielectric, metal and source. These are equal to one in the considered domain and zero elsewhere. Due to the continuity relationship ($E_{t1}=E_{t2}$ and $J_1 + J_2 = 0$ on the dielectric $E_{t1}=E_{t2}=0$ on the metal and finally $E_1=E_2=E_0$ on the source) in each point of the plane, it is easy to deduce from the equations (1) and (2) the scattering matrix (see appendix).

$$S_d = \begin{bmatrix} -H_m - H_s - \frac{1-N^2}{1+N^2} H_d & \frac{2N^2}{1+N^2} H_d \\ \frac{2N^2}{1+N^2} H_d & -H_m - H_s - \frac{1-N^2}{1+N^2} H_d \end{bmatrix} \qquad (3)$$

Where:

$$N = \frac{\sqrt{Z_{o1}}}{\sqrt{Z_{o2}}}$$

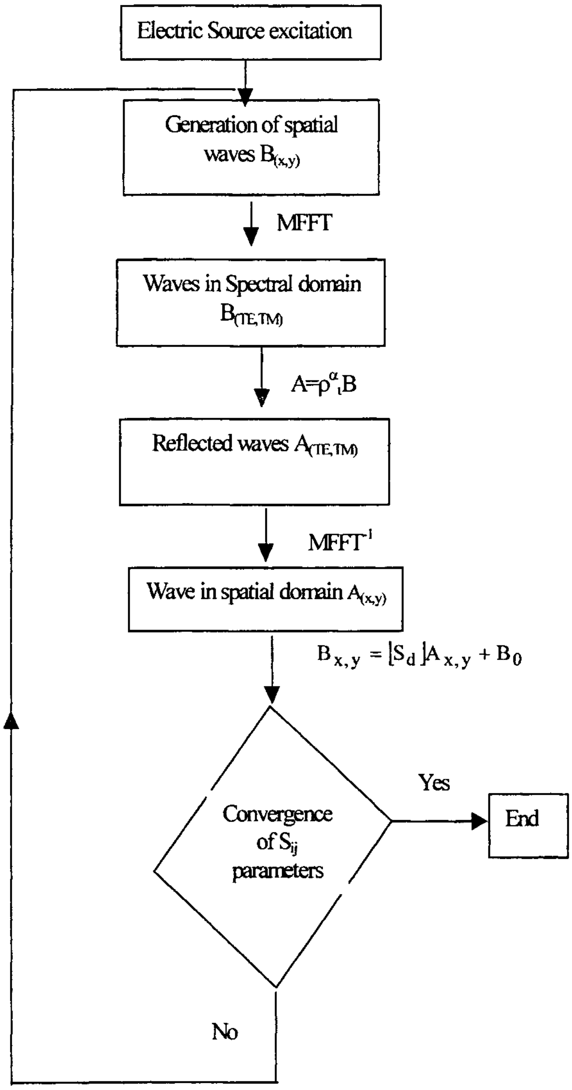


Fig.3 : Schematic description of the method
α = TE, TM mode.
ρ = reflection coefficient.

Each element of the scattering matrix S_d is an operator acting in the spatial domain. Let us note that the global operator S_d is unitary:

$$S_d^T S_d = H_m + H_d + H_s = 1 \qquad (4)$$

Consequently, it will be easy to introduce the known form of operator in a general formulation by associating to each cell of the spatial domain a matrix depending on the physical nature of the discontinuity (metal, dielectric, source).

