To-Average or Not-to-Average in FDTD Modeling of Dielectric Interfaces

Cynthia Furse, Craig Waterman, and Lance Griffiths

Electrical & Computer Engineering Dept, University of Utah, 50 S Central Campus Dr Rm 3280, Salt Lake City, UT 84112,

Abstract — Accurate Finite Difference Time Domain (FDTD) modeling of localized electromagnetic sources such as cellular telephones near the human body requires very precise modeling of the location of these devices. This paper discusses the effective location of near field sources when dielectric interface cells are made up of either averaged or unaveraged dielectric properties. It is shown that either method can accurately define the proximity between source and dielectric, but that the effective location differs by half an FDTD cell in the two methods.

I. INTRODUCTION

As Finite Difference Time Domain (FDTD) modeling of the human body and other complex dielectric systems becomes main stream [1], [2], many researchers have passionately defined a preference for either averaging or not averaging dielectric materials at boundaries. This paper discusses this issue from the point of view of very near field modeling, where the precise location of a source such as an antenna near the human head is critical to the accuracy of the results. This modeling issue has potentially significant implications in cellular telephone evaluation and the ongoing debate about if power deposition from cell phones is or is not higher in children than in adults [3]. Little or no difference between power absorbed from cellular telephones in adult and child head models is seen if the antenna is modeled an equal distance from the outer surface of both head models [4]. However, if the phone is modeled as being physically closer to the child's head due to the thinner ear of the child, significantly higher power is observed in the child than adult model [5]. This is not surprising, since the fields decrease rapidly (as 1/distance^3) very near the antenna, and small changes in the proximity of the phone can result in large changes in power deposition. In addition to the effect of considering or not considering the size of the ear, averaging or not averaging the dielectric properties at the outer surface of the head effectively moves the boundary half a cell, with potentially similar results. When evaluating near field effects such as the power deposition of a cell phone in a human head, it is critical to model the proximity of the source accurately. This requires an understanding of how the choice of averaging or not averaging FDTD dielectric boundaries affects the effective proximity of the source, which is the topic of this paper.

Several different methods have been used for averaging electrical properties at boundaries of dissimilar media. When a layer of dielectric is thinner than one FDTD cell, averaging is one method of accounting for the thin layer [6]-[10]. Averaging has also been used for boundaries in an effort to reduce unwanted effects from discontinuity of the charge at the boundary [11]-[14]. Interpolation of the fields rather than the model has been proposed as an alternative to averaging [15]. Averaging at the air-dielectric interface of microstrip circuits and antennas has been used routinely [16], [17]. In [18] it was found that averaging was not needed for buried layered structures with no conductor, but that it was helpful when a conductor rested on the air-dielectric interface. Three different types of averaging are used (arithmetic, geometric, and harmonic), and their relative accuracy has been shown to be dependent on the cell size [9], [13]. All of these papers were focused on the effect of the dielectric discontinuity on the model of the dielectric object where the distance between the boundary and source is large enough that a variation of half of an FDTD cell would be negligible. When models of antennas near a dielectric object are considered, the ability of averaging to improve the model is found to be minor, especially when compared to variation in the source location of the antenna [19]. It was also commented in [20] on the effect that averaging can have on displacing the boundary by half a cell.

This paper discusses the effect of averaging on the effective location of a source or boundary and demonstrates that either an averaged and non-averaged dielectric boundary can be used to model the source and boundary in their proper locations. This paper demonstrates the effect of averaging or not averaging the dielectric boundary by considering the phase of the reflection coefficient in front of the boundary. This derivation could have been done analytically as in [2], Section 3.6.8, and the same conclusion could have been drawn.

II. NEAR-FIELD FDTD MODELING

The analysis of modeling the proximity of a source to a dielectric object will be limited in this paper to one dimension in order to precisely demonstrate the effect of averaging, although the conclusion is easily generalized to 2D and 3D simulations. The one-dimensional TE FDTD grid is shown in Figure 1 for unaveraged cases (a) and (c) and averaged case (b). The locations of two different materials, 1 and 2, in the model are indicated as well as the average of materials 1 and 2 in the center of (b). The location of the physical boundary of each model is indicated by the arrow, as will be shown in the results section.

