Canonical Twodimensional Inverse Scattering Problem

The following twodimensional electromagnetic TE- or TM-scattering problem is considered: A circular cylindrical scatterer with cross-section in the xy-plane and infinitely long in z-direction is embedded in vacuum and is composed of 3 circular cylindrical concentric layers ($n = 3$) with radii

$$\begin{aligned} a_1 &= 2\,\lambda_0 \\ a_2 &= 4\,\lambda_0 \\ a_3 &= 6\,\lambda_0 \end{aligned}$$

with λ_0 being the vacuum wavelength. The relative permittivities ε_{rn}, permeabilities μ_{rn} and conductivities σ_n are given as follows

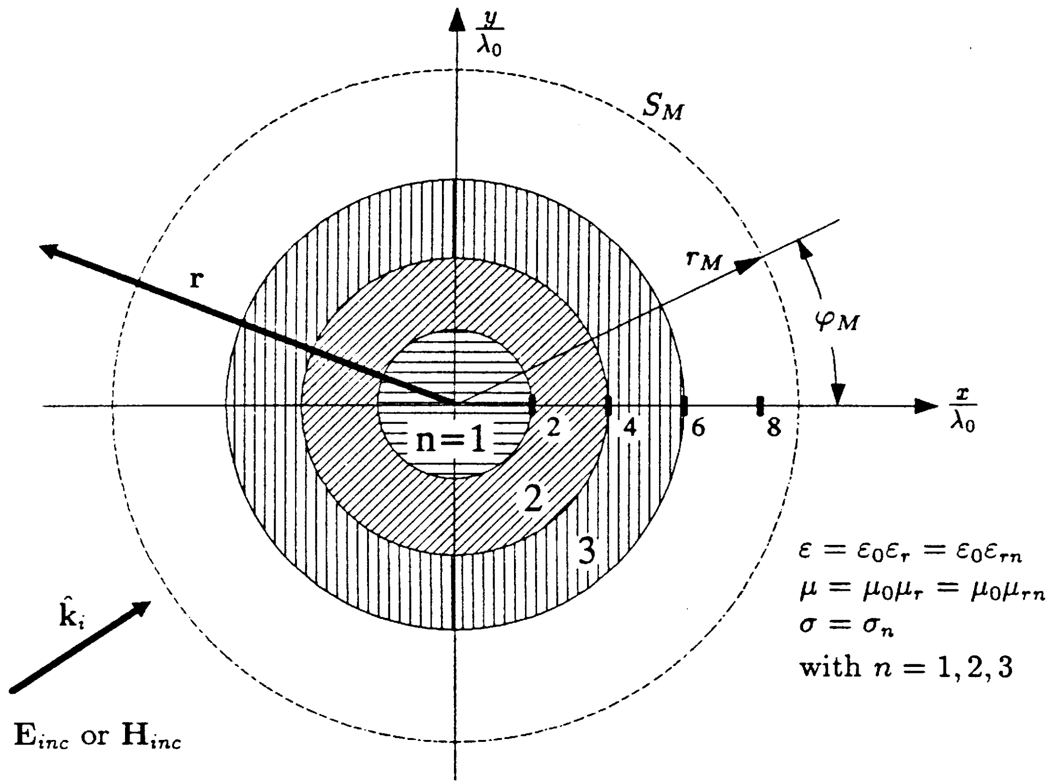
n	ε_{rn}	μ_{rn}	σ_n
1	1.02	μ	0
2	1.08	μ	0
3	1.05	μ	0

Notice: This is not a weak scatterer as it is assumed within the first-order Born approximation.

Either plane wave TE- or TM-excitation with wavenumber

$$k_0 = \frac{\omega}{c_0} = \frac{2\pi}{\lambda_0} \quad , \quad c_0 = \frac{1}{\sqrt{\varepsilon_0\,\mu_0}}$$

and propagation direction perpendicular to the cylinder axis is considered, i.e. the unit-vector of propagation $\hat{\mathbf{k}}_i$ is located in the xy-plane.



$\frac{y}{\lambda_0}$
S_M
r
r_M
φ_M
$\frac{x}{\lambda_0}$
2
4
6
8
n=1
2
3
$\hat{\mathbf{k}}_i$
$\mathbf{E}_{inc}$ or $\mathbf{H}_{inc}$
$\varepsilon = \varepsilon_0\varepsilon_r = \varepsilon_0\varepsilon_{rn}$
$\mu = \mu_0\mu_r = \mu_0\mu_{rn}$
$\sigma = \sigma_n$
with $n = 1, 2, 3$

Definition of the inverse problem:
Suppose the scattered field $\mathbf{E}_s, \mathbf{H}_s$ is known with "arbitrary" accuracy on a measurement surface S_M not necessarily in the far-field, i.e. it is supposed to be known as a function of the variables $\varphi_M, k_0, \hat{\mathbf{k}}_i$ for fixed but arbitrary r_M. Determine $\varepsilon_{r1}, \varepsilon_{r2}$ and ϵ_{r3}, together with a_1, a_2, a_3. The following "data acquisitions" might be considered:

Method	φ_M	k_0	$\hat{\mathbf{k}}_i$
angular diversity	$\epsilon[0, 2\pi)$	fixed	$\epsilon[0, 2\pi)$
frequency diversity	$\epsilon[0, 2\pi)$	$\epsilon[0, \infty)$	fixed
angular and frequency diversity	$\epsilon[0, 2\pi)$	$\epsilon[0, \infty)$	$\epsilon[0, 2\pi)$

Due to the complexity of the problem, discretization errors and noise are not yet to be considered.

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