2.2 Modal Fast Fourier Transformation

According to the schematic description in Fig.3, it is shown that the iterative process consists in combining the waves expressed in the spectral and spatial domain using the Modal Fast Fourier Transformation. It is developed to allow a high computational speed. The definition of this transformation leads to the following set of equations:

$$A_i^{TE} = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left\{ e_{xmn}\left\langle \mathrm{Cos}\left(\frac{2\pi m}{a}x\right)\mathrm{Sin}\left(\frac{2\pi n}{b}y\right)\middle| f_{mnx}^{TE}\right\rangle + e_{ymn}\left\langle \mathrm{Sin}\left(\frac{2\pi m}{a}x\right)\mathrm{Cos}\left(\frac{2\pi n}{b}y\right)\middle| f_{mny}^{TE}\right\rangle \right\} \quad (5a)$$

$$A_i^{TM} = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\left\{ e_{xmn}\left\langle \mathrm{Cos}\left(\frac{2\pi m}{a}x\right)\mathrm{Sin}\left(\frac{2\pi n}{b}y\right)\middle| f_{mnx}^{TM}\right\rangle + e_{ymn}\left\langle \mathrm{Sin}\left(\frac{2\pi m}{a}x\right)\mathrm{Cos}\left(\frac{2\pi n}{b}y\right)\middle| f_{mny}^{TM}\right\rangle \right\} \quad (5b)$$

$f_{mn;x,y}^{TE,TM}$ are the expansion functions, which are suitable for representing the box circuit in fig.1 and can be obtained by considering the modes of a wave guide bounded by four electric walls.

e_{xmn}, e_{ymn} are determined by the fast Fourier Transformation [11] of $E_0/(Z0_i)^{0.5}$ in which E_0 is the excitation source and x= a/M, and y=b/N.

m, n are the number mode of the metallic box

2.3 Spectral domain formulation

Using the Modal Fast Fourier Transformation requires to mesh the discontinuity plane (π) into small rectangular sub-domains (cells) which depend on the physical nature of the interface (metal, dielectric or source). In the spectral domain, the fields are therefore developed in N modes TE and N modes TM corresponding to N components for (x) direction and N components for (y) direction in the spatial domain. Moreover, the incident waves $A_i^{TE,TM}$ are reflected on the metallic box. This reflection is characterized by an operator reflection. It can be expressed in the following form[9]:

$$\rho_i^{\alpha} = \sum_{m\geq 1}\left| f_{mn}^{\alpha}\right\rangle \frac{1 - Z_{oi} Y^{\alpha}_{mn,i}\ \mathrm{Coth}\left(\gamma_{mn,i}\, h_i\right)}{1 + Z_{oi} Y^{\alpha}_{mn,i}\ \mathrm{Coth}\left(\gamma_{mn,i}\, h_i\right)}\left\langle f_{nn}^{\alpha}\right| \quad (6)$$

With:

$$Y_{mn,i}^{TE} = \frac{\gamma_{mn,i}}{j\omega\mu_o};\quad Y_{mn,i}^{TM} = \frac{j\omega\varepsilon_o\varepsilon_r}{\gamma_{mn,i}}$$

$$\gamma_{mn,i}^2 = \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2 - K_o\,\varepsilon_r \quad (7)$$

K_0 is the space wave number.

2.4 Inverse Modal Fast Fourier Transformation

To get the desired solution for microstrip structure characterization, an Inverse Modal Fast Fourier Transformation [IMFFT] must be done to return into the spatial domain [10]-[12].

$$\begin{bmatrix} A_i^{(x)} \\ \\ A_i^{(y)} \end{bmatrix} = \mathrm{IFFT}\left\{ M^{-1}\begin{bmatrix} A_{mn}^{TE} \\ \\ A_{mn}^{TM} \end{bmatrix}\right\} \quad (8)$$

With:

$$M = \left[\sqrt{\frac{ab}{2\sigma_{mn}}}\ \frac{1}{\sqrt{\left[\frac{m}{a}\right]^2 + \left[\frac{n}{b}\right]^2}}\begin{bmatrix} -\frac{m}{a} & \frac{n}{b} \\ \frac{n}{b} & \frac{m}{a}\end{bmatrix}\right]$$

And:

$$\sigma_{mn} = \begin{cases} 2 & \text{if } mn \neq 0 \\ 1 & \text{if } mn = 0 \end{cases}$$