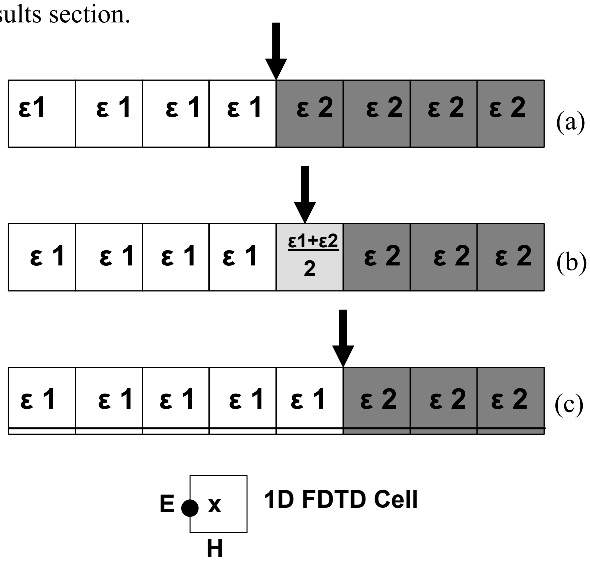

Fig. 1. FDTD model of a dielectric interface; (a) and (c) are unaveraged models, (b) is an averaged model. The location of the physical boundary that each model represents is shown with the arrow. The location of the fields in the 1D FDTD cell are shown as well. The two materials modeled were air (ε_{r1} = 1) and "water" (ε_{r2} = 40.0). Both materials are lossless.

Maxwell's differential equations in the time domain are

$$\nabla \times \bar{E} = -\mu \frac{\partial \bar{H}}{\partial t} \tag{1}$$

$$\nabla \times \bar{H} = \sigma \bar{E} + \varepsilon \frac{\partial \bar{E}}{\partial t} \quad . \tag{2}$$

They are simplified to two 1D scalar Maxwell's curl equations:

$$\frac{\partial E_y}{\partial t} = \frac{-1}{\varepsilon}\left(\frac{\partial H_z}{\partial x} + \sigma\, E_y \right) \tag{3}$$

$$\frac{\partial H_z}{\partial t} = \frac{-1}{\mu}\left(\frac{\partial E_y}{\partial x} \right) \tag{4}$$

where ε in (3) is defined either as averaged (Figure 1b) or unaveraged (Figures 1a and 1c) for cells at the boundary. For simplicity, σ = 0 in these test cases. In the non-averaged cases, (a) or (c), ε is ε_1 or ε_2 , depending on the cell. In the averaged case (b), the value of ε is ε_1 or ε_2 on either side of the boundary and the average value of the two materials ($\varepsilon_2 + \varepsilon_1$)/2 for the boundary cell. It should be noted that this averaged model is the same as modeling a 1-cell thick layer of ($\varepsilon_2 + \varepsilon_1$)/2 dielectric material sandwiched between two different dielectrics (ε_1 and ε_2), the boundaries of which would be where the arrows are shown in Figures 1a and c. This is not the most accurate way of modeling a thin layer, however, and is not recommended for applications where the thin layer is expected to have a significant effect on the performance of the system.

To find the phase of the reflection coefficient, the discrete Fourier transform (DFT) of the electric field in every cell was calculated at 1 MHz. This provided the total electric field, E^t. The DFT at each test cell was calculated for each of the three boundary arrangements, and it was also calculated with no dielectric boundary (i.e., with air in all of the cells). The latter gave the DFT of the incident wave alone (E^i), including any effective errors from numerical dispersion. The reflection coefficient at each point can be calculated using

$$\Gamma_{\ell} = \frac{E^t - E^i}{E^i} \tag{5}$$

where E^t and E^i are the DFTs of the total and incident wave, respectively. Analytical values for the test points were calculated using

$$\Gamma_{\ell} = \left|\Gamma\right| e^{j2\beta\ell}$$

where ℓ is the distance from the center cell (where the arrow is shown in Figure 1a) to the test point and

$$\left|\Gamma\right| = \frac{\sqrt{\varepsilon_{r1}} - \sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}} + \sqrt{\varepsilon_{r2}}} \, . \tag{6}$$