2.5 Iterative process

The implementation of the iterative process consists on establishing a recurrence relationship between the waves in media (1) and (2) using the reflection in the spectral domain and boundary conditions at the dielectric metal discontinuity in spatial domain. A successive set of iterations corresponding to the circuit plane (dielectric or an aperture in absorbing material) is considered. The process continues up to the convergence. It leads the following relations:

Spectral domain :

$$\begin{cases} B_{1,(q)}^{\alpha} = \rho_1^{\alpha}\, A_{1,(q-1)}^{\alpha} \\ B_{2,(q)}^{\alpha} = \rho_2^{\alpha}\, A_{2,(q-1)}^{\alpha} \end{cases} \quad (9)$$

Spatial domain :

$$\left\{\begin{pmatrix} A_{1,(q)} \\ A_{2,(q)} \end{pmatrix} = \left(S_d\right)\begin{pmatrix} B_{1,(q)} \\ B_{1,(q)} \end{pmatrix} + \begin{pmatrix} N \\ 1 \end{pmatrix}\frac{E_o}{\sqrt{Z_{o2}}}\right. \quad (10)$$

Where: q is the number of iterations.

Consequently, using equation (1) and (2), it is possible to calculate the electric field and the current density at the interface plane.

$$E_{i,q} = \sqrt{Z_{oi}}\,(A_{i,q} + B_{i,q})$$
$$J_{i,q} = \frac{1}{\sqrt{Z_{oi}}}(A_{i,q} - B_{i,q}) \qquad (11)$$

At this stage the Sij parameters for two port circuit of interest can be obtained by the following [9]:

$$[S_{ij}] = [1 - [Y]][1 + [Y]]^{-1}$$

$[Y]$ is the admittance matrix.

3 RESULTS AND ILLUSTRATIONS

The waves concept iterative process has been successfully applied to the analysis of several circuits, one of which is the stepped-impedance microstrip low pass filter presented in Fig.4.

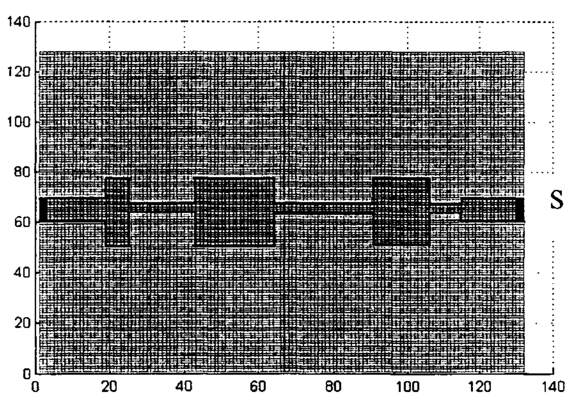


Fig.4 : Microstrip filter layout
S : Source.

The thickness and dielectric constant of the substrate are 1.57 mm and 2.33 respectively, and the height of the shielding box is 11.4 mm. We can model the source as an electric field E_0 equivalent to a magnetic current density [13], and defined by:

$$\vec{M} = \hat{z} \times \vec{E}_0 \qquad (12)$$

The expression of this electric field is $E_0 = \delta(x)$, with δ defined on the metallic domain, of width w and centered on zero.

$\delta(x) = \psi$, $y \in [0, d]$ and $x \in [-w/2, w/2]$
d: dimension of the source.

The value of ψ is chosen to satisfy the condition $\langle E_o, E_o \rangle = 1$.

A grid $N_x = 128$ and $N_y = 128$ is adopted for easiness in the computation of Fast Fourier Transformation. The dimensions of shielding box are 67.5mm x 67.5mm x 11.4mm and $\Delta x = \Delta y = 0.527$mm.

In order to demonstrate the validity and advantages of the iterative approach, a simulated program on Matlab is developed on a Personal computer Pentium II (200 MHz).
The convergence of mag. (S_{11}) is achieved in 195 iterations as depicted in Fig.5. It is shown to converge in few hundred iterations

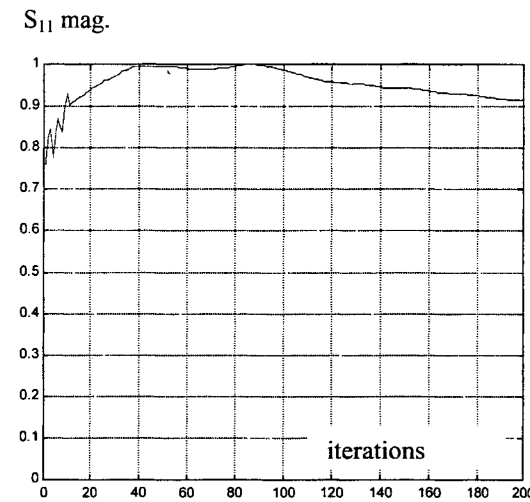


Fig.5 : S_{11} mag. as a function of iteration number

In Fig. 6-7, we show a comparison on two S parameters magnitudes obtained by iterative method and the published results [1]. It is seen that the error between them is less than 5%.

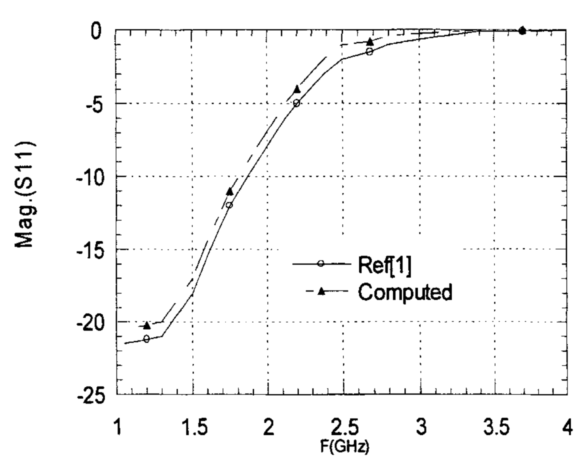


Fig.6 : S_{11} Magnitude as a function of frequency

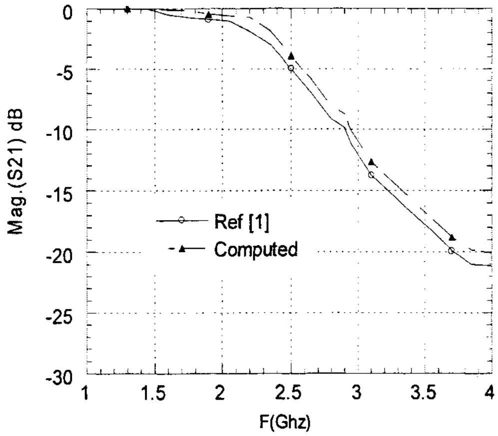


Fig. 7 : S_{21} Magnitude as a function of frequency

A second structure used, as a demonstrator is the one studied in [2][14]. Dimensions are given in Fig. 8.

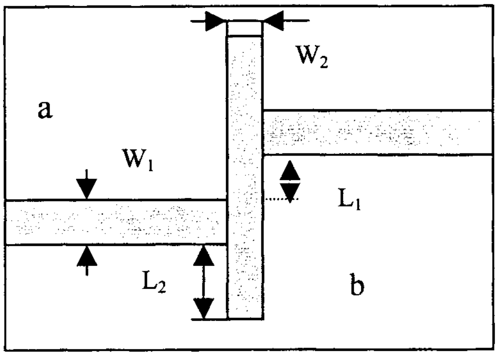


Fig. 8 : microstrip low pass filter
W_1=W_2=2.540mm; L_1=5.715mm, L_2=3.81mm
$\varepsilon_r = 2.2$; $h_2 = 0.794$ mm
Box 27.093mm x 10.16mm.x.12mm

In figures 9 and 10, numerical results are compared with success to those obtained from references [2][14]. The error between them is less than 10 %. There is a good agreement, showing the accuracy of the new method.