III. RESULTS

Figure 2 shows a comparison of the phase as a function of distance from the boundary (where the arrow is in Figure 1b) for all three models along with the analytical solution. The uppermost and lowermost lines were produced by the unaveraged models (a) and (c), respectively. The middle line was produced by (b). The analytical value for a boundary at the location of the arrow in Figure 1b is also give. Since the line for the averaged model is halfway between lines for the unaveraged model, we can see that averaging has the effect of shifting the boundary one half-cell away from the source. When the analytical value is calculated at the location of the arrows shown in Figures 1a and 1c, the match with those curves is as good as that shown here for Figure 1b. The only difference between the averaged and unaveraged models is the phase of the reflection coefficient (which is indicative of the effective location of the dielectric boundary). All three models can properly model the location of the boundary and will give the same phase as the analytical solution if the location of the boundary is taken to be wherever their respective arrows indicate.

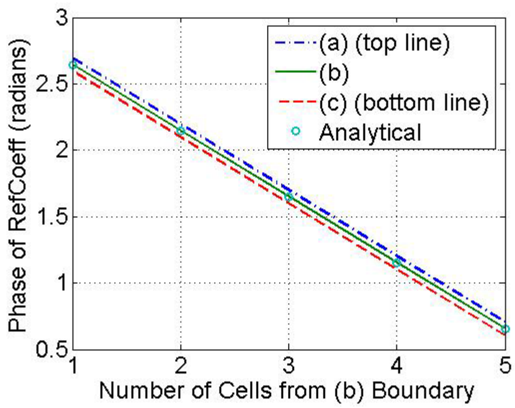

Fig. 2. Phase of reflection coefficient as a function of normalized distance from the (b) boundary (where arrow is seen in Figure 1b). Phase is shown for unaveraged (a and c) and averaged (b) FDTD models. The analytical solution is shown for a boundary where the arrow is shown in Figure 1b. When the analytical solution is calculated relative to the (a) or (c) boundaries (where the arrows are located in Figs. 1a and 1c), it lines up precisely with those lines, instead. This provides proof that both averaged and unaveraged models can be used to accurately model boundaries in FDTD, but that changing the averaging scheme changes the effective location of the boundary.

IV. CONCLUSIONS

Near field FDTD simulations such as models of a cell phone near the human head depend significantly on the proximity of the antenna feedpoint to the head. Properly modeling this requires precise attention to the distance between the source and model, which varies by half a cell depending on if the electrical properties on the boundary are averaged or not. In this paper it was shown that both averaged and unaveraged models can be used with equal accuracy, as long as the model takes into account the location of the boundaries for each case, shown as arrows in Figure 1.

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Dr. Cynthia Furse, Associate Professor; Electrical & Computer Engineering Department. Dr. Furse received her B.S. in electrical engineering with a mathematics minor in 1985, M.S. degree in electrical engineering in 1988, and her Ph.D. in electrical engineering from the University of Utah in 1994. She is currently an associate professor at the University of Utah and has taught electromagnetics, wireless communication, computational electromagnetics, microwave engineering, antenna design, and introductory electrical engineering. Dr. Furse works to interest young students, particularly women and minorities in engineering and routinely volunteers in Utah's K-12 schools as an engineering mentor, science educator, and engineering career guidance counselor and is active with the Society of Women Engineers, Junior Engineering State, Expanding your Horizons, School-to-Careers, MESA, Girl Scouts and Boy Scouts. Dr. Furse was the Professor of the Year in the College of Engineering at Utah State University for the year 2000. She is the Director of the Center of Excellence for Smart Sensors, an active, funded research program including electromagnetics for biology and remote sensing.

Dr. Lance Griffiths. Dr. Griffiths received his B.S. in electrical engineer-ing with a computer science minor in 1999, and an M.S. degree in electrical engineering in 2002, from Utah State University. He received the Ph.D. in electrical engineering from the Univer-sity of Utah in 2005. He is currently an RF design/systems engineer at L-3 Communications in Salt Lake City, Utah. Previously he worked at Trans-Lux Sports in Logan, Utah as a production engineer. He was presented with the Outstanding Teaching Assistant Award from the College of Engineering at the University of Utah in 2005. He is also a member of Who's Who Among Students in American Colleges and Universities, and is a volunteer for the Boy Scouts of America. His current research interests include numerical methods in electromagnetics, broad band antenna design, and random design and analysis.