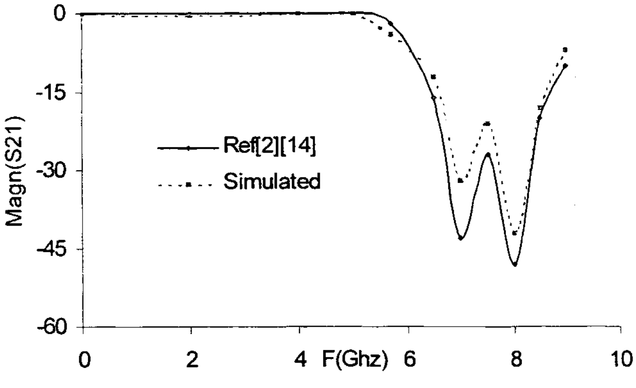


Fig. 9: S_{21} Magnitude as a function of frequency

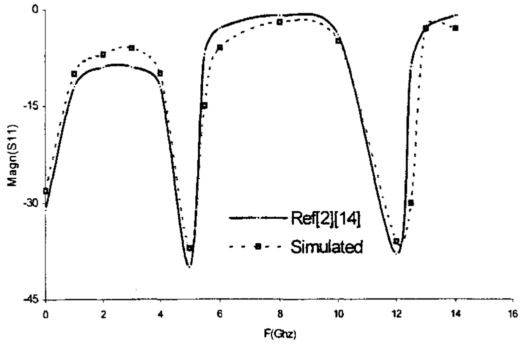


Fig. 10 : S_{11} Magnitude as a function of frequency

A similar, program is developed to determine the phase of S_{11}. The simulated results are illustrated in Fig; 11.

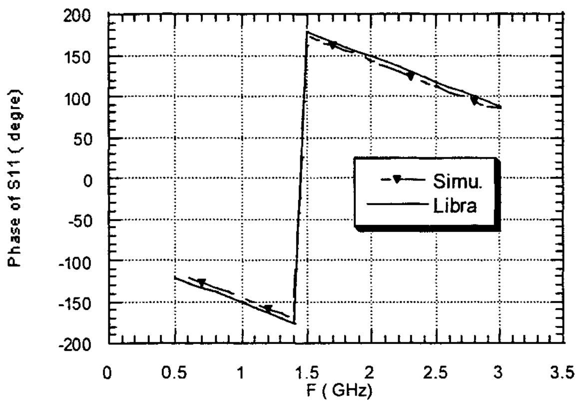


Fig. 11 : Phase of S_{11} as a function of frequency

As a conclusion, an excellent agreement is observed for each of the results, which depend on the adjustment of a uniform mesh dimension and on the couple between source and circuits.

4 COMPARISONS OF TIME AND MEMORY CONSUMPTION

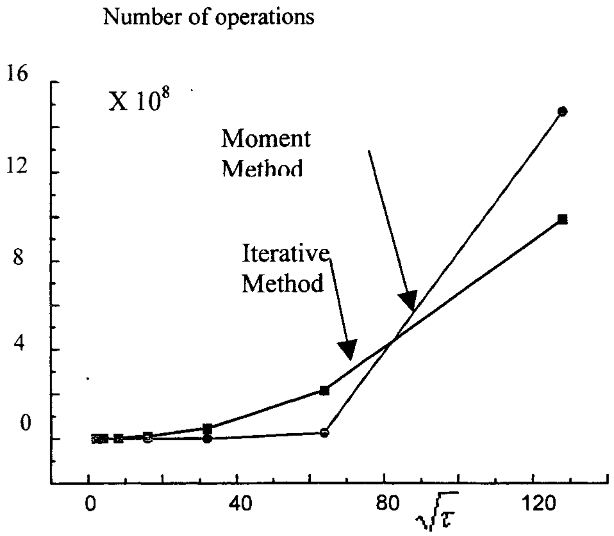
Let consider τ the total number of cells meshing the interface plane (π), and φ the number of cells on metallic domain (related to the number of rooftops in a moment method). The operation numbers of the two methods are presented in the following table[10]:

Iterative method	Operation number
Spatial domain	τ
M.F.F.T	$3\ \tau\ \log \tau$
Spectral domain	τ
I.M.F.F.T	$3\ \tau\ \log \tau$
Spatial domain	τ

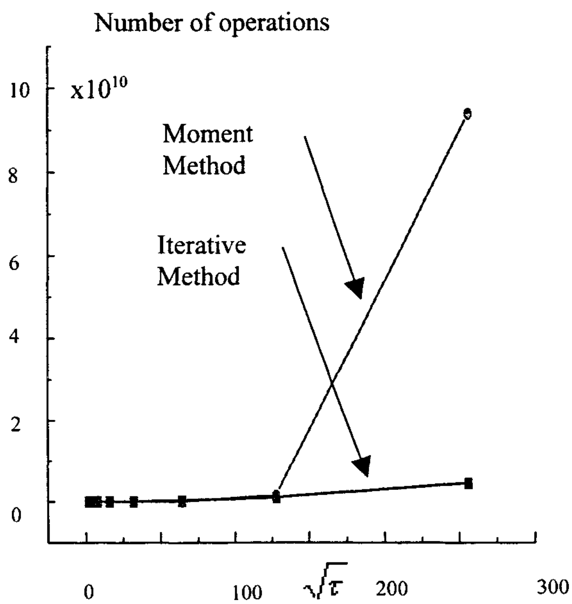
⇓

Number for q iteration requires the convergence (Iterative method)	$q(3\tau + 6\tau \log \tau)$
Number for q iteration requires the convergence (moment method)	$\tau^3 \varphi^3/3$

Consequently (see Fig. 12), the iterative method becomes interesting above 64x64 or 128x128 cells because the number of iterations required to obtain the convergence was always less than 500.



-a : if $\sqrt{\tau} < 150$



-b: if $\sqrt{\tau} < 256$

Fig. 12 : Comparison in number of operations between the iterative procedure and moment method for 500 iterations.

To demonstrate this theory, a program implemented with symbolic calculation using moment method is developed to characterize the studied structures. It is interesting to find that time saving factor is about 20 compared to the conventional method.

5 CONCLUSION

A general implementation of the iterative method based on the concept of waves has been presented. It takes the advantage of the simplicity, which does not involve bases functions and inversion of matrix. It is capable to analyze longer bodies[10]. Moreover, the introduction of the Modal Fast Fourier Transformations allowed simplifying calculations and accelerating the convergence with a reduced CPU time. The good agreement between computed and published results justifies the design procedure and validates the present analysis approach. Consequently, the present approach will be investigated for further new applications such as air bridges, diodes, active elements, etc...

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6 REFERENCES

[1] G. V. Eleftheriades, J. R. Mosig and M.Guglielmi, 'An efficient Mixed Potential Integral Equation Technique For the Analysis of Shielded MMIC'S ', *European Microwave Con. Proc. Vol.2, pp825- 829, date 1995.*

[2] D. M. Sheen, S. M. D. Abouzahra, and J. A. Kong, 'Application of the three-dimensional finite-difference time-domain method to the analysis of planar micro-strip circuit', *IEEE Trans. On MTT, vol.38, pp.849-857, July 1990.*

[3] T. Shibata, T. Hayashi and T. Kimura, 'Analysis of Microstrip Circuits Using Three Dimensional Full-Wave Electromagnetic Field Analysis in the Time Domain' *IEEE trans on MTT, Vol. 36, No. 6, pp. 1064-1070, June 1988.*

[4] R. F. Harrington, *'Field Computation by Moment Methods'* New York, Macmillan, 1968.

[5] O. Axelsson and V. A. Barker, *'Finite Element Solution of boundary value problems'* Theory and Computation New York: Academic Press, 1984.

[6] E. B. Becker, G. F. Carey and J. T. Oden, *'Finite Elements'* An introduction, Vol. 1, Englewood Cliffs, NJ : Prentice-Hall 1981.

[7] N. Sboui, A. Gharsallah, A. Gharbi, H. Baudrand, 'Analysis of Double Loop Meader Line By Using Iterative Process' *Microwave and Optical Tech. Letters, Vol.26, N°6, 20 Sept.2000,pp396-399.*

[8] M. Azizi, H. Aubert and H. Baudrand, 'A new iterative method for scattering problems', *1995 European microwave conf, proc, vol.1, pp255-258.*

[9] R. S. N'Gongo and H. Baudrand, 'A new approach for Micro-strip active antennas using modal FFT algorithm', *IEEE AP-URSI Symposium, July 11-16, 1999, Orlando, Florida, USA, pp. 1700-1703.*

[10] R. S. N'Congo and H.Bandrand, 'Modélisation des circuits actifs planaires de formes arbitraires par une méthode itérative' *J.T'99, Tunis 29,30 et 31 Janv.1999.*

[11] E. O. Brigham, *'The fast Fourier Transform'*, Prentice-Hall, Englewood cliffs, NJ, 1974.

[12] L. P. Dunleavy and P. B. Katehi, 'A General Method For Analyzing Shielded Microstrip discontinuities' *IEEE trans on MTT Vol. 36, No.12, pp. 1758-1766, December 1988.*

[13] J. C. Rautio and R. F. Harrington, 'An Electromagnetic Time-Harmonic Analysis of Shielded Micro-strip circuits' *IEEE Trans. on Microwave Theory Techniques, vol. MTT-35, N°8, 726-730, August 1987.*

[14] S. A. Meade and Chris J. Railton, ' Efficient Implementation of the Spectral Domain Method Including Precalculated Corner Basis Functions' *IEEE Trans. on Microwave Theory Techniques, vol. MTT-42, N°9, 1678-1684, September 1994.*

7 APPENDIX: DERIVATION OF S_d MATRIX

The boundary conditions at the discontinuity (plane π) are given as:

7-1 on the metal domain (M):

$E_1=E_2$

Using equation (1) and (2), it is easy to obtain:

$$\sqrt{Z_{01}}\left(A_1+B_1\right)=\sqrt{Z_{02}}\left(A_2+B_2\right)=0$$

Moreover

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix}=\begin{bmatrix} -H_m & 0 \\ 0 & -H_m \end{bmatrix}\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

where as in the perfect conducting domain M, we can deduce:

$$\begin{cases} H_m = 1 & \text{on the metal} \\ H_m = 0 & \text{elsewhere} \end{cases}$$

7-2 On the dielectric domain D :

$$J_1+J_2=0$$
$$E_1+E_2\neq 0$$

imply:

$$\frac{1}{\sqrt{Z_{o1}}}(A_1-B_1)+\frac{1}{\sqrt{Z_{o2}}}(A_2-B_2)=0$$
$$\frac{1}{\sqrt{Z_{o1}}}(A_1-B_1)=\frac{1}{\sqrt{Z_{o2}}}(A_2-B_2)$$

After that, it is possible to deduce:

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix}=\begin{bmatrix} -\frac{1-N^2}{1+N^2}H_d & \frac{2N^2}{1+N^2}H_d \\ \frac{2N^2}{1+N^2}H_d & -\frac{1-N^2}{1+N^2}H_d \end{bmatrix}\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

Consequently, The complete scattering matrix given by the relation (3) can be deduced.

7-3 On the source domain S

In the source field domain S, on can deduce (15) from (1), (2) and (14).

$$E_l= E_l=E_o \qquad (14)$$

$$B_1=\frac{E_o}{\sqrt{Z_{o1}}}-A_1$$
$$B_2=\frac{E_o}{\sqrt{Z_{o2}}}-A_2 \qquad (15)$$

imply:

$$\begin{bmatrix} A_1 \\ A_2 \end{bmatrix}=\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}+\begin{bmatrix} \frac{E_o}{\sqrt{Z_{o1}}} \\ \frac{E_o}{\sqrt{Z_{o2}}} \end{bmatrix}$$

Finally, the boundary conditions on each domains of the discontinuity surface can be expressed in the condensed form presented by the equation (10).